MODELLING AND IDENTIFICATION OF ROAD VEHICLE BODY DEFORMATION

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Received: Oct. 30, 2003

Abstract

The modeling of car body deformation is crucial in crash analysis, together with the determination of the energy which was absorbed by the deformation. This energy can help us determine the corresponding EES value. These parameters are of key importance, but on the other, hand their precise determination is a very difficult task. However, by utilizing the results of the crash tests and the method of digital image processing, it is possible to state the absorbed energy through the digital processing of the photos.

Keywords: car body deformation, digital image processing, energy equivalent speed, interpolation.

1. Introduction

Traditional engineering abandoned the field of crash analysis. It was A. Brüderlin who changed this custom when in 1941 he published his work [1] on the mechanics of road accidents. Later on, new methods dealing with EES were developed, especially by CAMPBELL, RÖHRICH, RAU and SCHAPER [1]. The first model was prepared by Kenneth Campbell in 1974 and was further developed by RÖHRICH in 1976 [1], [2].

Crash analysis is very interesting for more reasons. One of them is the fact that even though the crash situations are random probability variables, the deterministic view plays an important role in them. Crash situations occurring most frequently are chosen from statistics and are used as outputs (inputs) of crash tests. These tests are quite expensive, because to construct an optimal car body structure, more crash-tests are needed. Only some hundred tests per factory are realized annually, which is not a sufficient amount for accident safety. Therefore, these tests are supplemented by computer-based simulations and so the number of analyzed cases is increased to 1-2 thousands.

The work of experts of road vehicle accidents requires data which are as close to the reality as possible, therefore crash analysis is very helpful for them. These simulations are carried out by quite complicated computer programs. Digital image processing, which experiences its most intensive development in the recent years, is of key importance in the mentioned programs. The objective of the computer-based image processing is to analyze and evaluate the visual information from the environment with the help of a computer software. Using these methods, we can obtain information by which the security systems of a vehicle can be inproved in order to make the passenger security more and more effective [4], [5], [17], [18].

The usage of these new methods in crash analysis opens new dimensions and opportunities. They help us to gain important information (concerning the process of accident) interactively.

By using the digital image processing and the fuzzy neural networks [6], the EES value determination can be more exact and dynamic. In this paper a new EES value detection based on crash analysis method is introduced using 3D image information and an edge detection algorithm. The paper is organized as follows. Section 2 describes conventional methods of EES values determination. Section 3 focuses on the possibilities of using the digital image processing to determine car body deformation. Section 4 shows a method to detect boundary surface of the deformation. Section 5 describes how to determine the deformation energy. Section 6 shows the experimental results, and finally, section 7 reports conclusions.

2. Overview of Analyzing Methods

A. Crash Analysis

Car crash analysis uses different calculating methods, which can be divided into two basic groups:
Counting forward – The crash analysis is realized by computer programs using the input data of the crash. The possible before-crash data (directions, speed) are compared to the real accident. The counting and changing of parameters lasts until the input data correspond with reality, the brake traces, vehicle deterioration, etc.
Counting backward – Before-crash parameters of the vehicle are determined by the after-crash position and after-crash data (injury marks, brake traces, etc.). In this case the determination of the energy absorbed by the car body plays an important role. The EES method we are going to discuss in the following part belongs to this group.

These two methods absolutely complement each other.

B. On the EES/Energy Equivalent Speed/ Method

This method, based on the principle of the conservation of the energy, examines the balance between the before-crash E_K,E_R kinetic and rotational energies and the after-crash E_K kinetic, E_R rotational, E_{DEF} deterioration and E_H other (sound, heat) energies.
Let us consider the main known basic relationships:

Kinetic energy:

$$E_K = \frac{1}{2} m v^2, \tag{1}$$

where: m – mass of the vehicle, v – speed of the vehicle.
Rotational energy:

$$E_R = \frac{1}{2} J \omega^2, \tag{2}$$

where: J – inertial moment, ω – angular frequency.
Deformation energy:

$$E_{DEF} = \frac{1}{2} \cdot m \cdot EES^2, \tag{3}$$

where: EES – energy equivalent speed. The balance of the crash energy is therefore:

$$E_{K1} + E_{R1} + E_{K2} + E_{R2} = E'_{K1} + E'_{R1} + E'_{K2} + E'_{R2} + E_{DEF1} + E_{DEF2} + E_H, \tag{4}$$

where: $E_{K-1,2}$ – kinetic energies before the crash
$E_{R-1,2}$ – rotational energies before the crash
$E'_{K-1,2}$ – kinetic energies after the crash
$E'_{R-1,2}$ – rotational energies after the crash
E_{DEF} – deformation energy
E_H – the energy of the sound wave created during the crash.

After substituting (1)–(3) into (4) and transforming the equation to a v_1 quadratic equation:

$$v_1 = \frac{m_2}{m_1 + m_2} \cdot \left[\frac{m_1}{m_2} \cdot v'_1 \cdot \cos(\nu'_1 - \nu_1) + v'_2 \cdot \cos(\nu'_2 - \nu_1) + \sqrt{K + 2X\left(\frac{1}{m_1} + \frac{1}{m_2}\right)} \right], \tag{5}$$

where: v_1, v_2 – speed of vehicles before the crash
m_1, m_2 – mass of vehicles
v'_1, v'_2 – speed of vehicles after the crash

$$K = v_1'^2 - 2 \cdot v'_1 \cdot v'_2 \cdot \cos(\nu'_1 - \nu'_2) + v_2'^2 - \left(\frac{m_1}{m_2} \cdot v'_1 \cdot \sin(\nu'_1 - \nu_1) + v'_2 \cdot \sin(\nu'_2 - \nu_1) \right)^2,$$

$$X = \frac{1}{2} \cdot \left(J_1 \omega_1'^2 + J_2 \omega_2'^2 \right) + \frac{1}{2} \left(m_1 \cdot EES_1^2 + m_2 \cdot EES_2^2 \right) - \frac{1}{2} \cdot \left(J_1 \omega_1^2 + J_2 \omega_2^2 \right).$$

The equations clearly show that in order to determine the results we need to know the after-crash movements of the vehicles, as well as the EES values of the deterioration.

C. Deformation Energy

The determination of the deformation energy is a crucial task in crash analysis. However, until 1978 the vehicle's impact speed was determined by the so called Impulse thesis only. Although, later on, with the development of the computing methods a new device was introduced, namely the energy absorbed by the deformation started to be used for computing. This new method was applied by such experts as CAMPBELL, RAU and SCHAPER [1], [7].

2.1. Campbell's Method

Campbell based his method on a series of frontal crashes which were directed to a stiff wall. According to his researches Campbell inferred that after a crash a certain permanent change of form arises, which is determined by the impact speed. A new term was introduced, the EBS (Equivalent Barrier Speed).

Campbell assumed that the energy was distributed through the width of the vehicle in an equal proportion [1], [2].

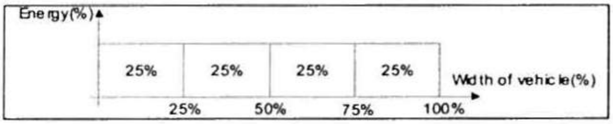


Fig. 1. Distribution of the energy according to the width of vehicle in Campbell's model

2.2. Röhrich's Energy-net

One of the most important features of this model is that the energy within the cross-section of the vehicle is not distributed in an equal proportion [1]. RÖHRICH defined the percentual data of the energy distribution according to different crash test results.

2.3. Rau and Schaper's Method

This method is a further development of the previously mentioned ones. The development is that in this case the energy within the cross-section of the vehicle is not

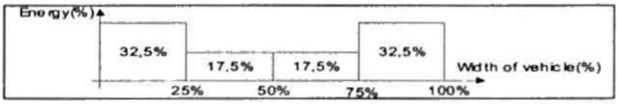


Fig. 2. Energy-distribution within the cross-section of an Opel type vehicle

distributed in an equal proportion. The percentual data of the energy distribution is defined according to different crash test results with overlapping. If the weights of the research vehicle and the analyzed vehicle differ, it is important to multiply the raster values with the weight proportion of the two vehicles [1].

Using this method, the energy values of the fields which are involved in the deformation should be summed.

3. Evaluation of the Applied Methods

The previously mentioned method utilizes the top view of the car body to determine the extent of the deformation. Unfortunately, in some cases, the top view analysis cannot be used. To eliminate this problem, it is necessary to construct the spatial model of the deformed car body elements. During a local accident analysis pictures are taken of the damaged car bodies from different points of view. If we take photos from all points of views, we can construct the spatial model of the deformed car body by using digital image processing.

As we know, computers can efficiently analyze digital photos. In contrast to the classic (analogical) photos, digital ones are constructed of pixels (in other words, it is defined by certain points of the plane only, and not in the space between these points). In addition, the brightness or colour of these pixels does not change continuously, only certain colours occur. Each point has its brightness code, the digital picture is therefore a matrix, which has as many lines as the picture, and within a line as many elements as picture points in a line. This means that the methods of mathematic algebra and analysis can be applied to it.

Let us analyze the top view of a deformed car body. Our aim is to construct an algorithm with the help of which we can construct the curve which defines the deformation from this specific view. If we repeat this process with the other pictures (from the other points of view), we can easily construct – by interpolation – the surface which will correspond with the boundary surface of the deformation.

4. Detection of the Boundary Curve

The most important objective of image processing is the recognition of objects and actions by a computer software. In typical images, edges characterize object boundaries so they are useful for segmentation, registration and identification of objects in a scene. The first step towards it is to limit/delimit the objects in the picture from each other. As we know, an edge will appear on the limit of the deformation. By the computer-based recognition of the edge points it is possible to construct a curve on the edge. The edge points are determined by the sudden change of the intensity codes [8].

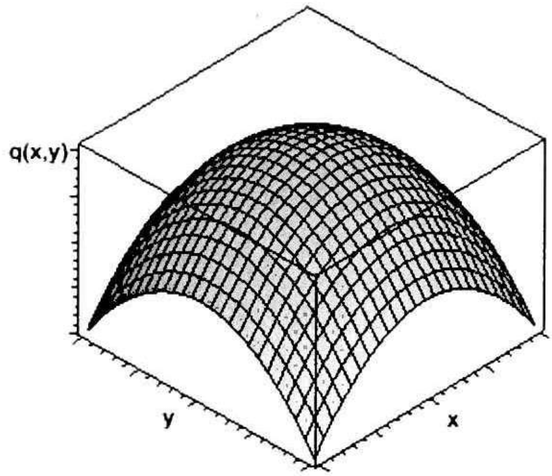


Fig. 3. An example of intensity function $z = q(x, y)$

An ideal edge is a discontinuity. The first derivative assumes a local maximum at an edge. For a continuous image $q(x, y)$, where x and y are the row and column coordinates, respectively, we typically consider the two directional derivatives $\partial_x q(x, y)$ and $\partial_y q(x, y)$.

$$\frac{\partial q(x, y)}{\partial x} \cong \Delta_x = \frac{q(x + d_x, y) - q(x, y)}{d_x}, \tag{6}$$

$$\frac{\partial q(x, y)}{\partial y} \cong \Delta_y = \frac{q(x, y + d_y) - q(x, y)}{d_y}. \tag{7}$$

Of particular interest in edge detection are two functions that can be expressed in terms of these directional derivatives: the gradient magnitude and the gradient orientation. The gradient is a vector whose components measure how rapidly pixel values are changing with distance in the x and y directions.

The gradient magnitude is defined as

$$|\nabla q(x, y)| = \sqrt{\Delta_x^2 q(x, y) + \Delta_y^2 q(x, y)}, \tag{8}$$

and the gradient orientation is given by

$$\angle \nabla q(x, y) = \text{ArcTan}\,(\partial_y q(x, y)/\partial_x q(x, y)). \tag{9}$$

Local maxima of the gradient magnitude identify edges in $q(x, y)$. When the first derivative achieves a maximum, the second derivative is zero. For this reason, an alternative edge-detection strategy is to locate zeros of the second derivatives of $q(x, y)$.

In (discrete) images we can consider d_x and d_y in terms of numbers of pixels between two points. Thus, when $d_x = d_y = 1$ (pixel spacing) and we are at the point whose pixel coordinates are (x,y) we have

$$\begin{aligned} \Delta_x &= q(x+1, y) - q(x, y), \\ \Delta_y &= q(x, y+1) - q(x, y). \end{aligned} \tag{10}$$

Instead of finding approximate gradient components along x and y directions, we can also approximate gradient components along directions at 45° and 135° to the axes. In this case the following equations are used:

$$\begin{aligned} |\nabla q(x, y)| &\approx |\Delta_x q(x, y)| + |\Delta_y q(x, y)| \\ &= |q(x, y) - q(x-1, y)| + |q(x, y) - q(x, y-1)|, \end{aligned} \tag{11}$$

or

$$\begin{aligned} |\nabla q(x, y)| \approx\ & |q(x, y) - q(x+1, y+1)| \\ & + |q(x, y+1) - q(x+1, y)|. \end{aligned} \tag{12}$$

This form of operator is known as the *Roberts edge operator* and was one of the first operators used to detect edges in images [8].

Many edge detectors have been designed using convolution mask techniques, often applying 3×3 or even larger mask sizes. An advantage of using a larger mask size is that errors due to the effects of noise are reduced by local averaging within the neighbourhood of the mask. An advantage of using a mask of odd size is that the operators are centred and can therefore provide an estimate that is biased towards a centre pixel (x, y). One important edge operator of this type is the *Sobel edge operator* (see $\mathbf{T}_X$ and $\mathbf{T}_Y$) [8].

$$\mathbf{T_X} = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \qquad \mathbf{T_Y} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}.$$

In practice the usage of other operators is possible, as well.

A. Edge Correction

Neither of the edge detecting methods is correct. Smooth and closed edges can be created only by the analysis and correction of the scattering of the edge-points.

The aim is to delete the false edge-points and link the tears in the edge. During interpolation, a polynom, which has the form of

$$p(x) = a_0 + a_1 x + \ldots + a_n x^n \tag{13}$$

is adjusted to the $(n + 1)$ edge point, and so this curve is considered to be the real edge (see *Fig. 4*).

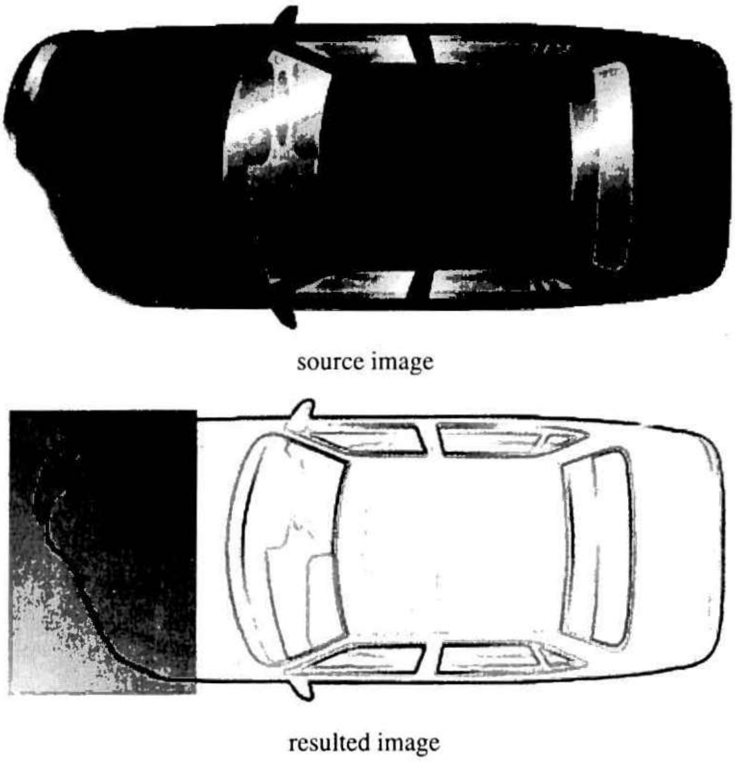


Fig. 4. The source image and the resulted image after edge recognition

We can determine the limit curve defined by some given points by two kinds of interpolation: either the Hermite or the Spline interpolations [9]. From these two types of interpolations we have chosen Hermite's. This interpolation solves the contacting task locally, which means that each section of the characteristic framework is analyzed separately. However, we need the values of the tangent vector in the check points.

The analytic curve connecting two neighbouring checkpoints can be defined by a cubic vector function with one variable:

$$\mathbf{r}(u) = \mathbf{a}_3 u^3 + \mathbf{a}_2 u^2 + \mathbf{a}_1 u + \mathbf{a}_0, \qquad u \in [0, 1]. \tag{14}$$

The two ends of the curve have to meet the two checkpoints ($\mathbf{p}_0$ and $\mathbf{p}_1$) in the end points, the tangent has to correspond with the given values ($\mathbf{p}_0^u$, $\mathbf{p}_1^u$):

$$\mathbf{r}(0) = \mathbf{p}_0, \qquad \mathbf{r}(1) = \mathbf{p}_1 \tag{15}$$

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}u}(0) = \mathbf{p}_0^u, \qquad \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}u}(1) = \mathbf{p}_1^u. \tag{16}$$

Consequently, the cubic weight functions can be determined, which fulfil the mentioned hypotheses (preconditions):

$$\begin{aligned} f_1(u) &= 2u^3 - 3u^2 + 1, \\ f_2(u) &= -2u^3 + 3u^2, \\ f_3(u) &= u^3 - 2u^2 + u, \\ f_4(u) &= u^3 - u^2. \end{aligned} \tag{17}$$

The determination of the curve is therefore:

$$\mathbf{r}\,(u) = f_1\mathbf{p}_0 + f_2\mathbf{p}_1 + f_3\mathbf{p}_0^u + f_4\mathbf{p}_1^u. \tag{18}$$

If the tangent is the same in the termini of the neighbouring curves, the continuity of the tangent is maintained.

The bend in the points of contact is usually not continuous. The shape of the curve can be modified by the alteration of any of the four boundary conditions. Although the determination of the tangents can be quite difficult for the user/expert, this problem can be solved if we determine the tangent of all the inner checkpoints as a vector connecting the two neighbouring checkpoints.

Therefore:

$$\begin{aligned} \mathbf{p}_0^u &= \mathbf{p}_1 - \mathbf{p}_0, \\ \mathbf{p}_i^u &= \mathbf{p}_{i+1} - \mathbf{p}_{i-1}, \\ \mathbf{p}_n^u &= \mathbf{p}_n - \mathbf{p}_{n-1}. \end{aligned} \tag{19}$$

B. Limit sphere determination

If we construct complicated surfaces, usually we construct first the parts of the whole and after that we adjust them. In the case of interactive construction the curves connecting the checkpoints devide the surface into patches, which will make the adjustment smooth.

Let us have a $[0, 1] \times [0, 1]$ range of parameters defining the spatial quadrangle part of a $\mathbf{q}(u, v)$ surface and $\mathbf{q}(u, 0)$, $\mathbf{q}(u, 1)$ $u \in [0, 1]$, and $\mathbf{q}(1, v)$, $\mathbf{q}(0, v)$ $v \in$

[0, 1] curves as the sidecurves of the surface.

$$\begin{aligned}
\mathbf{q}(0, v) &= f_1(v)\mathbf{p}_{00} + f_2(v)\mathbf{p}_{01} + f_3(v)\mathbf{p}^v_{00} + f_4(v)\mathbf{p}^v_{01},\\
\mathbf{q}(1, v) &= f_1(v)\mathbf{p}_{10} + f_2(v)\mathbf{p}_{11} + f_3(v)\mathbf{p}^v_{10} + f_4(v)\mathbf{p}^v_{11},\\
\mathbf{q}(u, 0) &= f_1(u)\mathbf{p}_{00} + f_2(u)\mathbf{p}_{10} + f_3(u)\mathbf{p}^u_{00} + f_4(u)\mathbf{p}^u_{10},\\
\mathbf{q}(u, 1) &= f_1(u)\mathbf{p}_{01} + f_2(u)\mathbf{p}_{11} + f_3(u)\mathbf{p}^u_{01} + f_4(u)\mathbf{p}^u_{11}.
\end{aligned} \tag{20}$$

If we want to determine the patches, we have to construct/define the opposite direction tangents in the points of the limit curves (described above).

It is possible to determine them only if the quadratic mixed partial derivates in the acmes, called twist vectors, are known:

$$\mathbf{q}^{uv} = \frac{\partial^2 \mathbf{q}}{\partial v \partial u}(u, v). \tag{21}$$

By this the searched derivate values can be appointed/determined:

$$\begin{aligned}
\mathbf{q}^u(0, v) &= f_1(v)\mathbf{p}^u_{00} + f_2(v)\mathbf{p}^u_{01} + f_3(v)\mathbf{p}^{uv}_{00} + f_4(v)\mathbf{p}^{uv}_{01},\\
\mathbf{q}^u(1, v) &= f_1(v)\mathbf{p}^u_{10} + f_2(v)\mathbf{p}^u_{11} + f_3(v)\mathbf{p}^{uv}_{10} + f_4(v)\mathbf{p}^{uv}_{11},\\
\mathbf{q}^v(u, 0) &= f_1(u)\mathbf{p}^v_{00} + f_2(u)\mathbf{p}^v_{10} + f_3(u)\mathbf{p}^{uv}_{00} + f_4(u)\mathbf{p}^{uv}_{10},\\
\mathbf{q}^v(u, 1) &= f_1(u)\mathbf{p}^v_{01} + f_2(u)\mathbf{p}^v_{11} + f_3(u)\mathbf{p}^{uv}_{01} + f_4(u)\mathbf{p}^{uv}_{11}.
\end{aligned} \tag{22}$$

The determination of the surface with the help of the matrix of the boundary condition:

$$\mathbf{H} = \begin{bmatrix} \mathbf{p}_{00} & \mathbf{p}_{01} & \mathbf{p}^v_{00} & \mathbf{p}^v_{01} \\ \mathbf{p}_{10} & \mathbf{p}_{11} & \mathbf{p}^v_{10} & \mathbf{p}^v_{11} \\ \mathbf{p}^u_{00} & \mathbf{p}^u_{01} & \mathbf{p}^{uv}_{00} & \mathbf{p}^{uv}_{01} \\ \mathbf{p}^u_{10} & \mathbf{p}^u_{11} & \mathbf{p}^{uv}_{10} & \mathbf{p}^{uv}_{11} \end{bmatrix} \tag{23}$$

can be therefore constructed in the following way:

$$\begin{aligned}
\mathbf{M} &= [f_1(u), f_2(u), f_3(u), f_4(u)],\\
\mathbf{N} &= [f_1(v), f_2(v), f_3(v), f_4(v)]^T\\
\mathbf{q}(u, v) &= \mathbf{M} \cdot \mathbf{H} \cdot \mathbf{N} = \sum_{k=1}^{4} \sum_{l=1}^{4} f_k(u) f_l(v) h_{kl},
\end{aligned} \tag{24}$$

where: $(u, v) \in [0, 1] \times [0, 1]$.

5. Determining the Deformation Energy

Parts of the car body can be devided into so called energy cells. We use crash tests to specify the energy absorbed by these cells. If we have a frontal crash, for example, to state the energy absorbed by the deformation, we have to sum up the energies absorbed by the deteriorated cells. This step presupposes the knowledge of the character and the spatial shape of the deformation. This information can be obtained with the help of the above mentioned methods. Apart from the boundary surface we also need the spatial model of the undamaged car body, which we adjust to the deformed vehicle. In the case of totally deformed cells we utilize all their energies. Although, if the cell is just partly deteriorated, it is necessary to determine the volume of the deteriorated part. This operation can be completed by using the equation of the boundary surface which was described above. With the help of *Eq.* (24) we can determine the boundary surface of the deformed and undamaged parts of car body. We are interested in the part of the surface created by the common part of the given cell and the boundary surface.

Having stated the volume of the deformed cell part, we can determine how much of the cell's energy was absorbed by the deformation. Following this procedure for all of the deformed cells, and summing up all these values, we can get the energy absorbed by all the cells of the car body. Therefore, knowing the energy absorbed by the deformed car body, the EES can be easily determined.

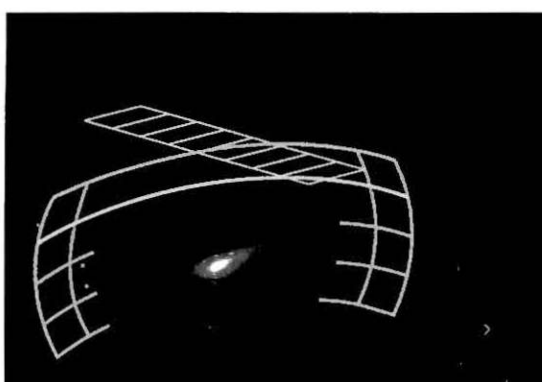
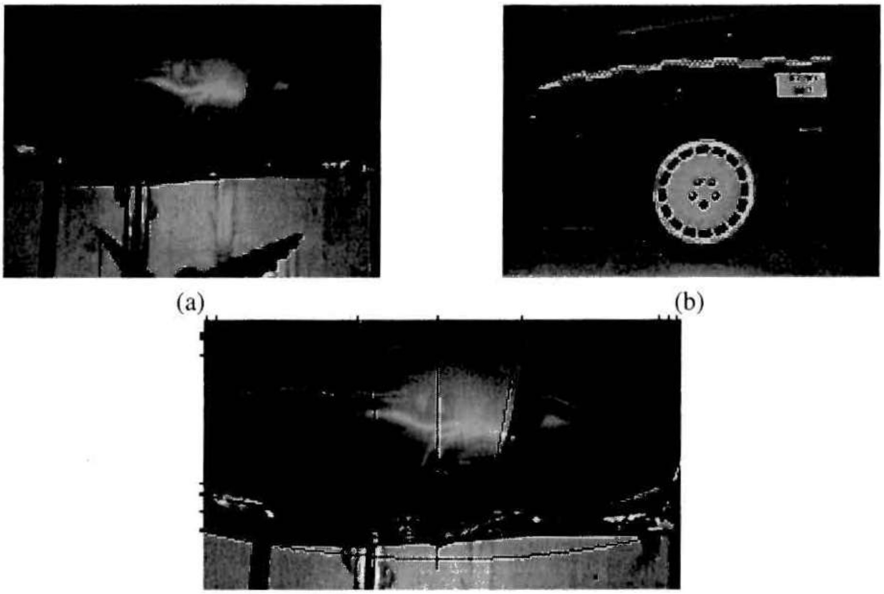


Fig. 5. The illustration of the common part of the limit sphere and the energy-cell

6. Performance Evaluation

To justify the EES values supplied by the method, we will analyze the deformation of the vehicle illustrated in *Fig. 6*. *Fig. 6(c)* shows the adjustment of the undamaged car body model to the analyzed vehicle.



(a) (b)

(c)

Fig. 6. (a) Top view of the tested vehicle, (b) Side view of the tested vehicle, (c) Illustration of the deformed volume

The energy values of the cells used in the computing are shown in *Table 1*. Using the boundary surface obtained from the limit curves and the undamaged car body model, we can determine the value of the absorbed energy.

The mass of the vehicle is: 1510 kg
The maximal depth of the deformation is: 205 mm
The real ESS value of the vehicle on Fig. 3 is: 15 km/h
The absorbed energy calculated by this method is: 11581.9 J
The calculated EES value by this method is: 14.1 km/h

In *Table 1* each cell represents an energy cell. The width of each cell is 1/8 part of the vehicle width, and the length of these cells is 10 cm. The values in the cells are given in Joule. The bottom part of *Table 1* means the front part of the vehicle.

We can see that the obtained value is very similar to the value obtained by the test. The method therefore can be really used to solve these kinds of problems. Of course, the method, like every other method, has some limits, so there are some cases where it can not appraise the deformation of the vehicle, but in the majority of cases it gives a very good approximation.

Table 1. Used energy values of energy cells taken from [2]

3000	5000	4000	8000	8000	4000	5000	3000
1575	2625	2100	4200	4200	2100	2625	1575
825	1375	1100	2200	2200	1100	1375	825
862	1438	1155	2300	2300	1155	1438	862
1088	1812	1450	2900	2900	1450	1812	1088
375	625	5000	1000	1000	5000	625	375

7. Conclusions

This article deals with a new method of Energy Equivalent Speed determination, and with the photo-based determination of the deformation's spatial shape. By using this method, it is possible to state the amount of the energy absorbed by the car body deformation. These EES data can be utilized by all crash analizing programs. Crash reconstruction can be improved by the further development of the methods which determine other important elements, for example: the analysis of brake traces and the quality of road-surface, and the influence of these elements on the process of braking, vehicle swinging and vibrating processes [19], [20], [22], [24].

Finally, we would like to mention that in most practical cases all aproached methods can ensure a more accurate description of the deformation values detection.

Acknowledgment

This work was supported by the Hungarian Fund for Scientific Research (OTKA 1999-2002 – T 030655 - Integrated fuzzy-stochastic modelling and identification of vehicle dynamic systems)

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