PERIODICA POLYTECHNICA SER. TRANSP. ENG. VOL. 29, NO. 1-2, PP. 47–58 (2001)

DYNAMIC ANALYSIS OF VEHICLE MANOEUVRES ON THE BASIS OF THE FINITE ELEMENT METHOD

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Received: Oct. 10, 2001

Abstract

In this paper the three dimensional dynamic analysis of vehicle motions is presented on the basis of the finite element method, therefore the distribution of elasticity and mass of the chassis and vehicle body is taken into account accurately. The mode superposition method is improved on the description of large horizontal displacements (in longitudinal and lateral directions), while the effects of dampers and other structural elements with non-linear characteristics are considered by the pseudoforce method. Shear forces arising between road surface and tyres are determined by the cosine version of magic formula tyre model. Examples, relating to cornering and braking, are presented by the use of a bus finite element model containing 2294 degrees of freedom.

Keywords: vehicle dynamics, finite element method, combined cornering and braking, μ-split braking, wheel locking.

1. Introduction

The mechanical properties of tyres have important role in the dynamic behaviour of vehicles. Therefore a large number of papers describe studies on tyre mechanics and present well-elaborated tyre models ([1, 2, 3, 4]). In other publications these detailed tyre models and/or sophisticated control strategies are applied to the dynamic analysis of (controlled) vehicles using relatively small and simple vehicle models ([5, 6]). Recently the commercial multibody programs have been preferred to the simulation of vehicle motions ([7]). Commercial finite element programs developed for general purposes can also be used effectively for structural dynamics of vehicles (MSC/NASTRAN, ANSYS, COSMOS/M, etc.). Other specific finite element programs applicable with great efficiency for the solution of special problems, such as MSC/DYTRAN, seem to be very useful to the investigation of the collisions of mobile machines. For example, ADINA, MARC, ABACUS, etc. are particularly well applicable in the analysis of large elasto-plastic deformations of complex (vehicle) structures.

The objective of this paper is to present examples of the non-conventional application of the finite element method, namely to the three dimensional dynamic analysis of the motions of road vehicles. The application of large finite element models, on the one hand, makes the accurate consideration of the elasticity and mass distribution of the structure of vehicles possible. However, on the other hand, the dynamic analysis of finite element models by step-by-step solution methods is very time consuming when their number of degrees of freedom is large. Consequently the application of any model condensation procedure is necessary. For this purpose, herein the mode superposition method is applied, since it is enough to use only a few low mode shapes to the structural dynamic analysis. It is well known that these mode shapes do not contain any damping and non-linear effects, therefore the pseudoforce method is applied to the description of the forces arising from dampers, springs and elastic deformation of other structural elements with non-linear characteristics. The material damping of the vehicle can be approximated by the application of Rayleigh and/or modal damping.

It is not enough to apply only the elastic natural modes, since to the horizontal motions of vehicles (longitudinal and lateral motions, finite rotations about the vertical axis) can be taken into account by the rigid body modes with zero natural frequencies. The tyre forces and self-aligning torques are described by the cosine version of the magic formula tyre model ([10]). Since the method used herein is a developed version of the computational procedure presented in publication [9], in this paper the necessary modifications will be discussed only.

Two examples are presented in order to demonstrate the effectiveness and applicability of the elaborated method and finite element program. The first problem relates to an asymmetric braking when the path of the vehicle is corrected by steering the front wheels. In the second example the locking of the right hand side wheels is studied. In both examples the same bus finite element model is applied with 2294 degrees of freedom.

2. An Extension of the Mode Superposition Method to the Description of Overall Motions

It is well known in structural dynamics that the mode superposition method in its original form can only be applied to linear structures in case of small strains and displacements. However, the vehicles during their motions carry out different manoeuvres with large horizontal displacements (in longitudinal and lateral directions) and rotations about the vertical axis. In order to take these large motions into consideration by the help of the mode superposition method the corresponding (three) rigid body modes with zero natural frequencies have to be included in the considered set of mode shapes. From the point of view of dynamic analysis it is important that the first three rigid body modes describe the large vehicle motions and the remaining natural modes with non-zero natural frequencies describe the small elastic vibrations of the studied vehicle structure (elastic modes). A detailed theoretical description of vehicle motions on the basis of the finite element method is given in paper ([8]).

The pseudoforces are the functions of the relative displacements and velocities produced by the small elastic vibrations of vehicle structures, they can therefore be determined directly from the elastic modes ([9]).

Considering that the vehicles perform large rotations about the vertical axis, the longitudinal and lateral tyre forces as external excitations have to be applied carefully to the instantaneous positions of the wheels (*Fig. 2*).

3. The Applied Bus and Tyre Model

The bus model (*Fig. 1*) used in this paper, except the tyres, corresponds to the bus model in paper [9], consequently only the differences relating to the tyres will be discussed here. In the previous bus model the springs and dampers representing the elasticity and damping of tyres were attached to the ground.

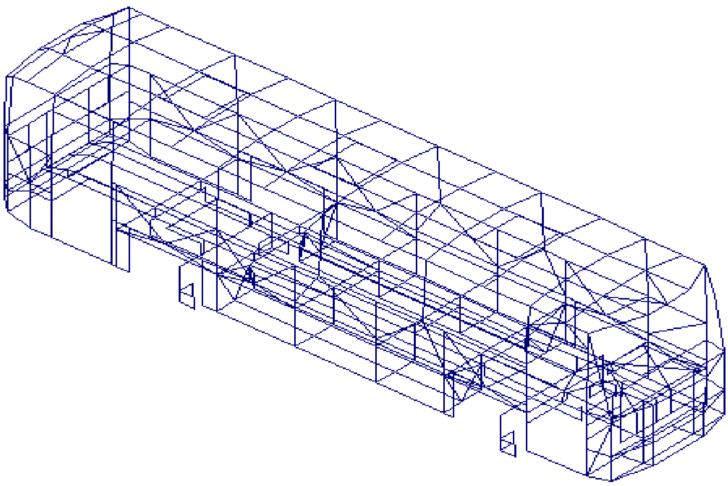

Fig. 1. Skeleton structure of the applied bus model

Now, these extremities of the springs and dampers are connected to the contact patches of tyres (*Fig. 3*). The motion of contact patches is constrained only in vertical direction hence the complete bus model can move and rotate freely in horizontal directions and about the vertical axis, respectively. This construction of contact patches can be taken into account as a possible finite element representation of the tyre model presented in paper [12]. Additionally the definitions of the slips and the gyroscopic effects arising from the tyre belt distortions ([12], equations 24–27) can also be used without any change.

Returning to the presented method, having prescribed the initial velocities the motion of the bus model can be controlled by driving and/or braking moments and steer angles applied to the wheels. Longitudinal and lateral tyre forces as

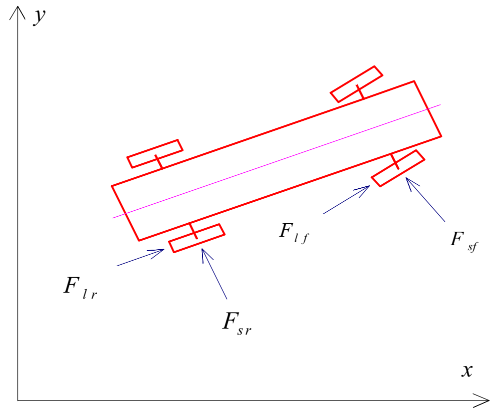

Fig. 2. Current position of the vehicle

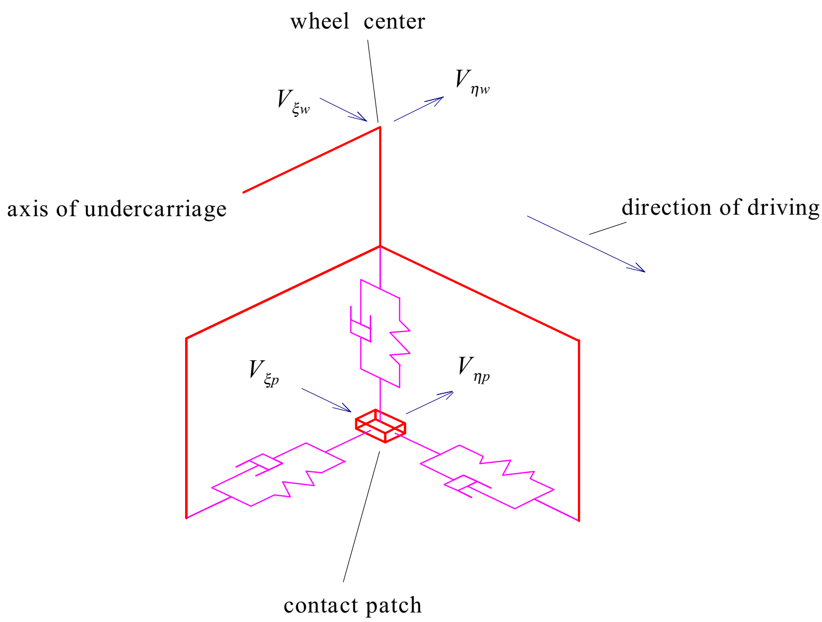

Fig. 3. Finite element model of tyres

the functions of combined slip (*Fig. 4.a*) and self aligning torques (*Fig. 4.b*) are determined by the cosine version of the magic formula tyre model ([10]).

The longitudinal and lateral slips, in correspondence with paper [12], are

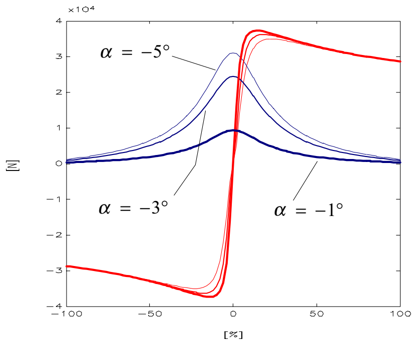

Fig. 4.a. Combined longitudinal and lateral tyre forces

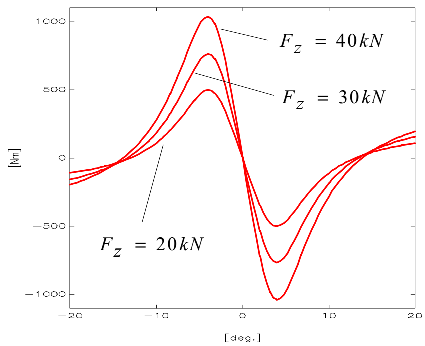

Fig. 4.b. Self aligning torques

defined as follows,

$$\kappa = -\frac{V_{\xi p} - \Omega\, R_e}{\left|V_{\xi W}\right|}, \tag{1a}$$

$$\alpha = -\frac{V_{\eta p} - V_{\xi w}\Psi_{g\,y\,r}}{\left|V_{\xi w}\right|}, \tag{1b}$$

where: κ is the longitudinal slip,
α is the lateral slip,
$V_{\xi p}$ is the longitudinal component of the velocity of the contact patch,
Ω is the angular velocity of the considered wheel,
R_e is the effective rolling radius,
$V_{\eta p}$ is the lateral component of the velocity of the contact patch,
$V_{\xi w}$ is the longitudinal component of the velocity of the wheel centre,
$|\cdot|$ is the sign of the absolute value,
$\Psi_{g\,y\,r}$ is the gyroscopic effect of the belt distortion exerted on the wheel yaw angle.

On the basis of paper [13] a rigid wheel model is applied to the consideration of the rotations of wheels described by the following differential equation,

$$\Theta_y\, \ddot{\varphi}(t) \;=\; M(t) - F_z\,(t)\,h(t) \;+\; F_l\,(t)\,R_e(t), \tag{2}$$

where: Θ_y is the mass moment of inertia of the given wheel with respect to its centroidal axis,
$\ddot{\varphi}(t)$ is the angular acceleration of the given wheel,
$M(t)$ is the driving or braking moment applied on wheels,
$F_z(t)$ is the tyre force in vertical direction,
$h(t)$ is the coefficient of rolling resistance,
$F_l(t)$ is the tyre force in longitudinal direction.

Eq. (2) applied to each wheel separately is solved together with the fundamental modal equations ([9]) therefore there is no need to use a separated numerical solution method. The initial angular velocities of the wheels are considered as input data and their initial angles of rotation are assumed to be equal to zero. On the basis of paper [14] a simple approximation has been elaborated to the determination of the effective rolling radius R_e as a function of the vertical tyre force $F_z(t)$. These effective rolling radii of wheels are modified by the square of the ratio of the instantaneous and initial angular velocities in such a manner that their magnitudes will be equal to the corresponding deformed radii in case of zero angular velocities.

The magnitudes of the contact patch masses are approximated in relation to the tyre belt distortion [15]. Considering the finite element representation of a single contact patch (*Fig. 3*), different mass magnitudes in different directions and moments of inertia about different axes can be defined, therefore the first rotational, camber and yaw mode of the rigid ring tyre model can be taken into account. In this paper the natural frequency of the (undamped) camber mode of tyres is ∼46 Hz, that of the rotational mode is ∼41 Hz. The yaw mode has not been considered yet.

4. Numerical Examples

4.1. *Asymmetric Braking and Correction of the Path of the Bus by Steering Wheels*

In this example the bus is driven along a straight and dry road with a speed of 20 m/s when to the left hand side wheels 8000 Nm and to the right hand side wheels 2500 Nm braking moments are applied (*Fig. 5*). The considered tyre force characteristics and self-aligning moments are shown in *Figs. 4.a–4.b*. In *Fig. 7* can be seen the path of the bus, where in consequence of the applied asymmetric braking moments it moves into the opposite traffic lane of the road. If the front wheels are steered according to the graph given in *Fig. 6*, then the lateral motion of the bus is corrected and it remains in its own lane as it is demonstrated in *Fig.8*. The longitudinal velocity of the centre of gravity of the bus is given in *Fig.9*, while in *Fig. 10* its lateral velocity is shown, where the thin and thick lines refer to the pure asymmetric braking and to the combined cornering and asymmetric braking, respectively. In *Figs. 11–15* the thin and thick lines are used in the same sense as they are applied in *Fig. 10*. The longitudinal tyre forces applied on the right hand side front wheel are given in *Fig. 11*, and the corresponding longitudinal slips are presented in *Fig. 12*. The related lateral tyre forces and lateral slips are shown in *Figs.* 13–14, respectively. At the first sight it is surprising that the shapes of the graphs of lateral tyre forces are very similar to the shapes of the graphs of the corresponding lateral slips, however it is not so surprising if we realise that in this example the slips remain in the stable (almost linear) range of the longitudinal tyre force characteristics and at the same time the lateral tyre forces have almost constant magnitudes (*Fig. 4.a*). At last *Fig.* 15 shows the change of the effective rolling radius of the right hand side front wheel.

Finally it is necessary to mention that the rolling resistance of wheels is considered in this example as it can be seen in *Fig.* 9 between 4 and 6.5 seconds.

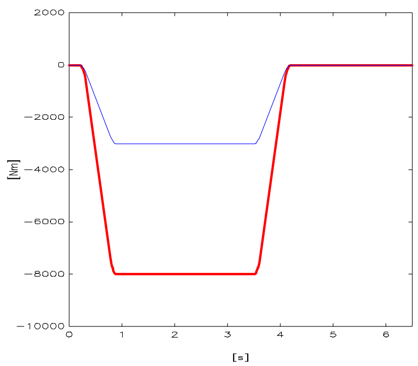

Fig. 5. Braking moments

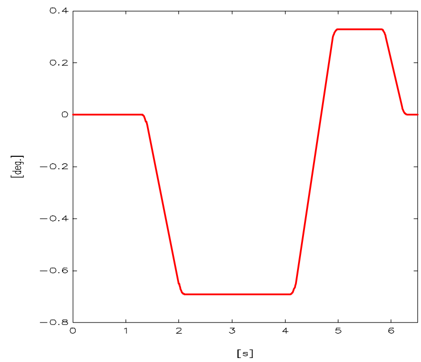

Fig. 6. Steer angle diagram

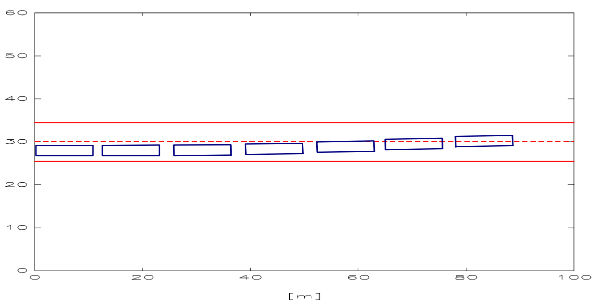

Fig. 7. Path of the bus in case of asymmetric braking

4.2. *Demonstration of Wheel Locking*

In this example a wet road surface is considered when the friction between tyres and road surface is decreased to ∼35% of the friction of dry road surface. The initial

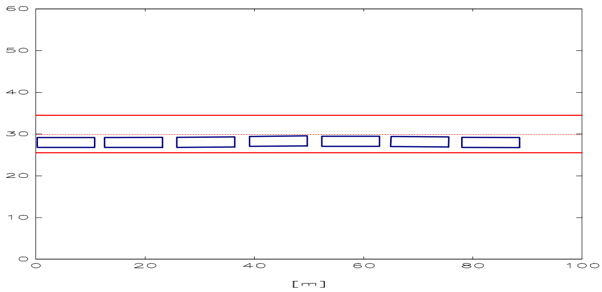

Fig. 8. Path of the bus in case of combined braking and cornering

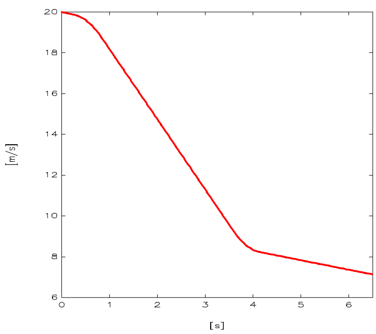

Fig. 9. Longitudinal velocity of the CG of the bus

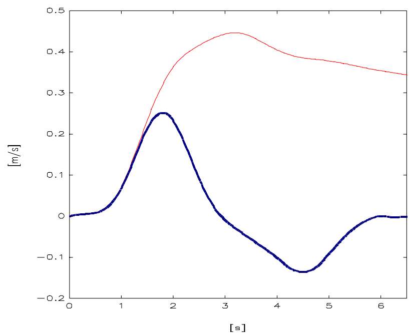

Fig. 10. Lateral velocity of the CG of the bus

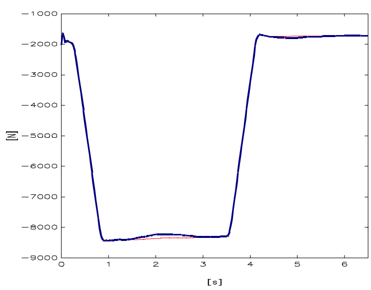

Fig. 11. Longitudinal tyre forces of the right hand side front wheel

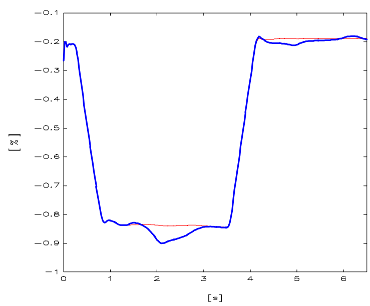

Fig. 12. Longitudinal slips of the right hand side front wheel

velocity is assumed to be equal to 20 m/s. The magnitudes of braking moments applied to the right and left hand side wheels are equal to 5500 Nm and 4500 Nm, respectively (*Fig. 16*). In spite of the fact that the magnitude of braking moments is larger at the right hand side wheels, the bus is displaced to the left in lateral direction since the right hand side wheels are locked during braking. It can be seen in *Figs.17–18*, which represent the lateral displacement and velocity of the centre of gravity of the bus, respectively.

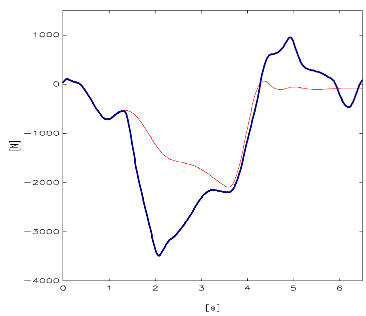

Fig. 13. Lateral tyre forces of the right hand side front wheel

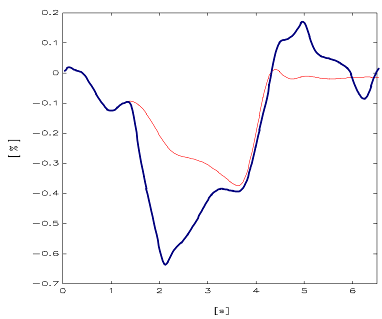

Fig. 14. Lateral slips of the right hand side front wheel

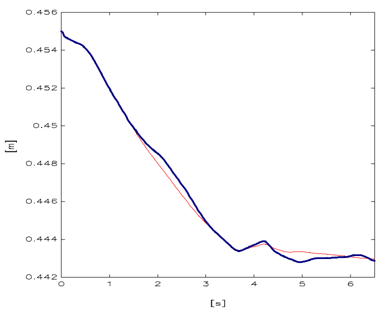

Fig. 15. Effective rolling radius

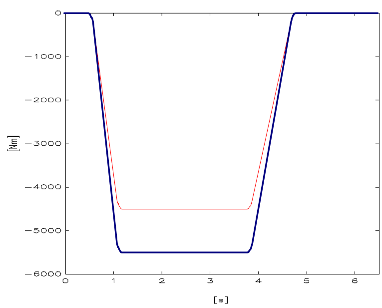

Fig. 16. Braking moments

The thick and thin lines in *Figs. 19–23* refer to the right and left hand side front wheels, respectively. *Fig. 19* shows the longitudinal tyre forces and in *Fig. 20* lateral tyre forces are demonstrated. In these figures the decrease of tyre forces because of wheel locking (thick lines) can be seen clearly. The corresponding longitudinal and lateral slips are given in *Figs. 21–22*, respectively. In *Fig. 22* it can be realized that there is no large difference between the lateral slips of the front wheels. At last *Fig. 23* shows the angular velocities of the front wheels.

In each example the fourth order Hamming method [9] was applied to the

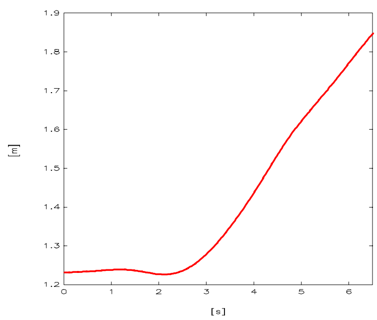

Fig. 17. Lateral displacement of the centre of gravity of the bus

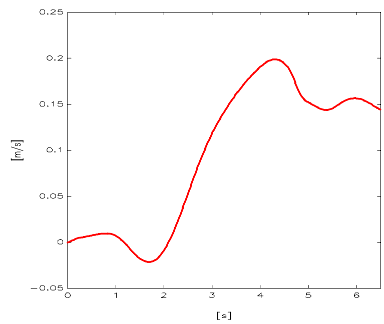

Fig. 18. Lateral velocity of the centre of gravity of the bus

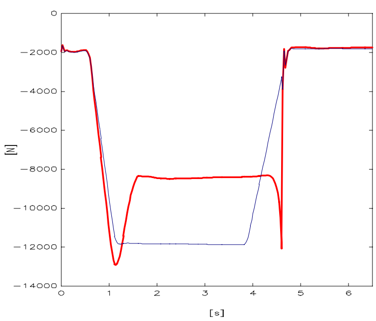

Fig. 19. Longitudinal tyre forces of the front wheels

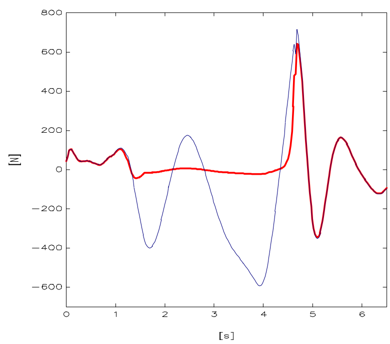

Fig. 20. Lateral tyre forces of the front wheels

numerical step-by-step solution with 2.5×10^{-4} (s) step size. The total solution time for 6.5 (s) time interval was equal to 68.32 (s).

5. Conclusions

The objective of this paper was to present a procedure to the three dimensional dynamic analysis of vehicle motions. It is based on the finite element and the mode superposition methods, while the effects of dampers and other structural elements with non-linear properties are considered by the pseudoforce method. Tyre forces and self-aligning moments are taken into account by the cosine version of the magic formula tyre model. It is possible to use very large finite element models, since almost all of the vehicle model can be elaborated by the application of an

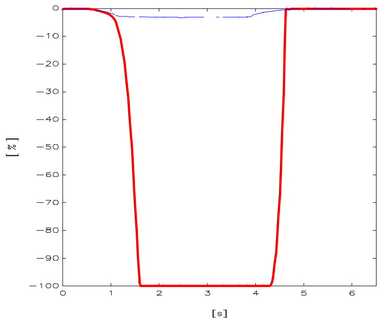

Fig. 21. Longitudinal slips of the front wheels

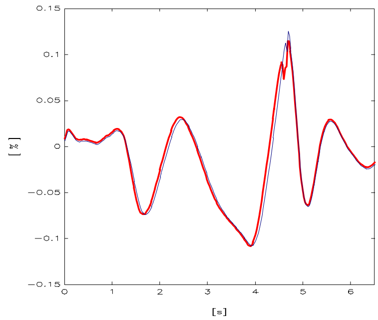

Fig. 22. Lateral slips of the front wheels

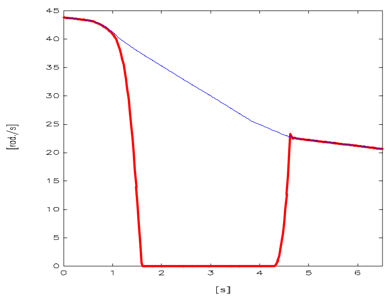

Fig. 23. Angular velocity of the front wheels

appropriate commercial finite element program. The first example demonstrates the effectiveness and applicability of the presented method for combined cornering and braking and the second one relates to wheel locking.

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