

# Maneuverability and Movement Stability of Buses of the Especially Large Size Class

Volodymyr Sakhno<sup>1</sup>, Igor Murovanyi<sup>2</sup>, Galina Filipova<sup>1</sup>, Victor Poliakov<sup>1</sup>, Valerii Dembitskyi<sup>2\*</sup>

<sup>1</sup> Department of Automobiles, Faculty of Automotive and Mechanical Engineering, National Transport University, Mykhaila Omelianovycha - Pavlenka Str. 1., 01010 Kyiv, Ukraine

<sup>2</sup> Department of Automobiles and Transport Technologies, Faculty of Transport and Mechanical Engineering, Lutsk National Technical University, Lvivska Str. 75., 43018 Lutsk, Ukraine

\* Corresponding author, e-mail: [dvm2@meta.ua](mailto:dvm2@meta.ua)

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## Abstract

One of the main problems of modern big city is the global crisis of normal urban environment operation as a result of structural growing of automobilization level, oversaturation of road-street network by the transport streams. Buses of the especially large size class occupy, at the least, 20% in the structure of a modern bus park. The buses of especially large size classes, with the aim of necessary maneuverability, as a rule are made articulated.

One of the ways of improving the maneuverability of articulated buses is the usage of guided towed sections. Solving the mentioned problem usually needs the mathematic models for predicting the vehicle behavior.

The aim of introducing the trailer direction is to increase the overall traffic lane of an articulated bus that ensures a necessary maneuverability. The aim is gained by a choice of a rational transmitting ratio of trailer axle gear and necessary value of braking moment on one of its wheels.

The solving of the problem concerning the stability of an articulated bus with a guided trailer axle was made by the way of analyzing the linear approaching of stationary vehicle movement.

An influence of the construction parameters of the bus and the trailer, a transmitting ratio of control gear and a value of braking moment on the parameters of bus maneuverability were analyzed. It was proved that for an articulated bus, the normal maneuverability parameters can be ensured at the trailer direction by a way of braking of one of the wheels.

## Keywords

articulated bus, trailer, control system, maneuverability, stability, critical speed

## 1 Introduction

One of the main problems of modern big cities is the global crisis of normal urban environment operation as a result of structural growing of automobilization level, oversaturation of road-street network by the transport streams. It leads to an abrupt deterioration of indices of public transport service, appearance of traffic jams, increasing noise level and air pollution, a practical decrease of movement speed, an increase of power consumption, an increase in the number of traffic accident victims (Omelnysky, 2018). According to the data of international public transport union, the urban surface public transport needs, at the same carriage capacity, 20 times less surface of road network in comparison with individual passenger cars. A modern bus pollutes the environment 5 times less

and needs 3 times less power consumptions when calculating per one carried passenger in comparison with an individual passenger car (Omelnysky, 2018). Along with this the bus passenger capacity should correspond to a passenger flow. The buses of the especially large size class occupy, at the least, 20% in the structure of a modern bus park. The passenger capacity of especially big two-section buses is from 150 to 200 passengers at general bus lengths of up to 20 m and general weights of up to 28 t. The buses of the especially large size class, with the aim of necessary maneuverability, as a rule are made articulated. An effectiveness of using the buses of the especially large size class greatly depends on the state of public transport network, one of the development possibilities of which is a new bus

system "Bus rapid transport" (BRT). Within this system the rolling stock is called metrobus.

When comparing with the underground railway, the project BRT has evident advantages: less cost of net production, less cost of rolling stock, mobility etc. These advantages become apparent, first of all, at maximum passenger capacity usage of metrobuses and when they travel at maximum speed. As a result, nowadays the urban bus manufacturers design high capacity constructions that consist, as a rule, of separate sections/units articulated with each other. A swing joint makes the long vehicles to be universal in usage and admits a quick maneuvering even in tense urban environment. Such passenger articulated vehicles have found their usage in BRT systems.

Thus, the main advantages of BRT systems are a comparatively low cost of production, the speed of line production, low cost of buses, the possibility to pliantly change the passenger flow due to movement intensiveness, the possibility to partly use a line BRT for another special transport. It can use separate lanes and partly move by the existing roads. In the case of separate lanes, it can develop a high traffic speed in a city. It can have different routes on one lane, in contrast to an underground railway. It decreases the usage of private transport, improves the transport situation and gives a possibility to refuse the short route buses in cities (Omelnytsky, 2018; Michalek, 2019; Hemily and King, 2008).

Along with irrefutable advantages of the articulated buses and trolleybuses, they have got some disadvantages – these are worse maneuverability and stability of movement when comparing with single-section buses. That is why the improvement of maneuverability and stability of articulated buses is actual. Solving the mentioned problem usually needs mathematic models for predicting the vehicle behavior. The models should be flexible in using, reliable and compact enough to be easily analyzed by the on-board (built-in) computing device. A module approach to modelling shown in a research work (Michalek, 2019) allows to build the compact non-holonomic kinematic models of multi-section buses that include an active bus and an arbitrary number of trailers, articulated by the passive turning joints and immobile or guided wheels of trailers.

## 2 Literature review

In a research work (Gottmann et al., 2018) the results of complex investigation of stability of a linear single-track model of two-sectional articulated vehicle in a plane-parallel movement are given. Two parts of a vehicle model are connected in a coupling point with the help of special

original joint. The equations of movement are made in analytical form that allows to examine the non-linear models (non-linear viscoelastic characteristicly functions of coupling). The most important parameters that control the beginning of a movement stability interruption, and the role played by an equivalent coefficient of rotation decreasing and the equivalent torsion hardness that characterize a coupling, are derived with the aim to find out its design criteria. In their research work (González et al., 2016) offered a uniform model that includes the dynamic of turn of an articulated vehicle and for any axle of both a tractive vehicle (bus) and a trailer section. In the model all the possible existing bus configurations, including a swing joint, a layout of power unit and a disposal of active chassis are presented. Such a research is motivated by a key question: how to develop a uniform model that would be inclusive and adjusted for all before-mentioned vehicle configurations? To develop a model step by step three layers of modelling process are presented. A cooperation of a tire with a supporting surface is described by a magic formula including a change of vertical load. An articulated bus is presented as a system of differential equations. A model variation is showed for expediency, effectiveness and convenience of the offered approach that can be adjusted for any articulated vehicle while calculating the stability parameters in a rectilinear movement. In a research work (Ljungqvist et al., 2019) a single-track dynamic model of an articulated bus was offered on the basis of which there are the predictions that the transversal accelerations of the front body part and the angles of turn of an articulated bus are small. Besides, the friction and the clearances in the swing joint are not taken into consideration. A system is intended for a strict description of a forward motion and of a rotation of a vehicle based on the differential equations of kinematic parameters that present the articulated buses. To calculate the equations, MATLAB-Simulink was used. The modelling results are the functions of kinematic and dynamic parameters that allow to determine the trajectory and the trajectory width of an articulated bus. The received results are a basis for the strict evaluation of the dynamic model and for the exploration of articulated bus dynamics on a higher and more complicate level. In a research work (Tian et al., 2020) the three-dimensional dynamic models of a vehicle and a trailer were elaborated, on the basis of which a train dynamic model was built. On the basis of an approximation theory of first order of the ordinary differential equations and the Hopf bifurcation theory, a linear and non-linear stability of every element and of the articulated vehicle in a total rectilinear movement

was explored. A lot of results show that for non-linear and linear models the critical speeds differ from each other a little. In a research work (Altafini, 2001) the equations of vertical and lateral dynamics of a vehicle with 6 degrees of freedom led to a matrix view. The movement of such instrument in vertical and lateral planes were investigated. There was shown that an elaborated method was applied for analysis of movement stability, especially for passenger trains. In a research work (Zakin, 1986) a multiversion extension of a method D2-IBC (Data Driven – Inversion Based Control) and its application for articulated vehicle stability control was observed and discussed in detail.

When developing any type of bus, including a metrobus of extralarge capacity, the mass and geometrical restrictions, predicted in the regulatory documents should be taken into consideration. Thus, according to Regulations No. 36-03 "Common technical provisions concerning an official approval of the passenger vehicles of large capacity regarding to a general construction" (Sakhno et al., 2021) an inner radius of turn should be 5.3 m, and the most exposed point from the center of a bus turn should describe an arc of radius of 12.5 m. These restrictions can be realized by an articulated bus (AB) of 18.5 m length. While increasing the general length of AB for more than 18.5 m, steering by the wheels of the trailer section is necessary. But the buses with a steerable rear axle are disposed to the transversal deviations of the trailer that should be taken into consideration while choosing and proving the arranging and mass parameters of AB. That is why during a practical realization of AB with a steerable trailer, there should be chosen a compromise law on the basis of complex analysis of different criteria of steering and their influence on the dynamic behavior of the articulated vehicle in both permanent and unstable movement modes.

The aim of the research work is to improve the parameters of maneuverability and stability of movement of an articulated bus with a steerable trailer section due to a choice of the rational arranging and mass parameters of separate sections and their control systems.

### 3 Research results

Previously it was determined that an articulated bus can be competitive with a metro at passenger capacity within 180–200 passengers. At such passenger capacity an overall length of a bus should be within 21.5–22.5 m. To regularly load the axles of an articulated bus, passenger distribution between the bus and the trailer should be within 2:1.5 at a bus base area within 5.5–6.5 m and a trailer base area – within 5.0–6.0.

An analytical model of AB is shown in Fig. 1.

In a driving section of an AB (articulated bus) the front axle has the steerable wheels, the middle angle of turn of which  $\theta$  is equal to a half-sum of the angles of turn of external and internal wheels. The rear axle is situated behind the bus mass center. The trailer section of an AB leans on an axle whose wheels can be both non-turning and turning (the most common case refers to the turning wheels). The angle of turn of these wheels is marked as  $\theta_2$ . Besides, to correct the trajectory of the trailer concerning the trajectory of the bus during a movement of the AB by the curvilinear trajectories a braking moment is used on one of the wheels of the trailer axle.

There is a known system of equations that describes a movement of an AB in an unsettled turn (Verbitskiy et al., 2013), supplemented by a braking moment on one of the trailer axle wheels which is written as:

$$\begin{aligned}
 & (m + m_2) \dot{V} - m_2 d_1 \sin \phi_1 \dot{\omega} + m_2 l_1 \sin \phi_1 \ddot{\phi}_1 \\
 & + m \omega U + m_2 \omega (U + \omega c) + m_2 l_1 \times \omega_1^2 \cos \phi_1 \\
 & = Y_1 \sin \theta - X_1 \cos \theta + Y_1' \sin \theta \\
 & - X_1' \cos \theta - \sum (X_{11} + X_{11}') \\
 & + \sum [(Y_2 + Y_2') \sin \theta_2 - (X_2 + X_2') \cos \theta_2] \\
 & - X_{2ran} \cos \theta_2; \\
 & (m + m_2) \dot{U} - (cm_2 + m_2 d_1 \cos \phi_1) \dot{\omega} \\
 & + m_2 l_1 \cos \phi_1 \ddot{\phi}_1 + (m + m_2) \omega V - m_2 d_1 \omega_1^2 \\
 & \times \sin \phi_1 = Y_1 \cos \theta - X_1 \sin \theta + Y_1' \cos \theta \\
 & - X_1' \sin \theta + (Y_{1i} + Y_1') + (Y_2 + Y_2') \cos \theta_2 \\
 & - (X_2 + X_2') \sin \theta_2 - X_{2ran} \sin \theta_2 \\
 & - cm_2 \dot{U} + (I + c^2 m_2) \dot{\omega} + cm_2 d_1 \cos \phi_1 \ddot{\phi}_1 \\
 & - cm_2 \omega V + cm_2 d_1 \omega_1^2 \sin \phi_1 = (X_1 \sin \theta - Y_1 \cos \theta) \\
 & \times (\varepsilon \sin \theta + a) + (Y_1' \cos \theta - X_1' \sin \theta (a + \varepsilon \sin \theta)) \\
 & + (Y_1 \sin \theta + X_1 \cos \theta) (H + \varepsilon \cos \theta) \\
 & - (Y_1' \sin \theta + X_1' \cos \theta) (H + \varepsilon \cos \theta) \\
 & - c(Y_{11} + Y_{11}') b_2 - c(Y_2 + Y_2') \cos \theta_2 \\
 & - (X_2 + X_2') \sin \theta_2 - X_{2ran} \sin \theta_2 \times B/2 \\
 & + M_{cT1} + M_{p1} + M_{h1}; \\
 & m_2 d_1 \sin \phi_1 \dot{V} + m_2 d_1 \cos \phi_1 \dot{U} \\
 & - [I_1 \dot{\omega} + (I_2 + m_2 d_1^2)] \ddot{\phi}_1 \\
 & + [V \cos \phi_1 - (U - \omega c) \sin \phi_1] m_2 d_1 \\
 & = \sum l_1 (X_2 + X_2') \sin \theta_2 + (Y_2 + Y_2') \cos \theta_2 \\
 & + M_{cT2} + M_{p2} + M_{h2} - M_{ran}.
 \end{aligned} \tag{1}$$

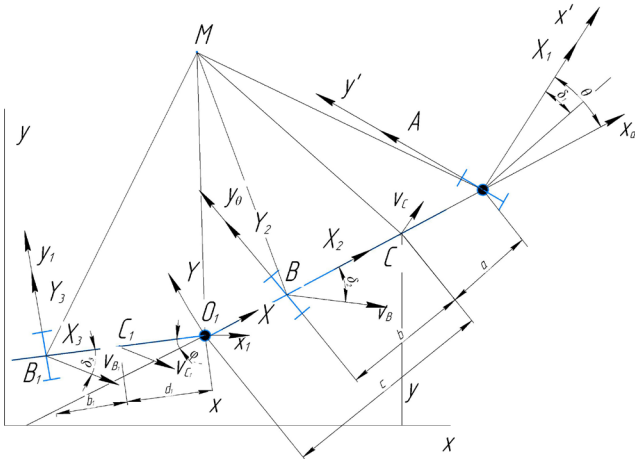


Fig. 1 Scheme of an unsettled turn of AB

where:

- $m, I$ : the mass and the central moment of the bus section concerning a vertical axle;
- $m_2, I_2$ : the same for the trailer;
- $\vartheta, \vartheta_1$ : the course angles of the bus and the trailer;
- $\phi_1$ : the folding angle between the bus and the trailer;
- $v, u, v_2, u_2$ : the longitudinal and lateral projections of speed of the mass center of the bus and of the trailer on an axle of a moving frame that are continuously linked with them;
- $\omega$ : the angle bus speed;
- $M_{\text{tra}} = f(\varphi_K, \dot{\varphi}_k)$ : the braking moment on the trailer axle wheel that is used to correct the articulated vehicle movement trajectory;
- $X_{1,2,3}, Y_{1,2,3}, X'_{1,2,3}, Y'_{1,2,3}$ : the longitudinal and lateral reactions of the road bed on the wheels on the left and right side of the bus and the trailer.

The interaction of the wheels with the area of bearing in lateral direction is described by a reaction of the road bed as a function of the withdrawal angle by non-linear hypothesis:

$$Y = k_1 \delta_1 - k_2 \delta_2 + k_3 \delta_3. \quad (2)$$

The stabilizing moments of tires are also described as a non-linear dependence from the withdrawal angle:

$$M_c = \sigma_1 \delta_1 - \sigma_2 \delta_2 + \sigma_3 \delta_3, \quad (3)$$

where  $k_1, k_2, k_3, \sigma_1, \sigma_2, \sigma_3$  are the given characteristics of the wheels of an axle KKM.

The moments of the viscous friction in the steering are proportional to the angles of turn of the given wheels:

$$\begin{aligned} M_{h1} &= h_1 \times \dot{\theta} \\ M_{h2} &= h_2 \times \dot{\varphi} \end{aligned} \quad (4)$$

where:

- $h_1$  and  $h_2$ : coefficients of the viscous friction in the steering details;
- $M_{p1}, M_{p2}$ : moments of elasticity in the steering of front and rear axles are proportional to the angles of turn of the given wheels.

$$\begin{aligned} M_{p1} &= \chi_1 \times \dot{\theta} \\ M_{p2} &= \chi_2 \times \dot{\varphi} \end{aligned} \quad (5)$$

where  $\chi_1$  and  $\chi_2$  are the coefficients of rigidity of the actuator.

The resistance coefficient to the lateral withdrawal  $k$  is calculated as:

$$k = k_o \frac{\sqrt{1 - (X/\varphi G)^2}}{1 + 0.375 X/G}, \quad (6)$$

where:

- $k_o$ : the resistance coefficient to the lateral withdrawal in the case of absence of longitudinal forces on the wheel;
- $G$ : the vertical load on the wheel;
- $X$ : the value of longitudinal force that is given by correlation.

$$X = \begin{cases} M_T/r, & \text{if } M/r < \varphi G \\ \varphi G, & \text{if } M/r \geq \varphi G \end{cases}, \quad (7)$$

where  $M_T$  is the braking moment applied to the trailer wheel.

The withdrawal angles of the axles of the AB are written as:

- for KKM of the bus:

$$\delta_1 = \theta - \arctg \frac{u + \omega(a - c \times \cos \theta) - c \times \dot{\theta} \times \cos \theta}{V + c(\omega + \dot{\theta}) \sin \theta}, \quad (8)$$

- for bus skeleton with unsteerable axle:

$$\delta_2 = \arctg \frac{-u + b\omega}{v - \varepsilon\omega}, \quad (9)$$

- for steerable trailer axle:

$$\delta_3 = -\arctg \frac{V \sin \varphi + (u - \omega \varepsilon) \times \cos \varphi - (\omega - \dot{\varphi}) d}{V \cos \varphi - (u - \omega \varepsilon) \sin \varphi} - \theta_2. \quad (10)$$

The values  $V, U, \omega, \varphi_1$  can be found out of the dynamic equations of the articulated vehicle. The mass center coordinates and the course angle of the bus are calculated with the help of the kinematic equations:

$$\begin{aligned} \dot{x} &= V \cos \theta - U \sin \theta \\ \dot{y} &= V \sin \theta + U \cos \theta \\ \dot{\theta} &= \omega \end{aligned} \quad (11)$$

The purpose of introduction of the trailer control is to decrease the overall traffic line of an AB that ensures a necessary maneuverability, for example at a circular motion, for what the following equation must be satisfied  $R_B = R_E$ . The speed ratio of the control-gear of pivoted axle of (wheels of) the trailer (Fig. 2) is calculated in such a succession.

By solving the system of linear equations that determine a stationary mode, one can get the next values of phase variables in a general view (Verbitskiy et al., 2013):

$$u = \frac{k_1 (-v^2 (m a L_1 + m_1 b_1 (a + c)) + l b k_2 L_1) v}{v^2 \begin{pmatrix} (b k_2 - a k_1) (m L_1 + m_1 b_1) \\ -c m_1 b_1 (k_1 + k_2) \end{pmatrix} + l^2 k_1 k_2 L_1 + 2 k_1 k_2 L_1 a b} \times \theta;$$

$$\omega = - \frac{k_1 k_2 l L_1 v}{\begin{pmatrix} (k_1 a - k_2 b) L_1 m \\ + \begin{pmatrix} (a + c) k_1 \\ + (c - b) k_2 \end{pmatrix} b_1 m_1 \end{pmatrix} v^2 - k_1 k_2 l^2 L_1} \times \theta;$$

$$\varphi = - \frac{\begin{pmatrix} \begin{pmatrix} (-b_1 k_3 (a + c) + d_1 k_2 l) m_1 \\ -k_3 a m L_1 \end{pmatrix} v^2 \\ + ((l - c) a + (b + c) b - l) k_2 k_3 L_1 \end{pmatrix} k_1}{(1 + \text{PCH}) k_3 \begin{pmatrix} \begin{pmatrix} (m_1 b_1 + m L_1) \\ \times (k_2 b - k_1 a) \end{pmatrix} v^2 \\ - m_1 k_2 L_1 c \\ + k_1 k_2 l^2 L_1 \end{pmatrix}} \times \theta.$$

By the found values of phase variables the parameters of movement trajectory of characteristic points of the trailer can be determined:

$$\begin{aligned} R_D &= \frac{v}{\omega}; \quad L_D = \frac{u}{\omega}; \quad R_{D1} = \frac{v_1}{\omega}; \quad L_{D1} = \frac{u_1}{\omega}; \\ R_B &= \sqrt{R_D^2 + (CB - L_D)^2}; \\ R_E &= \sqrt{R_{D1}^2 + (C_1 E - L_{D1})^2}. \end{aligned} \quad (12)$$

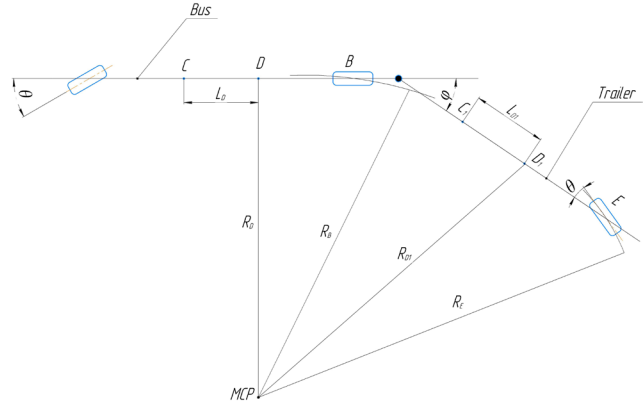


Fig. 2 Scheme of AB with a steerable trailer

When considering either the movement speed (while determining the parameters of maneuverability it is accepted that  $v = 5$  m/s), or the turn angle of the steerable bus wheels ( $\theta = 0.75$  rad) as stable, one can calculate the speed ratio of the control-gear of the axle (wheels) of the trailer (PCH).

As the outgoing data the next parameter values are taken:  $g = 9.81$ ;  $a = 3.40$ ;  $b = 2.0$ ;  $b_1 = 2.085$ ;  $d_1 = 2$ ;  $c = 4.39$ ;  $L = 22.0$ ;  $m = 16,000$ ;  $k_f = 0$ ;  $m_2 = 9,900$ ;  $k_1 = 160,000$ ;  $k_2 = 326,000$ ;  $v = 5$ ;  $k_3 = 165,000$ ;  $kk_1 = 2,600$ ;  $kk_2 = 200$ ;  $h_1 = 30$ ;  $h_2 = 30$ ;  $\varphi = 0.8$ ;  $\theta = 0.5$  rad;  $\theta_2 = \kappa \times \theta$  rad;  $\kappa = 0, 0.05, 0.10, 0.15, 0.20$ ;  $\varphi_1 = 0$ ;  $k_0 = 2.1$ ;  $\sigma_1 = \sigma_2 = 20$ .

The next graphs of change of the speed ratio of the trailer control-gear correspond to the numerical values of these AB parameters, Fig. 3.

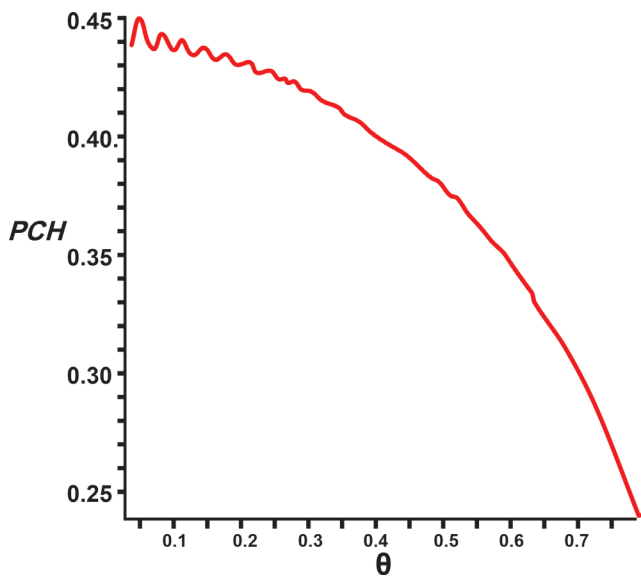
#### 4 Discussion and interpretation of received results

Based on the results of analysis of rational values of speed ratios, that ensure an approximate coincidence of the characteristic points trajectories of the vehicle-tractor and the semi-trailer, a value of a speed ratio (PCH) = 0.45 was chosen, that ensures a minimum value of overall traffic lane (OTL).

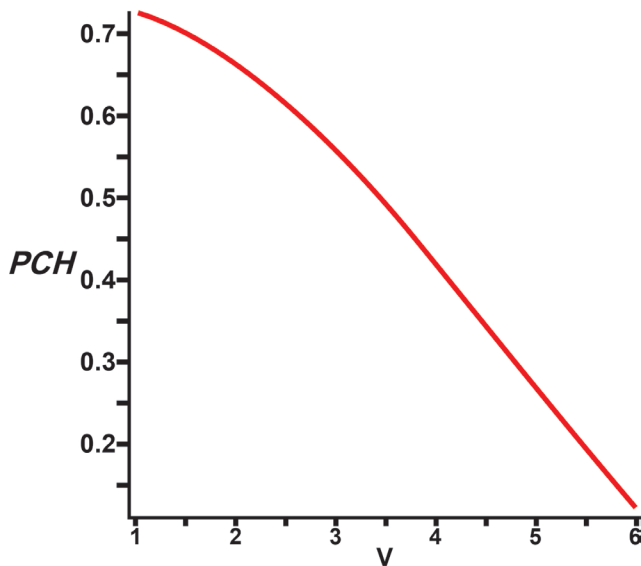
At a determined speed ratio of the axle (wheels) control gear of the trailer, an integration of system of Eq. (8) was made. At integration the radiuses of the trajectories of bus and trailer characteristic points were determined using control system 1, that includes kinematic and dynamic (braking of one of the wheels) ways, and using control system 2, that includes braking of one of the trailer wheels, Fig. 4.

Using the mass center trajectory parameters of the bus and the trailer, an OTL was determined of angles of folding and turn of the trailer's steerable wheels, of a value of braking moment on the wheels of one trailer side, while performing different maneuvers by the AB, Table 1.



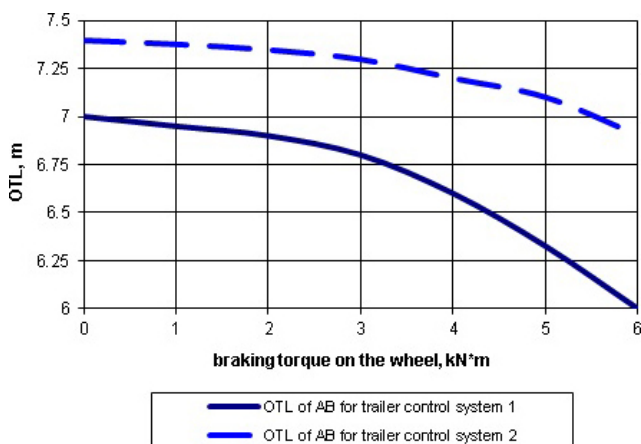


(a)



(b)

**Fig. 3** Change of the speed ratio of the control-gear of a semi-trailer axle; (a) dependence of  $\theta$  (at a speed of  $v = 5$  m/s); (b) dependence of  $v$  (at a turn angle of  $\theta = 0.75$  rad)



**Fig. 4** OTL of AB at different trailer control systems

**Table 1** OTL of AB at different trailer control systems

Type of AB	Overall traffic lane (m)		
	Movement in a circle	Turn at 90°	Turn at 180°
With a non-steerable trailer	7.4/7.2*	6.9/6.6*	7.2/7.0*
With a steerable trailer	7.0/6.8*	6.5/6.3*	6.8/6.5*
With braking of trailer wheels	7.0	6.6	6.9
With steering and braking of trailer wheels	6.8	6.4	6.6

Note: \* OTL for AB on the laterally rigid wheels

Analyzing the graphs in Fig. 4, and the data of Table 1 shows that it is possible to control an AB trailer not only by steering its steerable wheels, but also by braking of wheels on one side. Along with that a normative value of OTL is reached at a braking moment on one of the wheels within 5.5 kN m.

The effect of trailer control by partial braking of wheels on one side is equivalent to, in first approximation, the effect from the appearing moment of friction forces. Along with that, the value of the turn radius of the trailer section depends not only on the turn angle of its wheels, but on the value of braking moment and the wheel on which it is applied. It is very important for the construction of hybrid buses with electrical drive on an axle of the trailer section, because in such a way it is possible to correct not only the trajectory of the AB sections, but also the stability of an articulated bus.

While investigating such a difficult dynamic system steerable by an axle of the AB trailer, it is keenly necessary to examine its movement stability, and the folding resistance is added to the traditional sidewise skidding and rollover stability for single vehicles. The steerable articulated vehicles have characteristics that are not allowed for separate vehicles. In general, the stability of the whole system is determined not only by the stability of separate sections, but it is necessary to make an analysis, taking into consideration the influence of the section on each other. But it is related to a calculating of differential movement equations of sixth order.

To solve the problem of AB stability with a steerable trailer axle, it is necessary to make an equation of disturbed movement. According to Lyapunov, in uncritical cases it is possible to restrain by an analysis of linear approaching of an articulated vehicle's stationary movement, for which the system of Eq. (8) allows the solution  $U = 0$ ,  $\omega = 0$ ,  $\varphi_1 = 0$ , ( $\theta = 0$ ) and to which, on a road plane, corresponds to the movement of all the points of the articulated vehicle

with a speed  $v$  along a straight  $\theta = const$ . Let's accept such movement as undisturbed. For such movement the critical speed of the AB is determined as:

$$v_{kp}^2 = k_t (1 - \lambda) [k_1 k_2 (1 - \lambda) + k_1 \sigma_2 - k_2 \sigma_1] F^{-1}, \quad (13)$$

where:

$$F = k_t [m(k_1 a - k_2 b) - (m + m_1)(\sigma_1 + \sigma_2)] - m_1 k_2 (1 - \lambda) - m(k_1 \lambda + \sigma_1) \times (k_2 b + \sigma_2) - m_1 \sigma_1 (k_2 l + \sigma_2).$$

By an appropriate choice of rigidity  $k_t \leq k_{t*}$  of an operating wheel module of the bus at non-steerable wheels of the trailer, one can reach an absence of critical speed that is characteristic for modern vehicle constructions:

$$k_* = \frac{m(k_1 \lambda + \sigma_1)(k_2 b + \sigma_2) + m_1 \sigma_1 (k_2 l + \sigma_2)}{m(k_1 a_1 - k_2 b) - (m + m_1)(\sigma_1 + \sigma_2) - m_1 k_2 (l - \lambda)}. \quad (14)$$

For an AB with a steerable axle (wheels) on the trailer, it is possible to increase the critical movement speed by a choice of a caster effect  $\lambda$ . Taking into consideration the results of a research work (Verbitskiy et al., 2013) for the caster effect of trailer wheels, one can get:

$$\lambda = \frac{k_t [m(k_1 a_1 - k_2 b) - (m + m_2)(\sigma_1 + \sigma_2) - m_2 k_3 l] - \sigma_1 [m(k_1 b + \sigma_2) + m_2 (k_3 d_1 + \sigma_2)]}{k_t (k_1 m - k_3 m_2) + m k_1 (k_3 d_1 + \sigma_2)}.$$

In Fig. 5 the dependence of critical speed of the AB from the caster effect of trailer wheels is shown.

The research of influence of construction and exploitation factors on the maneuverability and stability of AB movement was performed, an evaluation of which was effected according to the critical movement speed using a method of experiment planning (Verbitskiy et al., 2013).

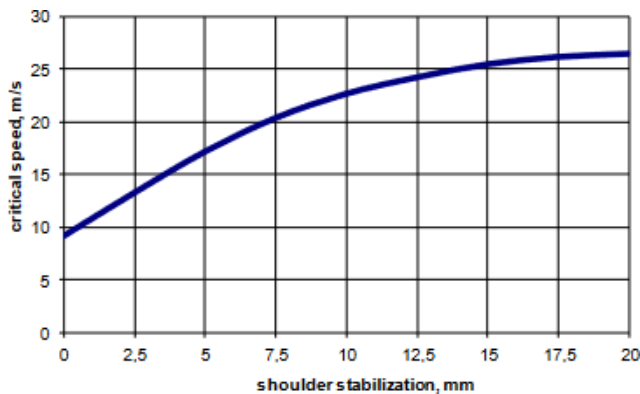


Fig. 5 Dependence of critical speed of AB from the caster effect of trailer wheels

The research was made in two stages, 12 factors were chosen that correspond to the conditions of independence, steering attitude and unambiguity, at the same time their totality corresponds to the compatibility and absence of linear correlation.

In the first stage, by the principle of full factor experiment, four factors were chosen: the trailer base area, the trailer weight, the turn angle of trailer wheels and the resistance coefficient to lateral extraction of trailer axle wheels. The check of significance of the coefficients of polynomial model by Student's t-test showed that the coefficients of double action can be neglected. That is why it is possible to use a one-factor experiment in the second stage, while determining the influence of the factors on maneuverability and stability of the AB.

In a purpose to extend the research results for ABs of different arranging schemes, calculations using the dimensionless parameters were made.

A range of change of the dimensionless parameters was chosen taking into consideration the construction specialities of existing ABs and the requirements to the geometric and mass parameters that are valid in EU countries.

The influence of the following parameters that changed within the chosen limits was investigated:

- Geometric parameters:

$$a' = \frac{a}{L} = 0.2 - 0.8;$$

$$L' = \frac{L}{D} = 0.35 - 0.80;$$

$$c' = \frac{c}{L} = 0.0 - 0.4;$$

$$d' = \frac{d}{L} = 0.0 - 0.8;$$

$$L'_2 = \frac{L_2}{D_2} = 0.13 - 0.38;$$

where  $L, L_1$  are the bus and trailer base area;  $D, D_2$  are the general length of bus and trailer.

- Mass parameters of bus and trailer:

$$m' = \frac{m}{m + m_2} = 0.35 - 1.10;$$

$$m'_2 = \frac{m_2}{m + m_2} = 0.35 - 0.80;$$

- Resistance coefficients to lateral extraction of the tires of the articulated vehicle axles:

$$k'_1 = \frac{k_1}{k_0} = 0.6 - 1.2;$$

$$k'_2 = \frac{k_2}{k_0} = 0.6 - 1.5;$$

$$k'_3 = \frac{k_3}{k_0} = 0.6 - 1.5.$$

According to the calculation results of influence of geometrical and mass factors on OTL, the following conclusions were made:

- the bus and trailer base areas have the most important influence on OTL value. An increase in these factors increases the OTL of the articulated vehicle.
- such factors as the position of the mass centers of the articulated bus sections and the position of the coupling point of bus and trailer, the mass parameters of the articulated vehicle and the resistance coefficients to extraction of its wheels have a non-significant influence.

The analogical calculations were made also for the examination of influence of construction and exploitation factors on the indices of articulated bus movement stability. According to the results of calculations the next conclusions were made:

- the base area of the trailer and its length practically do not influence the CMS (critical movement speed) of the articulated vehicle;
- the bus weight practically does not influence the CMS;

## References

- Altafani, C. (2001) "Some properties of the general n-trailer", *International Journal of Control*, 74(4), pp. 409–424.  
<https://doi.org/10.1080/00207170010010579>
- González, D., Pérez, J., Milanés, V., Nashashibi, F. (2016) "A Review of Motion Planning Techniques for Automated Vehicles", *IEEE Transactions on Intelligent Transportation Systems*, 17(4), pp. 1135–1145.  
<https://doi.org/10.1109/TITS.2015.2498841>
- Gottmann, F., Wind, H., Sawodny, O. (2018) "On the influence of rear axle steering and modeling depth on a model based racing line generation for autonomous racing", In: 2018 IEEE Conference on Control Technology and Applications (CCTA), Copenhagen, Denmark, pp. 846–852. ISBN 978-1-5386-7699-8  
<https://doi.org/10.1109/ccta.2018.8511508>
- Hemily, B., King, R. D. (2008) "Uses of higher capacity of buses in transit service", The National Academies Press. ISBN 978-0-309-09804-5  
<https://doi.org/10.17226/13919>
- Ljungqvist, O., Evestedt, N., Axehill, D., Cirillo, M., Pettersson, H. (2019) "A path planning and path-following control framework for a general 2-trailer with a car-like tractor", *Journal of Field Robotics*, 36(8), pp. 1345–1377.  
<https://doi.org/10.1002/rob.21908>
- Michalek, M. M. (2019) "Modular approach to compact low-speed kinematic modelling of multi-articulated urban buses for motion algorithmization purposes", In: IEEE Intelligent Vehicles Symposium (IV), Paris, France, pp. 2060–2065. ISBN 978-1-7281-0561-1  
<https://doi.org/10.1109/IVS.2019.8814281>

- a decrease in the trailer weight increases the CMS of the articulated vehicle;
- the CMS of the articulated bus mostly depends on the rigidity of tires of the rear bus axle;
- an increase in the resistance coefficients to lateral extraction of the tires of the articulated vehicle front axle causes an increase in the CMS;
- an increase in the tires' rigidity of the trailer axle decreases the CMS of the articulated vehicle.

While regulating the air pressure in the tires of an articulated vehicle, it is possible to change the resistance coefficient to lateral extraction and in such a way to increase the CMS of the articulated vehicle.

## 5 Conclusions

A mathematical model of an articulated bus with a trailer section control system for both kinematic and dynamic way of turning was developed. According to the developed model the indices of maneuverability and stability of bus movement were determined. The influence of construction parameters of the bus and trailer were analyzed, i.e., the speed ratio of steering actuator and the value of braking moment on the indices of the maneuverability. It was proved that for an articulated bus of a total length of 22 m, the normed indices of maneuverability can be assured by trailer control by the way of braking of one of the wheels. The value of braking moment was found. The results of investigations of the maneuverability and stability of movement and the different ways of trailer control can be used when developing a hybrid articulated bus of a total length of 22 m for its application in a BRT system.



- Omelnytsky, O. E. (2018) "What Kind of Transportation can be an Alternative to the Metro in Ukraine and Quickly Push Out "Minibuses" from Our Cities", *Avtoshlyakhovyk Ukrainy*, 3, pp. 7–11.  
<https://doi.org/10.33868/0365-8392-2018-3-255-7-11>
- Sakhno, V. P., Polyakov, V. M., Sharay, S. M., Murovanyu, I. S., Omelnytskyu, O. Y. (2021) "Шарнірно-зчленовані автобуси. Маневреність та стійкість: монографія" (Articulated buses. Maneuverability and stability: monograph), ІВВ Луцького НТУ, Lutsk, Ukraine. (in Ukrainian)
- Tian, X., He, R., Sun, X., Cai, Y., Xu, Y. (2020) "An ANFIS-Based ECMS for Energy Optimization of Parallel Hybrid Electric Bus", *IEEE Transactions on Vehicular Technology*, 69(2), pp. 1473–1483.  
<https://doi.org/10.1109/TVT.2019.2960593>
- Verbitskiy, V. G., Sakhno, V. P., Kravchenko, A. P., Kostenko, A. V., Danilenko, A. E. (2013) "Автомобили. Устойчивость: Монография" (Cars. Stability: monograph), Ноулджд, Lugansk, Ukraine. (in Russian)
- Zakin, Y. (1986) "Маневренность автомобиля и автопоезда" (Maneuverability of the car and road train), Транспорт, Moscow, Russia. (in Russian)