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Determination of the Minimum Number of Possible Testing Situations in Autonomous Driving Using Critical Phenomena

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Abstract

The main task of the article is to define the critical minimum number of tests for the dynamics of a traffic node in autonomous driving based on critical phenomena, i.e. the percolation theory. The critical (minimum) number of tests of a node means how we can represent the traffic dynamics of a node with critical, percolating path using a "state-following state" system on the graph. The test cases along the percolation path, i.e., those involved in the formation of the new phase, represent the entire test system and are minimal. In the article we show that only less than 10 of the 640 tests to be performed have to be realized and are representative for release processes.

Keywords

highly autonomous driving (HAD), critical phenomena, traffic situations, testing, minimizing

1 Introduction

Highly autonomous driving (HAD) is expected to have a positive impact on the global transport environment. According to the state of research, more than 80% of traffic accidents are caused by human error. By replacing the human operation with a technical-mathematical solution, it is possible to reduce the number of accidents (Kiss and Berecz, 2019).

Driver-free vehicles allow, for example, the reinterpretation of taxi services and the modernization of logistics.

Nowadays, the development of autonomous vehicles has been in full swing for years. OEMs (car manufacturers) promise to develop vehicles with a higher degree of autonomy in the coming years (Szabó and Bakucz, 2021).

The driver of an autonomous vehicle can be deactivated, it cannot be taken into account by the operator of the control operations. As a result, there will be very high reliability requirements for safety, reliability and security (Derbel et al., 2012).

For a practical interpretation and implementation of the safety requirements for self-driving vehicles, it is necessary to understand what reliable and safe behavior really means (Bede and Péter, 2011). For example, a HAD car must be able to handle traffic rules, the geometry and topology of its surroundings, and must be able to interpret the meteorological system, but also rare, difficult-to-predict road hazards (Kiss, 2022). A strategy must then be devised to check that the vehicle has actually reached the required level of safety (Kiss, 2020). The problem in the development of HAD in the coming years is the release of the systems: how the completed hardware-software and algorithm can be put on the market and how to sell the system.

The basic priority in highly automated driving is the testing of diverse predefined issues (Kiss, 2019; Kiss and Berecz, 2021). If the demand for testing is extremely high, the economics of implementing highly automated driving will be on the agenda.

In order to solve various engineering reliability tasks, it is essential to determine the minimum number of necessary testing situations for a traffic node. In our article, we use the phase transition theory of theoretical physics.

We say, that the test-set is a random system, and in the phase transition interpreted on the graph the participating test cases can only be considered after the path of the new phase on the graph.

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Then the system is:

- "percolated";
- "a breakthrough is realized";
- "a new phase is occupied";
- "is mathematically demonstrable".

At this point, when the phase transition occurs, going through the critical test cases between two distinguished points defined in percolation theory, we arrive at the minimum number of test cases.

Consequently, in this article a method is introduced to define the minimum number of traffic situations at a traffic junction based on these synonyms of percolation.

The critical traffic situation means that the geometry and traffic dynamics of the node are analyzed and quantified using paths. The dynamics of a transport node is determined by discretizing the node and recording the movements of each traffic participant in a "state - next state" system.

In our process, we look for the minimal, critical path between "state"-to-"next state" nodes, knowing that criticality in this graph can be characterized by a fractal dimension in Mathlab 2020b computer algebra software (Mathworks Inc., 2020).

In the present project, in which we present the tests applied in the Autonomous Driving System of the University of Óbuda, the corner radar sensor time series (Fig. 1) plays the role of the perception tasks. Due to the importance of HAD time series analysis, it is a challenge to characterize the extremal value of time series and the dynamics of such extremes, and to develop an efficient approach for embedded systems to understand the formation of time series. Recently, significant progress has been made in understanding interfaces through the application of fractal concepts and the development of the theory of dynamic scaling. The dynamic scaling introduced in fractal growth has become an indispensable tool for the characterization and fractal-based, nonlinear analysis of the morphology and evolution of interfaces can also be applied in the theoretical study of surfaces.

In this journal article, we review the basic ideas of dynamic scaling and multifractal analysis, as well as how to characterize the maximum value of autonomous radar sensor time series (Hammerschmidt, 2017). To illustrate the application of these ideas, we used data from 13593 autonomous radar sensors in the HAD database for a driving scenario (Fig. 2).

In Section 2 we are dealing with fractal dynamic scaling, with the first-order consideration of radar extremities.

Fig. 1 Corner radar sensor test (Source: own research)

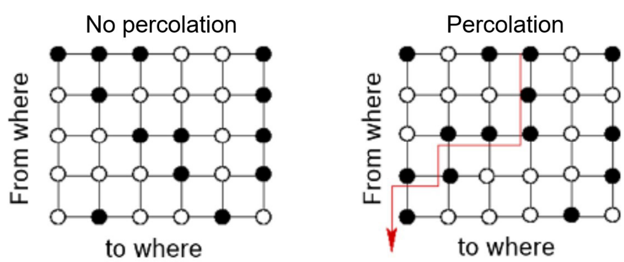

Fig. 2 Minimum number of study sites (Source: own research)

The methods for multifractal analysis are described in Section 3. This is followed in Section 4 by the first results on corner radar, and in Section 5 by a summary of past and future work.

2 Critical phenomena, the applied method

Few indicators of HAD deal with time series and their non-linear properties in many systems (Anderson et al., 2016).

The corner radar sensor ultimately measures distance, but taking into account the parameters shown in Fig. 1, it is an essential part of sensor detection accuracy. In our Óbuda University Autonomous Driving research project, we look for the critical (minimum) number of testing scenarios to be necessarily in the corner radar release issues.

In certain applications, the goal is to produce a time series with a specific physical or technical property, but time series are often inherent in industrial and natural processes. In fact, Mandelbrot pointed out that some time series are best approximated by a fractal geometry system (Mandelbrot, 1982). This recognition led to the development of a dynamic scaling system that describes not only a given morphology but also the internal dynamics of time series, including extremes (Bianchi et al., 1992). In this approach, we consider the time evolution of time series in a d-dimensional space, starting from an initial corner radar time series.

The essence of the method is the physical experience that growing surface-instabilities with the same scaling factor are physically identical, i.e. they can be scaled together (Grassberger and Proccacia, 1983).

Our article is engaged with minimum problem, where an $m \times n$ grid of nodes is given as shown above (Bakucz and Kiss, 2021). The nodes are arranged, grid-like, into m rows and n columns. Each node is either "a car is present → 1" or "a car is not present → 0" with probability *p*. Clearly, if *p* is small (close to 0), then few nodes will be "on" whereas if *p* is large (close to 1) then most nodes will be "on" (Abraham et al., 1986).

A vertical percolation path is a path from a node in the top row to a node in the bottom row consisting entirely of "on" nodes (Farmer and Sidorowich, 1987). Then, a vertical percolation is said to occur if there is a vertical percolation path (Fig. 2).

Fig. 2 determination of the minimum number of possible testing situations in a traffic node, based on percolation. Black dots represent tests (vehicle movement) from the vertically marked position to the horizontally marked position (see Fig. 3).

If we can "just" pass from the top down, percolation takes place, a new phase is created, and this state completely characterizes the vehicle tests interpreted in the node. It is sufficient to perform only these "percolating" tests.

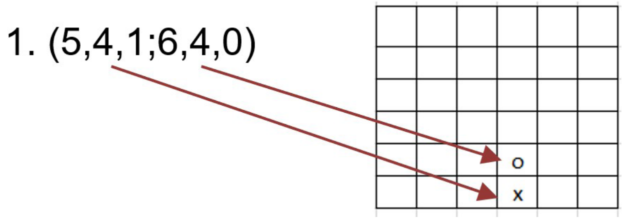

Fig. 3 Vector position of the car (Source: own research)

A percolation of test cases exists if either a vertical or horizontal percolation exists then the new phase has emerged, i.e. the phase transition took place, and it is at this point, by the breakthrough, that the shape on which the percolation takes place is a fractal shape (Arizmendi et al., 1995; Goldberg et al., 1988). This condition is characteristic of the test system as a whole, and the execution of the tests associated with the breakthrough shape characterizes the entire test system (Figs. 3 and 4).

Notation of Fig. 2: The "o" means occupied by a car. The "x" indicates ("one of") the next positions. In the vector notation the symbol "o" has position 5,4 and 1 indicates occupied. For the symbol "x" the *x*,*y* coordinates are 6,4 and 0 indicates one of the next position. We collect the possible "from" "to" scenes, which is represented in a system where the coordinates "from where?" are displayed on the vertical axis and the coordinates "to where?" on the horizontal axis (see in Fig. 4).

Collecting the possible "from" "to" scenes, which is represented in a system where the coordinates "from where" are displayed on the vertical axis and the coordinates "to where" on the horizontal axis. The "o" means occupied by a car. The "x" indicates (one) the next positions. Slowly filling the system (left side) with blue dots (test cases) when we reach the criticality of the system, i.e. when percolation takes place, the tests along the critical path (or forming the critical path) represent the entire test system.

Thus, if we generate a random grid (that is, we create a grid and then turn nodes "on" according to probability *p*), then there may or may not be a percolation path (Fig. 5) (Peitgen et al., 1992).

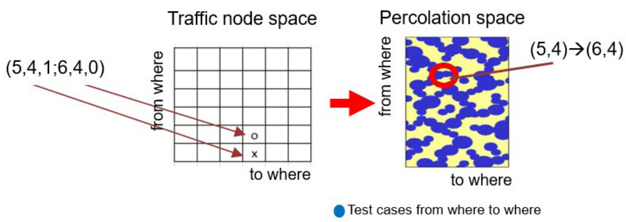

Fig. 4 Possible vehicle positions (Source: own research)

Let Q be the probability that a percolation is found. Since this depends on the seed probability p, we'll write this as $Q(p)$. Your goal is to plot the graph $Q(p)$ vs. p for a 6×6 mesh. We have to estimate this function using a sufficiently large number of samples (repeated simulations).

We use the percolation, which is a model of a porous medium, and is a paradigm model of critical phenomena in statistical physics. Think of the bonds in an infinite graph that are not removed as indicating whether water can flow through this part of the medium. Then, the interesting question is whether water can percolate, or, alternatively, whether there is an infinite connected component of bonds that are kept. As it turns out, the answer to this question depends sensitively on the fraction of bonds that are kept. When we keep most bonds, then the kept or occupied bonds form most of the original graph. In particular, an infinite connected component may exist, and if this happens, we say that the system percolates. On the other hand, when most bonds are removed or vacant, then the connected components tend to be small and insignificant.

Thus, percolation admits a phase transition. Despite the simplicity of the model, the results obtained up to date are far from complete, and many aspects of percolation, particularly of its critical behavior, are ill understood.

The key challenge in percolation is to uncover the relation between the percolation critical behavior and the properties of the underlying graph from which we obtain percolation by removing edges.

While in percolation the random network under consideration naturally lives on an infinite graph, in random graph theory one considers random finite graphs (Peters, 1991). Thus, all random graphs are obtained by removing edges from the complete graph, or by adding edges to an empty graph. An important example of a random graph is obtained by independently removing bonds

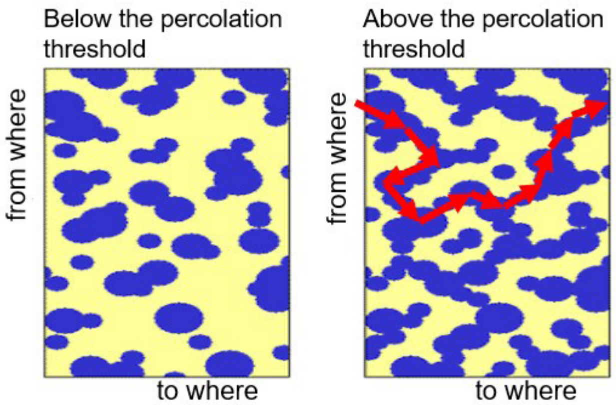

Fig. 5 Increase the number of test cases (Source: own research)

from a finite graph, which makes it clear that there is a strong link to percolation. However, other mechanisms are possible to generate a random graph (Schroeder, 1991).

We shall discuss some of the basics of random graph theory, focusing on the phase transition of the largest connected component and the distances in random graphs. The random graph models studied here are inspired by applications, and we shall highlight real-world networks that these random graphs aim to model to some extent.

We now start by introducing some notation.

Let $G = (V,E)$ be a graph, where V is the vertex set and $E \subseteq V \times V$ is the edge set.

In our case we define testcases on the traffic junction and the blue dots (Fig. 6) represent the "from cell (state)" to the "to cell (state)" testing experiments.

For percolation, the number of vertices, denoted by $|V|$, is naturally infinite, while for random graphs, $|V|$ is naturally finite.

A random network is obtained by a certain rule that determines which subset of the edges E is occupied, the remaining edges being vacant.

Let $v \in V$, and denote by $C(v)$ the set of vertices which can be reached from v by occupied edges (Eq. (1)). More precisely, for $u \in V$, we say that $u \longleftrightarrow v$ when there exists a path of occupied edges that connects u and v, and we write

$$C(v) = \{u \in V : u \longleftrightarrow v\} . \quad (1)$$

The central answer in the study of minimum test cases involves the cluster size distributions, i.e., for percolation whether there exists an infinite connected component, and for random graphs what the distribution of the largest connected component is.

3 Algorithm, results

The new procedure can be described by the following steps:

- A → A traffic node dynamics is given, where the movement of vehicles with the given initial and boundary conditions (topology of the junction, cyclists, pedestrians, traffic light) is received by a radar system and released for self-driving.
- B → Discretization of the node with rectangles is necessary in order to carry out the percolation process. Percolation is a perturbation scheme interpreted on a square cell element. The elements of the square cell are either loaded or unloaded, i.e. where movement can occur or where it cannot. It is most simply described as a seepage process in soil, where the gap is where the fluid can flow and the grain is where it

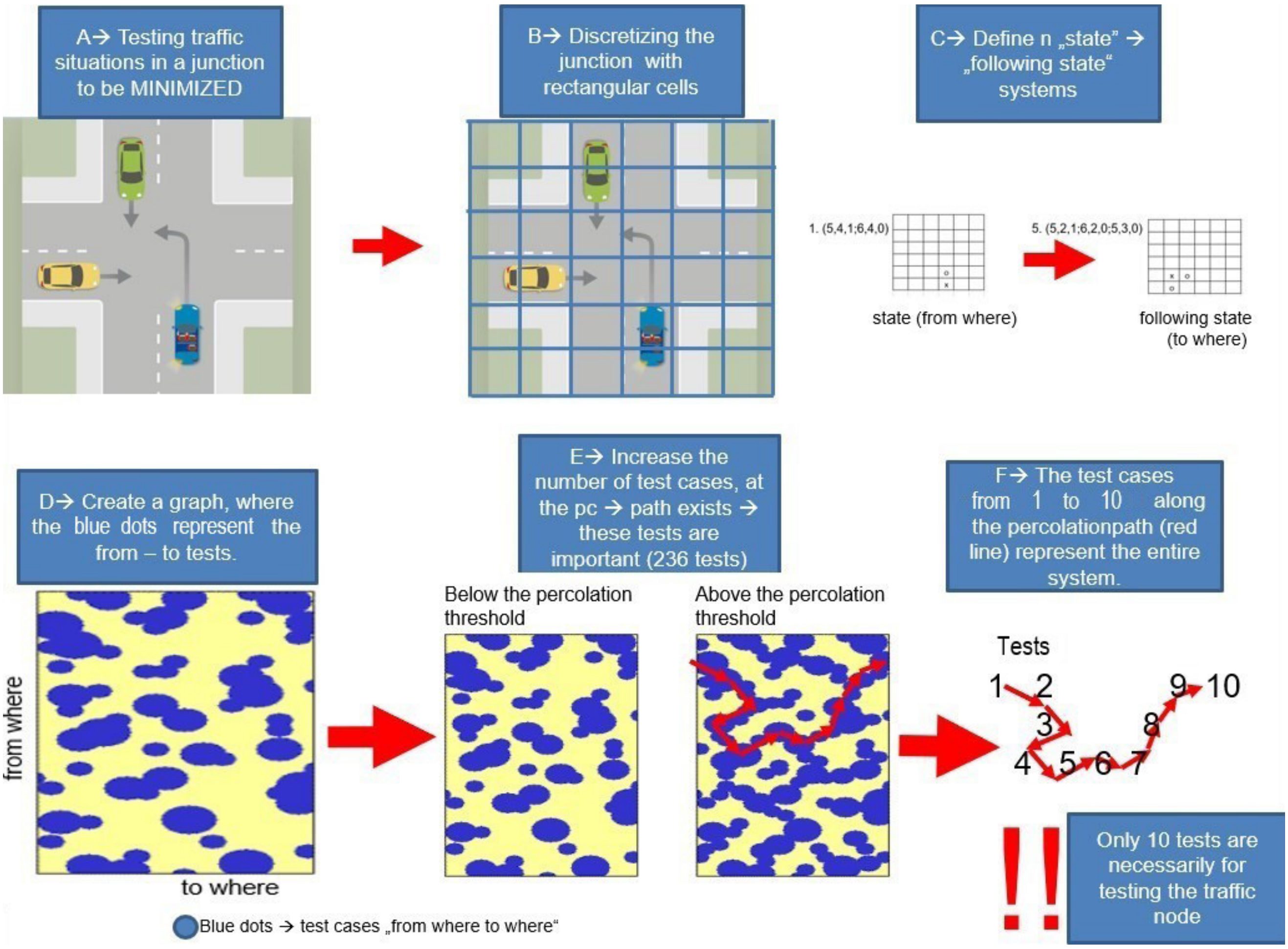

Fig. 6 Óbuda University Autonomous Driving Software Radar analysis: determination of possible minimum number of testing situation based on percolation theory (Source: own research)

cannot. In our case, the loaded cell is where transport can get space (pavement) ("o" occupied) and the unloaded cell is where it cannot (pavement, divider).

- C → The interpretation of each element (cell) in the dissociated node is following the notation of Fig. 3. The "o" means occupied by a car. The "x" indicates ("one of") the next positions. For the symbol "x" the x,y coordinates are 6,4 and 0 indicates one of the next possible positions. We collect the possible "from" "to" scenes (see later), which is represented in a system where the coordinates "from where" are displayed on the vertical axis and the coordinates "to where" on the horizontal axis. In the cellular system, the transport dynamics of a node are interpreted as creating a "from where to where" system. This is interpreted in Fig. 6, as indicating the "from where" state (coordinates) on the left and the "to where" state (coordinates) on the right.
In the squares on the left and in these cells are the starting coordinates of the vehicle. We need to test this vehicle unit (e.g. radar). That is, we can imagine the whole test process by breaking down the vehicle movements to be tested into elements, atoms, from the initial position (the squares on the left) to the „where" positions on the right. For example, given 4000 test tasks, we can break each of them down into atoms and write down the initial position and the final position. The point is that mathematically the 4000 tests are redundant and there is a subset representing the 4000 tests. This subset is searched by the critical path definition band using percolation theory.
- D → We want to represent the "from where to where" system with bubbles to apply percolation theory. To do this, however, the two cooordinates (e.g. 4,5 from which a test car starts) need to be downdimensionalized, and this transformation is done in step D → the test coordinates are transformed into a single number. For example, (5,6) to 1 and (8,2) to 2 can be used to indicate the tests to be performed. Interpreting the "from where to where" system in a figure, the test-coordinates (from where to where) are shown on the

horizontal axis (from where) and the coordinates on the vertical axis (to where). Characterized by blue dots, which are interpreted as necessary tests in the original system of requirements.

- E → In the dot system, we are now in the classic percolation theory backstage. We can wander around on the blue dots. As percolation theory dictates, we increase the number of dots (the number of tests) then there exists a critical path between the left and right side of the system when we just achieve the breakthrough, we get percolation and we reach the critical number of tests. By increasing the test cases, placing the percolation network at the appropriate coordinates becomes visible. By indicating the test cases in the system (the blue dots) and increasing their number, there is a state when the path marked in red appears in Fig. 5, i.e. we can go from the left side of Fig. 5 to the right side of the path marked in red. Then the new phase is said to appear, and in this state the whole test system can be represented by performing the tests belonging to the line, i.e. it is not necessary to perform the sometimes hundreds of thousands of tests, but it is sufficient to perform only these few tests.
- F → If the percolation breakthrough has taken place on the blue dots, then in the coordinate system of the test experiments the tests (the coordinates) along the percolation path will be selected and these will be the elements of the minimum number of tests selected on the basis of the critical phenomena.

4 Used test data-base

The tests are defined by the customer in the requirements. The possible tests were recorded with the dSpace autonomous driving simulation environment (dSpace (2022) is an Autonomous DrivingSimulation Tool develoded by dSpace GmbH, in Paderborn, Germany). In total, for the radar experiments, more than 4000 experiments were defined and virtually tested and decomposed into elementary units for a given approach situation (Fig. 7).

5 Conclusion

Determination of critical traffic tests based on percolation theory is an important application area. However, the exact determination of the reliability of traffic nodes based on percolation paths is not well established in the literature (Kratmüller, 2010).

The advantages of the method are the following:

1. The percolation path method defines a minimum number of critical test experiments that replaces the entire testing system mathematically in a completely exact and derivable way.
2. With help of dynamic scaling if r = 124 (ms) then $c(124,t) = 82$ and then when $t \to \infty$

$$\alpha = \frac{\lg(124)}{2\lg(82)} = 0.5269 \tag{2}$$

then the extreme value is w = 129.1 constantly [Radar_unit].

The next phase of the project is the fixed-point Matlab C++ Code Generation and its testing with real data (Sun et al., 2020).

6 Future work

Within the framework of the Self-Driving Automotive Platform Project running at the University of Óbuda, we analyzed nonlinear methods for predicting the probable maximums, the so-called extremes of autonomous driving vibration signal of the corner radar sensor time series.

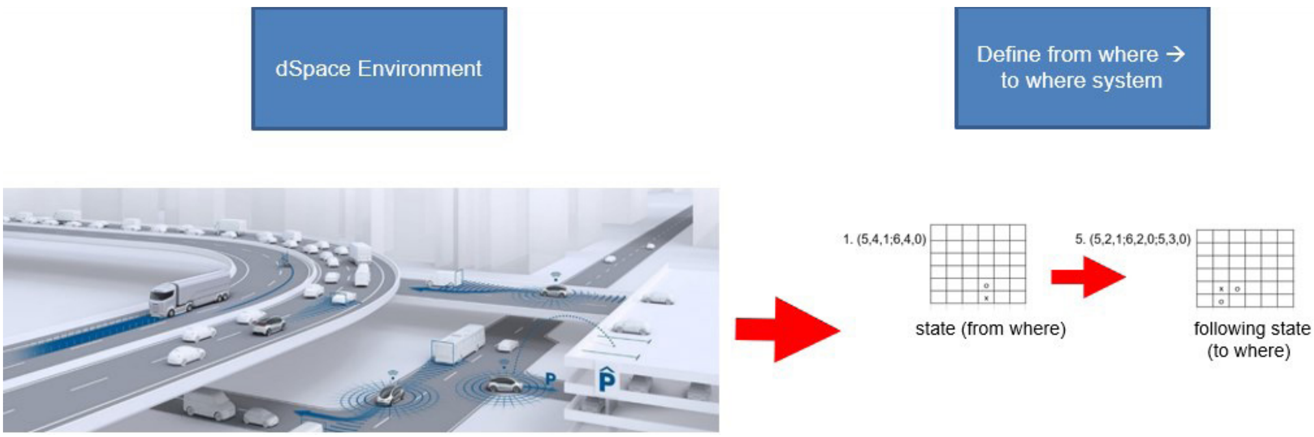

Fig. 7 Óbuda University Autonomous Driving Software Radar analysis in dSpace simulation environment in August 2022 (dSpace, 2022)

The first conclusions are the following:

- Determining the extreme values of a radar signal can be significantly simplified by nonlinear time series analysis, thereby making it embeddable.
- The need for testing can be reduced if collisions can be predicted by analyzing the extreme values of the radar. sensor parameters.
- New test results can be integrated into the system.

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