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The Relationship Between Loading Frequency and Truck Speed on Asphalt Pavements

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Abstract

The mechanistic-empirical road pavement design methods necessitate knowledge of the dynamic modulus master curve of asphalt mixtures. This requires the designer to accurately define both the load frequency and the temperature of the pavement structure. The parameter of load frequency is of great importance in laboratory fatigue tests. However, it presents a challenge since road structures are loaded in the time domain, not the frequency domain. This complexity has prompted researchers to develop time-frequency conversion methods to estimate the corresponding frequency from the duration of traffic-induced stress or deformation pulses. Recent findings indicate that the time-frequency equivalence factor is considerably lower than previously assumed. In this study, we introduce a new four-parameter empirical RAMBO material model, derived from the Ramberg–Osgood equation, which offers parameters to determine the time-frequency equivalence factor for asphalt mixtures. The relationship between the moving wheel load velocity and the load impulse time is analysed, primarily based on experimental data and computational methods from existing literature. A method for determining the wavelength of the load impulse is proposed. The theoretical correlation indicates that a wheel load travelling at 100 km/h results in a dynamic deformation frequency of 10 Hz. However, for a strain impulse, the same speed corresponds to a much lower frequency of 3–4 Hz. This study offers a refined understanding of the time-frequency equivalence factor, which is of crucial importance for more accurate road pavement design and fatigue testing.

Keywords

wheel load, time-frequency, impulse, master curve, conversion, material model

1 Introduction

It is widely known to experts in the field of flexible pavement structures that the viscoelastic behavior of asphalt mixtures is highly dependent on temperature and load frequency. At low temperatures and high load frequencies, the elastic behavior is more dominant, whereas at high temperatures and low load frequencies, the viscous behavior is more dominant. Therefore, mechanistic based pavement design procedures now require knowledge of the dynamic modulus master curve of asphalt mixtures for the design. The dynamic modulus master curve is generated from laboratory test data at different temperatures and frequencies. The dynamic modulus captures well the behavior of viscoelastic materials under periodic excitation (Pellinen et al., 2003). However, road structures are subjected to the loads of moving vehicles in the time domain rather than in the frequency domain (Romeo et al., 2024). The duration of wheel loading, and hence the frequency of loading, depends on traffic speed, tire contact area and shape, load magnitude, and roadway structure (Cheng et al. (2022) in their work: *Fatigue Test Setups and Analysis Methods…*). The interaction of these factors ultimately results in the desired load frequency, which is not an easy task to determine. The load frequency is a crucial set-up parameter for laboratory fatigue tests, since it influences the stiffness modulus of the asphalt mixture being tested (Pereira et al., 1997). It is evident that its accurate determination is particularly important for mechanistic-empirical road pavement design methods. The structure of the load frequency estimation models known from the literature is very similar, firstly, they provide the relationship between

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the wheel load velocity V and the impulse time t_p in the structure, and then they incorporate a time-frequency conversion method. For example, the load time used in the MEPDG (Mechanistic-Empirical Pavement Design Guide) is equal to the impulse time of the vertical compressive stress in the pavement structure on site due to the moving wheel load, as shown in Fig. 1. In dynamic laboratory deformation tests, however, the loading time is given by the period time of continuous sine or haversine loading waves, which differs to some extent from the field condition considered in the MEPDG method.

Researchers have developed different time-frequency conversion methods to accurately calculate the corresponding frequency from the time duration of the stress or strain pulses induced by the traffic. Among these conversion methods, the $f = 1/t_p$ and $f = 1/2\pi t_p$ equations are widely used to calculate frequency, where f is the frequency in Hz and t_p is the impulse time in s (Shafiee et al., 2015). Many research papers suggest that the second approach is the most appropriate, as it is widely used in the field of rheology. Nevertheless, so far, there is no consensus among researchers on the issue of direct conversion between time and frequency domains (Al-Qadi et al., 2008; Christensen, 1982; Dongre et al., 2006; Katicha et al., 2008; Sias Daniel and Richard Kim, 1998). The most recent results on this issue are found in a study by Romeo et al. (2024), where the optimal conversion factor for the four-parameter sigmoid function introduced by Witczak and Fonseca (1996) is given as $f \approx 0.0673/t_p$, based on data from 30 asphalt samples. This represents a much lower loading frequency than is usual in the literature. This raises the possibility that the existing models give distorted frequency values due to inaccurate time-frequency conversion factors. The issue is further complicated by the fact that the vertical σ_z compressive stress and the longitudinal ε_y strain loading times are significantly different. Fig. 2 clearly shows that the impulse time between points 2–3 is much smaller than the one defined by points 1–4. Therefore, a universal loading frequency cannot be given for the entire road structure, only for selected structural response.

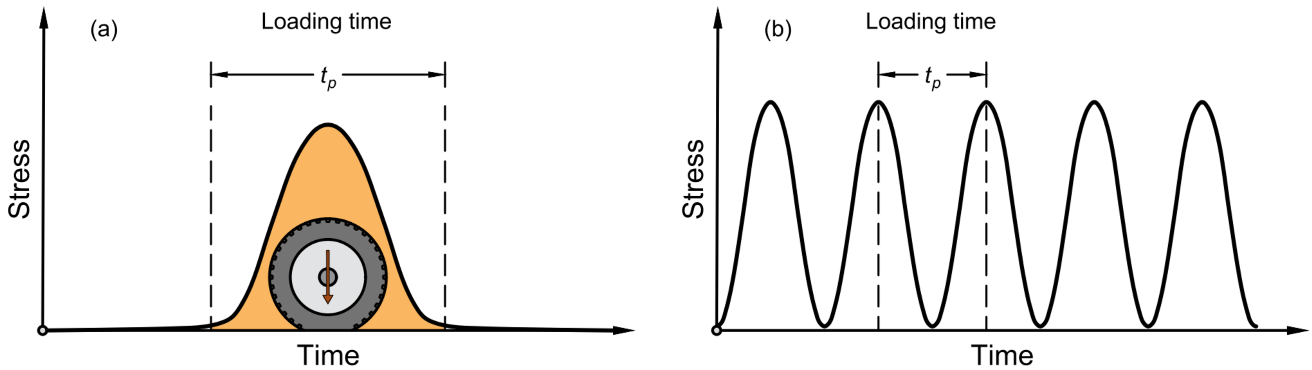

Fig. 1 Schematic diagram of loading time determination: (a) in MEPDG and (b) in laboratory tests (Cheng et al. (2022) in their work: *Effects of Using Different Dynamic Moduli…*)

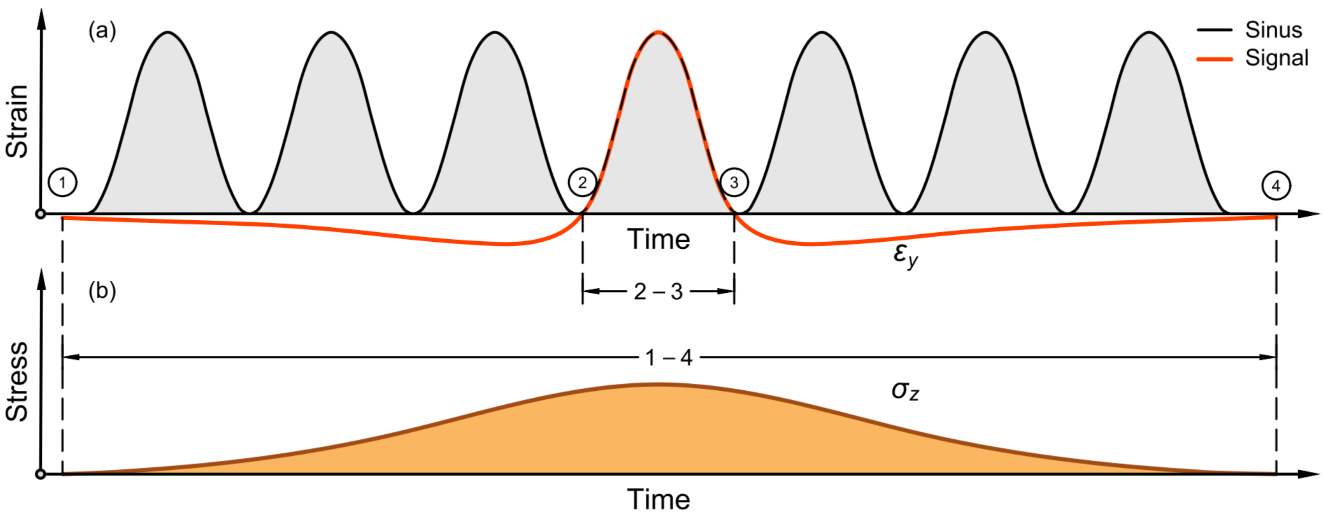

Fig. 2 Interpretation of loading time as a function of structural response: (a) asphalt loading periods and associated stretches and (b) maximum of the vertical compressive stress's impact interval on the subbase

To solve this problem, we present a new 4-parameter empirical RAMBO material model based on Ramberg-Osgood equation, whose parameters can be simply determined in the frequency domain and then used invariantly in the time domain to describe the material behavior of the asphalt mixture. To determine the factors required for the time-frequency conversion, we perform model calculations using the approximate procedure of Ninomiya and Ferry (1959). After the exact time-frequency relationship has been established, the relationship between the velocity of the moving wheel load and the load impulse time is discussed. Here we rely mainly on experimental data reported in the literature and on already developed computational methods. Our aim is to give a theoretical formula between wheel load velocity and load frequency. The theoretical formula provides the possibility to separate the effects of the parameters that influence the load frequency, which often appear in the form of an experimental constant in the developed models.

2 Related works

The loading time of vertical compressive stress in flexible pavement structures was first investigated by Barksdale (1971). In his work, he analyzed the stress impulse data series from AASHO (American Association of State Highway Officials) road tests, including inertial and viscous effects, to find a relationship between wheel load traveling at speeds up to 72 km/h (45 mph) and equivalent load impulse duration at different depths. The relationship developed by Barksdale (1971) is still used today by some design systems to calculate the duration of load impulses.

Brown (1973) further analyzed the data reported by Barksdale and others to establish a relationship between representative load impulse duration and wheel load velocity as a function of asphalt layer thickness h:

$$\log\left(t_p\right) = 0.5h - 0.2 - 0.94\log\left(V\right), \tag{1}$$

where t_p is the load time (s), h is the asphalt thickness (m) and V is the wheel load speed (km/h). Note that Eq. (1) gives the load time for the middle of the layer of thickness h. According to Ullidtz (1987), the impulse time at the centre of the asphalt layer under study is obtained by dividing the effective length L_{eff} by the moving wheel load velocity:

$$t_p = \frac{2a_c + h}{V}, \tag{2}$$

where a_c is the radius (m) of the load-bearing surface and $L_{eff} = 2(a_c + h/2)$.

Loulizi et al. (2002) studied the compression stress impulse, which is generated by the moving wheel load and the FWD device, on the Virginia Smart Road at different depths below the pavement surface. The measured impulse times ranged from 0.02 s (70 km/h vehicle, 40 mm depth) to 0.1 s (10 km/h vehicle, 597 mm depth). For the FWD device, however, the 0.03 s load time was found to be a good approximation of the induced stress impulse at any depth below the pavement surface. It was also found that the test temperature (15–25 °C) did not significantly affect the normalized compressive stress impulse times.

Based on full-scale road experiments, Chenevière et al. (2005) proposed the following simple relationship:

$$f = 0.46V. \tag{3}$$

Based on the measurements, the effect of temperature on frequency was estimated to be 10% between 5 °C and 30 °C. Losa and Di Natale (2012) theoretically determined representative frequency values by minimizing the discrepancy between the maximum straining in linear elastic and viscoelastic path models. Representative frequency values, calculated for 215 roadway structure variations, were correlated with the significant variables using multivariate regression. The model was developed separately for each of the three directions according to the anisotropic behavior of the moving wheel load (Eq. (4)):

$$\begin{gathered}
f_x = 0.027V\left(1/2a + 1/2b\right)e^{-3.14z+\alpha(T)}, \\
f_y = 0.042\left(V/2a\right)e^{-3.34z+\beta(T)}, \\
f_z = 0.043\left(V/2a\right)e^{-2.65z+\beta(T)}, \\
\alpha\left(T\right) = 2.12\times10^{-5}T^3 - 2.6\times10^{-3}T^2 + 12.8\times10^{-2}T, \\
\beta\left(T\right) = 1.25\times10^{-5}T^3 - 1.6\times10^{-3}T^2 + 9.20\times10^{-2}T,
\end{gathered} \tag{4}$$

where f_i frequency in i-th direction (Hz) (where i is an auxiliary variable representing the x, y and z directions), z depth in asphalt concrete layer ($0.075 < z < 0.30$ m for f_x and f_y, $0.037 < z \leq 0.30$ m for f_z) (m), $\alpha(T)$ the effect of asphalt concrete temperature (T, in °C) in -direction, and $\beta(T)$ the effect of asphalt concrete temperature in y- and z-directions, a half-length of the rectangular footprint in the motion direction (m), b half-length of the rectangular footprint in the transverse direction.

Ulloa et al. (2013) also used theoretical calculations to develop equations for predicting representative frequency values for the pavement response being analyzed:

$$f_1 = 0.2187V, \tag{5}$$

$$f_2 = 0.4681V. \tag{6}$$

The correlations were highly dependent on vehicle speed (in km/h), but pavement structure and asphalt layer temperature were not found to be significant variables. Equations (5) and (6) are proposed for different types of impulses.

Bodin et al. (2015) established the following empirical relationship between wheel speed and laboratory test frequency:

$$f \approx 0.3V. \tag{7}$$

Equation (7) is valid only for the range of pavement temperatures (20–41 °C), which has been used in the design of it. In their study, Shafiee et al. (2015) analyzed the frequency of measured stress and strain impulses in situ using Fast Fourier Transform (FFT). Their research data confirm that the frequency calculation under moving wheel loading is highly dependent on the type of response of the structure. Equations (8) to (10) were given for the bottom of the asphalt layer:

$$f_x = 1.158V^{0.40}, \tag{8}$$

$$f_y = 1.165V^{0.45}, \tag{9}$$

$$f_z = 0.607V^{0.50}, \tag{10}$$

where f_x, f_y and f_z refer to the loading frequencies in transverse, longitudinal and vertical directions (Hz), and V is vehicular speed (km/h).

Cheng et al. (2020) investigated the relationship between traffic-induced strain impulses and load frequencies on flexible, semi-rigid and steel (asphalt pavement bridge) road structures. The measurements confirmed that the load frequency f increases approximately linearly with the wheel speed V, with only one common model being found for all three pavement types under analysis:

$$f = 0.127V. \tag{11}$$

The frequency values at temperatures above 35 °C exceeded those at lower temperatures, while in the temperature range 4–31 °C the correlations between wheel speed and load frequency were almost identical for the three road structure types investigated. Later, Cheng et al. (2022) in their work: *Bridging the Gap Between Laboratory…* complemented the relationship of load frequency with temperature:

$$f = 0.0984e^{0.0160T} \times V. \tag{12}$$

Based on previous work on the topic, it is evident that the load frequency increases with increasing vehicle speed, while it decreases with increasing pavement depth or wheel load contact radius. Temperature also has an effect on the load frequency, with an increase in frequency as the temperature increases. Among the significant variables listed, vehicle speed was found to be the most significant compared to pavement depth and load radius. This finding is in line with expectations, as vehicle speed directly affects the duration of traffic loading and thus crucially alters the loading frequency (Cheng et al. (2022) in their work: *Fatigue Test Setups and Analysis Methods…*). It is also worth noting that the load frequency is often derived by researchers from theoretically calculated response impulses from mechanical models. However, these theoretical calculations do not necessarily reflect the actual behavior of asphalt layers in the field. According to Cheng et al. (2022) in their work: *Fatigue Test Setups and Analysis Methods…*, these purely theoretical models significantly overestimate field measurements, which calls attention to the fact that it is advisable to determine the loading frequency of the asphalt mixture from field impulse data.

3 Theoretical backgrounds

3.1 Master curve of the dynamic modulus

The viscoelastic behavior of asphalt mixtures under harmonic loading was modelled using a master curve function proposed by Kweon (2008):

$$f_r = E_N f_r + C\left(E_N f_r\right)^R, \tag{13}$$

where f_r is the reduced frequency and E_N is the normalized dynamic asphalt modulus:

$$E_N = \frac{E^* - E_e^*}{E_g^* - E_e^*}, \tag{14}$$

where E_e^* is the asymptote of low frequencies or high temperatures and E_g^* is the asymptote of high frequencies or low temperatures, the parameter R affects the slope of the master curve and C moves it on the horizontal axis. Kweon (2008) used the general mathematical form of the Ramberg-Osgood (RAMBO) model as a starting point to formulate Eq. (13). A major advantage of the RAMBO model over the sigmoid function introduced by Witczak and Fonseca (1996) is that its parameters are truly independent of each other (see e.g., Kweon, 2008:p.38). The relation reported in Kweon's original paper is impractical for practical applications, as fitting it to experimental data is complicated. Therefore, for further calculations, it is used organized by the dynamic modulus E^*:

$$E^*\left(f_r\right) = E_e^* + \frac{E_g^* - E_e^*}{1 + Cf_r^{R-1}}. \tag{15}$$

We also provide the compact form of the RAMBO model with the parameters c_1, c_2, c_3 and c_4 to make it more convenient to work with later:

$$E^*(f_r) = c_1 + \frac{c_2}{1 + c_3 f_r^{c_4}}, \tag{16}$$

where c_1 defines the lower asymptote of the curve, c_2 defines the difference between the upper and lower asymptotes of the curve, c_3 shifts the whole curve horizontally (to the left or right depending on its value), c_4 defines the slope of the curve between the upper and lower asymptotes. The independence of the coefficients of the RAMBO model allows the full morphology of the master curve to be numerically characterized.

3.2 Master curve of the relaxation modulus

The time-dependent relaxation modulus of asphalt mixtures can also be approximated empirically by sigmoidal functions, similar to the dynamic modulus. Therefore, the RAMBO model can be applied for this purpose after appropriate modifications.

The relaxation function is therefore essentially the same form as Eq. (16), but with new coefficients:

$$E(t_r) = d_1 + \frac{d_2}{1 + d_3 t_r^{d_4}}, \tag{17}$$

where $E(t_r)$ is the time-dependent relaxation modulus at the reduced time t_r and d_1 to d_4 are the model constants.

Currently, there are two common methods in the literature for calculating the load time. The first proposed method converts the frequency directly into load time using the formula $t = 1/f$. The second method, on the other hand, first converts the frequency into ω angular frequency and then derives the load time $t = 1/2\pi f$. In the case where the relationship between time and frequency is not to be given specifically but in a general way, it is only the inverse proportionality between the two variables that is required (Lee, 2022):

$$f \approx \frac{\beta}{t}, \tag{18}$$

where β is the time-frequency equivalence factor. The above general relationship gives the time-frequency conversions $f = 1/t$ for constant $\beta = 1$ and $f = 1/2\pi f$ for $\beta = 1/2\pi$. We can now formulate the time-dependent relaxation modulus of the asphalt mixtures we are looking for in general form:

$$E(t_r) = c_1 + \frac{c_2}{1 + c_3 \beta^{c_4} t_r^{-c_4}}, \tag{19}$$

where $d_1 = c_1$, $d_2 = c_2$, $d_3 = c_3\beta^{c_4}$ and $d_4 = -c_4$, are used in Eq. (17). It follows from the above that if we use the relation Eq. (18) for the conversion, the sign of the coefficient c_4 changes and the total curve shifts horizontally by β^{c_4}. Finally, by substituting the coefficients $c_1 = E_e^*$, $c_2 = E_g^* - E_e^*$, $c_3 = C$ and $c_4 = R - 1$ into the relation Eq. (19) and further simplifying, the final relation of the relaxation modulus is obtained:

$$E(t_r) = E_e^* + \frac{E_g^* - E_e^*}{1 + C(\beta/t_r)^{R-1}}, \tag{20}$$

where $E(t_r)$ is the time-dependent relaxation modulus at the reduced time t_r. Therefore, in order to derive the time-dependent relaxation modulus from the frequency-dependent dynamic modulus RAMBO model, we need to be able to determine the value of the constant β in the equation.

3.3 Description of asphalt relaxation with Prony series

A preferred option for describing the relaxation response of viscoelastic materials is the generalized Maxwell model. The generalized Maxwell model consists of n Maxwell elements connected in parallel, and a Hooke element connected in parallel (Fig. 3).

The parallel-connected spring element with modulus E_∞ is needed because laboratory experiments have shown that some elastic deformation will remain in the material after a sufficiently long time. The total (combined) stiffness of the model, corresponding to the relaxation modulus $E(t)$, can be represented by a Prony series:

$$E(t) = E_\infty + \sum_{i=1}^{n} E_i e^{-\frac{t}{\tau_i}}, \tag{21}$$

where E_∞ is the modulus of elasticity of the fully relaxed material, and E_i and τ_i are the stiffness and relaxation time

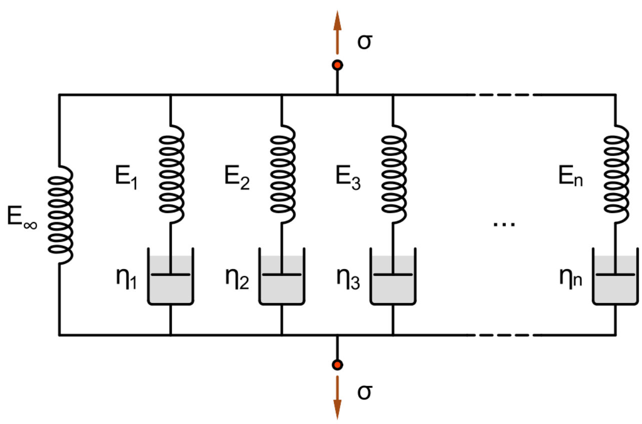

Fig. 3 The n-term generalized Maxwell model (E_i: elastic modulus of the i-th linear spring, η_i: viscosity of the i-th dashpot)

of the i-th viscoelastic Maxwell element. The complex modulus of the general Maxwell model is equal to the sum of the complex moduli of each branch due to the parallel coupling, i.e., the real and the imaginary parts have to be summed separately:

$$E'(\omega) = E_\infty + \sum_{i=1}^{n} E_i \frac{(\omega\tau_i)^2}{1+(\omega\tau_i)^2}, \tag{22}$$

$$E''(\omega) = \sum_{i=1}^{n} E_i \frac{\omega\tau_i}{1+(\omega\tau_i)^2}. \tag{23}$$

Note that the (τ_i, E_i) Maxwell parameters in Eqs. (22) and (23) are also included in the relaxation function Eq. (21), so they can be determined from the experimental data $E^*(\omega)$ at the respective ω frequencies:

$$\left|E^*(\omega)\right| = \sqrt{E'(\omega) + E''(\omega)}. \tag{24}$$

This is possible because the Laplace transforms of Eq. (21) and Eq. (24) exist in closed form (Romeo et al., 2024). Thus, the exact transformation between $E(t)$ and $\left|E^*(\omega)\right|$ can be performed if the Prony series is known for either quantity (Park and Schapery, 1999). Knowing the Prony series of asphalt mixtures, we can check the accuracy of the empirically determined time-frequency equivalence factor β.

4 Material and method

4.1 Description of the tested road structures

The load frequency calculation procedure presented in this paper was tested on three road pavement structures with different stiffnesses:

- Semi-rigid;
- Flexible;
- Especially flexible.

The deflection was measured with a KUAB type Falling Weight Deflectometer (FWD) at 5 m increments on the 200 m long selected road sections. The load magnitude was 50 kN and data from the second drop were processed. The pavement temperature varied between 20–22 °C. The structure of the pavements is summarized in Table 1.

Table 1 Pavement layering for the studied road structure types

Layer #	Semi-rigid	Flexible	Especially flexible
1	100 mm asphalt concrete	100 mm asphalt concrete	70 mm dense bitumen macadam
2	150 mm cement treated base	350 mm well graded crushed stone	250 mm macadam base course

4.2 Determination of the time-frequency equivalence factor

Model calculations were performed to determine the time-frequency equivalence factor. An empirical method was used to find a relationship between the coefficients of the RAMBO dynamic modulus master curve and the β constant in question.

As a first step, a theoretical $E^*(f_r)$ master curve database was constructed by varying the parameters of the dynamic modulus RAMBO model Eq. (15) over a wide range. All possible combinations of the parameters in Table 2 were used in the analysis. A total of 400 master curves were used to compile the analysis database. Subsequently, the relaxation modulus of each $E^*(f_r)$ and $\phi(f_r)$ master curve was determined using the approximate relations of Ninomiya and Ferry (1959):

$$E(t) \approx \left. \begin{array}{l} E'(\omega) - 0.40E''(0.40\omega) \\ +0.014E''(10\omega) \end{array} \right|_{t=1/\omega}. \tag{25}$$

The phase angle model $\phi(f_r)$ is derived from the approximate formula for Kramers-Kronig relations introduced by Booij and Thoone (1982):

$$\phi(f_r) \approx \frac{\pi}{2} \frac{d \ln\left|E^*(f_r)\right|}{d \ln f_r}. \tag{26}$$

Finally, the storage and loss modulus in Eq. (25) can also be calculated from the dynamic modulus and phase angle:

$$E' = \left|E^*\right| \cos\phi, \tag{27}$$

$$E'' = \left|E^*\right| \sin\phi. \tag{28}$$

Each master curve contained 50 data points within the frequency (or time) range 10^{-7} and 10^{+7}. Equation (20) was

Table 2 Parameter of the RAMBO model to establish the dynamic modulus and relaxation modulus master curves

Parameters	Value
E_e^* (MPa)	0, 100, 200, 300
E_g^* (MPa)	25 000, 30 000, 35 000, 40 000
R	0.5, 0.6, 0.7, 0.8, 0.9
C	1.0, 1.5, 2.0, 2.5, 3.0

fitted to each of the $E(t_r)$ master curves in the database, where only the constant β was unknown. The goodness of fit of the model was measured by the root mean square error (RMSE):

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \hat{Y}_i\right)^2}, \tag{29}$$

where Y_i is the measured value and $\hat{Y}_i$ is the predicted value. Regression analysis was applied to find the relationship between β constants and the RAMBO model parameters R and C. Finally, the equation formulated for the time-frequency equivalence factor β was simplified as much as possible while maintaining accuracy.

4.3 Performance test of the time-frequency transformation model

The suitability of the developed time-frequency equivalence factor was tested by comparative calculations. For the viscoelastic comparison calculations, we used the ELiCon v0.1 Microsoft Excel worksheet (Levenberg, 2018), which allowed us to perform an exact conversion between time and frequency domains in real time. The input to ELiCon v0.1 in the time domain is a four-parameter analytic function proposed by Smith (1971) to describe creep compliance:

$$D(t) = D_\infty + \frac{D_0 - D_\infty}{1 + \left(t/\tau_D\right)^{n_D}}, \tag{30}$$

where D_0 defines the upper asymptote of the curve, D_∞ defines the lower asymptote of the curve, τ_D shifts the whole curve horizontally to the left or right, n_D defines the slope of the curve between the upper and lower asymptotes. The ELiCon v0.1 worksheet uses the above equation to calculate the uniaxial creep compliance for a given time and gives the relaxation modulus for the same time. In the frequency domain, the output of the worksheet for certain frequencies is the absolute value of the complex modulus (the dynamic modulus) and the phase angle. The conversion is based on the Prony series of the generalized Maxwell model that has already been presented (Lv et al., 2019; Park and Schapery 1999).

For the parameters given in Table 3, the master curves for the dynamic modulus in the frequency domain and the relaxation modulus in the time domain were calculated with the ELiCon v0.1 worksheet. Equation (15) was fitted to the dynamic modulus data points E^* from ELiCon v0.1 and resulting RAMBO model parameters (E_e^*, E_g^*, R and C) were acquired. Then, substituting the already known parameters E_e^*, E_g^*, R and C and the time-frequency equivalence factor β into Eq. (20), we obtained the RAMBO master curve of the relaxation modulus. This derived master curve was compared with the relaxation modulus value calculated with the ELiCon v0.1 worksheet.

Table 3 Master curves model parameters calculated with ELiCon v0.1

Parameters	Value
D_0 (1/MPa)	1 / 25 000
D_∞ (1/MPa)	1 / 500
τ_D	10, 10^2, 10^3, 10^4
n_D	0.2, 0.3, 0.4, 0.5

For each regression model, we calculated the standard error of the estimate of variable Y, the standard error of the variable X (S_e), the standard deviation (S_y) and the so-called corrected coefficient of determination $\left(R_*^2\right)$. The corrected R_*^2 is a modification of R^2 that considers how many observations we have and how many explanatory variables we have:

$$R_*^2 = 1 - \left(S_e/S_y\right)^2. \tag{31}$$

Lower S_e/S_y and higher R_*^2 values show a better agreement between predicted and measured data. Previous research suggests that excellent model fit is achieved when S_e/S_y is less than 0.35 and R_*^2 is greater than 0.9 (Witczak et al. (2002) in their work: *Pursuit of the Simple Performance Test…*).

4.4 Relationship between moving wheel load and load frequency

Once the time-frequency relationship is formulated, the relationship between the speed of the moving wheel load and the impulse time can be defined. It is useful to start from the physics relation of distance/time, where the wheel speed V is equal to the travelled distance divided by the elapsed time:

$$V = \frac{2a_c}{t} \rightarrow t = \frac{2a_c}{V}, \tag{32}$$

where V is the wheel speed and a_c is the radius of the tyre contact area. Since the stress or strain impulse times in the pavement structure due to wheel loading are different, the above Eq. (32) can be given in general form:

$$t = \lambda \times V^{-k}, \tag{33}$$

where $\lambda = 2a_c$ is the wavelength and $k = 1$ is the experimental constant. Furthermore, substituting Eq. (33) into the general relationship frequency-time Eq. (18), we can obtain the formula for the load frequency:

$$f \approx \frac{\beta}{\lambda} \times V^k, \tag{34}$$

where λ is the wavelength of the stress or strain, β is the time-frequency equivalence factor, k is the experimental constant and V is the wheel speed. The wavelength is determined primarily experimentally. We can start from the deflection line of real pavement structures (Fig. 4).

Equation (35) fits the deflection data well (Primusz et al., 2015):

$$D(r) = D_0 \frac{4a_c^2}{cr^2 + 4a_c^2}, \tag{35}$$

where D_0 is the central deflection, r is the radial distance, a_c is the radius of the load plate. In the proposed function, c is the so-called sub-factor which affects the shape of the deformation line. Furthermore, the radius of curvature at the centre of the load at the bottom of the pavement layer with thickness (h) can be expressed by the strain ε_y (Jung, 1988):

$$\varepsilon_y(r) \approx h/\left[2R(r)\right], \tag{36}$$

where $R(r)$ is the radius of curvature at radial distance r, h is the thickness of the asphalt layer. The curvature can be well approximated from the curvature line from Eq. (35) using the relation $\kappa(r) \approx D''$ (Primusz et al., 2015).

$$\kappa(r) = 8\frac{D_0 a_c^2 c\left(3cr^2 - 4a_c^2\right)}{\left(cr^2 + 4a_c^2\right)^3} \tag{37}$$

The inflection point of the curvature function Eq. (37) can be considered as the load wavelength of the specific strain in the bottom layer of asphalt with thickness h due to Eq. (36) (Fig. 4):

$$\frac{\lambda}{2} \approx \frac{2a_c}{\sqrt{3c}} \rightarrow \lambda \approx \frac{4a_c}{\sqrt{3c}}. \tag{38}$$

According to the derived formula Eq. (38), the wavelength depends on the contact radius ac of the load wheel and the deflection line c as shape parameter. This allows non-destructive and fast inference of the load frequency from field measurements.

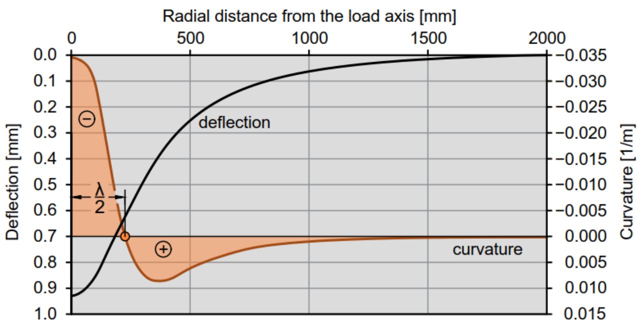

Fig. 4 Surface deflection basin and the curvature derived from it

5 Evaluation of results

To obtain the time-frequency equivalence coefficient, multiple linear regression was applied on the hypothetical master curve database data:

$$\beta = b_0 + b_1 R + b_2 C, \tag{39}$$

where b_0 is the axis intercept and b_1 and b_2 are the regression coefficients. The coefficients and statistical properties of the model were determined using the Microsoft Excel Data Analysis plug-in:

$$\beta = 0.0498 + 0.0549R + 0.00012C. \tag{40}$$

Since the value of the coefficient b_2 can be taken to be practically zero, the value of β is not, or only to a very small extent, explained by the parameter C. According to our previous research (Cho et al., 2020), the parameter C is directly related to temperature:

$$C = k_1 e^{k_2 T}, \tag{41}$$

where k_1 and k_2 are regression constants and T is the temperature. This implies that the effect of temperature on the conversion is very small. Therefore, the final relationship can be given in the following simpler form:

$$\beta = 0.0501 + 0.0549R. \tag{42}$$

Statistical properties of model Eq. (40) and simplified model Eq. (42) are summarized in Tables 4 and 5.

According to relation Eq. (42), the conversion of the viscoelastic material properties of asphalt mixtures from the frequency domain to the time domain depends primarily on the parameter R, which determines the slope of the master curve between the lower and upper asymptotes. A graphical representation of the model is shown in Fig. 5. Knowing the time-frequency equivalence factor β searched for, the conversion of the viscoelastic material properties from the frequency domain to the time domain can now be solved.

Considering also the typical parameter value $R = 0.65$ of asphalt mixtures investigated in Cho et al. (2020), we can say that the load frequency can be well approximated by one of the formulas $f \approx 0.9/t$ or a $f \approx 0.08/t$. Confirmation in the literature can be found in the work of Dongre et al. (2006) by examining the data. We can see that the approach of Daniel and Kim (1998) is almost identical to the β derived from the RAMBO model. A similar result is obtained if we apply Schapery's (1965) relationship to the conversion:

$$\omega = \frac{1}{2t}. \tag{43}$$

Table 4 Statistical characteristic of the multivariate regression analysis parameter β

	Coefficients	Standard error	t Stat	p-value	Lower 95%	Upper 95%
Intercept	0.04983125	0.000148450	335.6762384	0	0.049539403	0.050123097
X variable 1	0.05494250	0.000181361	302.9455912	0	0.054585952	0.055299048
X variable 2	0.00012125	3.62722E-05	3.342781356	0.000908183	4.99404E-05	0.00019256

Table 5 Statistical properties of the simplified model parameter β

	Coefficients	Standard error	t Stat	p-value	Lower 95%	Upper 95%
Intercept	0.05007375	0.000131163	381.7684337	0	0.049815892	0.050331608
X variable 1	0.05494250	0.000183664	299.1461406	0	0.054581426	0.055303574

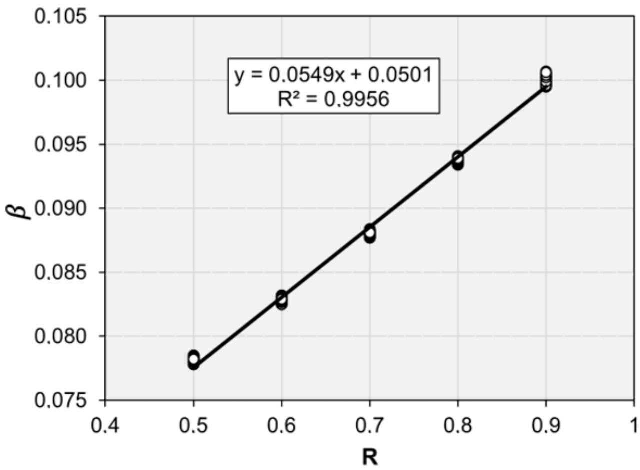

Fig. 5 The required β time-frequency equivalence factor vs. the R parameter

The approximate formula for the load time can be derived from the above formula by substituting $\omega = 2\pi f$:

$$t = \frac{1}{4\pi f} \approx \frac{0.08}{f}. \tag{44}$$

Further confirmation is obtained by examining the data in Dongre et al. (2006). We can see that the approach of Sias Daniel and Richard Kim (1998) is almost identical to the approximation derived from the RAMBO model.

We compared the presented calculation method with the Prony series, which was computed with the ELiCon v0.1 workbook based on the generalized Maxwell model. The RAMBO material model was fitted to the calculated data points and the goodness of fit was measured using the corrected coefficient of determination. In general, there were no master curves where R_*^2 was less than 0.96.

Once the time-frequency relationship is formulated, the relationship between the speed of the moving wheel load and the load time can be defined. Ulloa et al. (2013) showed that the load frequency is proportional to the vehicle speed, independently of the asphalt thickness and pavement temperature. A similar result was obtained by Sullivan et al. (2013) who found that asphalt layer thickness played little or no role in determining the actual load duration. Cheng et al. (2020) found that below 31 °C, the frequency values can be considered relatively constant as they are much less responsive to temperature. Therefore, neither the thickness of the asphalt layer nor the temperature of the asphalt layer will be considered in the following, as they only slightly modify the load frequency values.

The wavelength of the stress or strain λ in Eq. (38) can be determined experimentally or from theoretical considerations. For the deflections measured with the FWD equipment, fitting Eq. (35) gives the shape factor c. The average shape factor is 0.06 for semi-flexible pavement, 0.185 for flexible pavement and finally 0.516 for particularly flexible pavement based on field measurements. It is observed that the more flexible the structure, the higher the value of c. The values of the wavelengths λ calculated from the shape factors using Eq. (35) are summarized in Table 6.

Substituting into Eq. (34) the value $\beta \approx 0.085$ and the constant of the fractional conversion of the wheel speed V from km/h to m/s (1/3.6), we obtain the following simple relation:

$$f \approx \frac{0.024}{\lambda} V^k, \tag{45}$$

where V is the wheel speed (km/h), λ is the wavelength of the stress or strain (m) and k is an experimental constant. The wavelength depends on the response of the pavement structure (stress, strain, deflection) to the external load, as shown in previous research. However, the stresses induced by a moving wheel load are well known and can be assumed.

Table 6 Pavement characteristics defined by the FWD equipment on the studied road sections

Parameter	Semi-rigid	Flexible	Especially flexible
Shape factor (c)	0.060 ± 0.013	0.185 ± 0.039	0.516 ± 0.061
Wavelength (λ) in m	1.420 ± 0.160	0.800 ± 0.080	0.480 ± 0.030

Nikolaides and Manthos (2019) determined the stress wavelength λ at the bottom of the asphalt layer to be 0.2–0.4 m, depending on the loading configuration. It was observed that λ increased with increasing number of tires and axles. A similar result (0.3–0.4 m) is obtained by back-calculating the wavelength close to the surface (@40 mm) from the loading times measured by Loulizi et al. (2002). Considering that our previous research has shown that the angle of inclination of the stress cone from the load contact circumferential surface for asphalt pavements is only 20° ± 2° (Tóth and Primusz, 2022), compared to 34° often quoted in the literature (Rohde, 1994), we can approximate the wavelength well by $\lambda \approx 2a_c$. For a tire wheel model, $a_c = 0.12$ m, and $k \approx -1$ (Loulizi et al., 2002), substituting these characteristic data into the above relation Eq. (41), we obtain the following simplified relation:

$$f \approx 0.1V, \tag{46}$$

where V is interpreted in km/h. The slope of the theoretically derived relationship is close to the equation $f = 0.127V$ reported in Cheng et al. (2020), which was determined from field measurements. The approximate relationship Eq. (46) confirms the 10 Hz value proposed by Witczak et al. (2002) in their work: *Simple Performance Test…*, which suggests that at speeds typical of highways (~110 km/h), the load impulse time is approximately 0.1 s.

In the case of straining wavelengths, we can start from the work of Mollenhauer et al. (2009), where the following empirical model for the impulse time of in situ asphalt elongations was established:

$$t = 1.805V^{-0.944}, \tag{47}$$

where V in km/hr and t in ms. Substituting the above Eq. (47) into Eq. (45) and rearranging it, we obtain the approximate formula for the load frequency as a function of the moving wheel speed:

$$f \approx \frac{0.024}{1.805/3.6} V^{0.944} \approx 0.0373V. \tag{48}$$

From the relationship the straining wavelength $\lambda = 1.805/3.6 = 0.5$ m. This value is the same as the typical value for the particularly flexible pavement type in Table 6. Sullivan et al. (2013) recommend using a very similar intercept of slope $f = 0.0246V$ based on Australian road test data, as they find that the wavelength $\lambda = 1.8$ m is equal to the total length of the deflection basin. This interesting result is explained by the early work of Coffman (1967). The slope of the two models differs because the Australian researchers used a time-frequency equivalence factor of $\beta = 1/2\pi$, which is almost twice as large as the value of 0.085 that we propose.

Based on the results obtained so far, a general relation for the load frequency can be formulated when computing with a strain wavelength:

$$f \approx \frac{\sqrt{3c}}{4a_c} \beta V^k, \tag{49}$$

where β is the time-frequency equivalence factor, a_c is the tire load contact radius (m), k is the experimental constant (–), c is the deflection shape-factor (–) and V is the wheel speed (km/h).

Fig. 6 shows graphically the results of the general model Eq. (49) for the cases of fixed values $a_c = 0.15$, $\beta = 0.085$, $k = 1$. The results of the new model, varying the shape-factor parameter between 0.2 and 0.8, are between the load frequency values of Shafiee et al. (2015) and Sullivan et al. (2013). This can be explained by the fact that in our research we also used Falling Weight Deflectometer (FWD) equipment to investigate pavement structures. Sullivan et al. (2013) also used FWD data in their analysis, where the effect of asphalt thickness was not detected. Loulizi et al. (2002) also found that the stress impulse induced by FWD loading had a duration of 0.03 s at any given depth below the pavement surface. This suggests that the load frequency should be calculated differently for moving vehicles and for impulse-based load measuring devices. Assuming an average pulse duration $t_p = 0.03$ s and using the average λ wavelengths determined with the FWD equipment (Table 6), the moving wheel load V velocity simulated with the FWD equipment can be estimated. For a particularly flexible pavement structure, $V = 57.6$ km/h, while for a flexible pavement structure

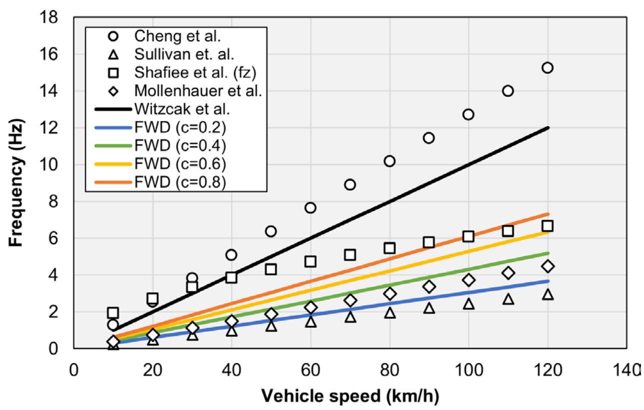

Fig. 6 Comparison of previous frequency calculations with the new theoretical model

a much higher $V = 69$ km/h is obtained. These results are close to the findings of Wang and Li (2016), who found that the equivalent speed of FWD loading with maximum elongation deformation is 24–80 km/h depending on the asphalt layer thickness and temperature.

6 Summary

In our research, we studied the relationship between the load frequency and the moving wheel load. A key element in solving this problem is to determine the time-frequency equivalence factor. The asphalt mixture $\left|E^*\right|$ dynamic modulus master curves were transformed into the relaxation modulus $E(t)$ using the RAMBO model. Theoretical calculations showed that the conversion of viscoelastic material properties from the frequency domain to the time domain depends on the parameter R, which determines the slope of the master curve between the lower and upper asymptotes. This implies that the time-frequency conversion factor is not a constant value, but depends on the material quality of the asphalt mixture. The developed empirical time-frequency equivalence factor β was validated using the Prony series approximation of the ELiCon v0.1 worksheet.

Previous experimental results reported in the literature show that the load frequency is proportional to the vehicle speed V, independently of the asphalt thickness and pavement temperature. Therefore, the relationship between load frequency and speed can be described theoretically. The difficulty is caused by the necessity to obtain the wavelength of the stress or strain λ or the impulse time t_p. The value of the wavelength can be estimated from the curvature function derived from the deflection basin, or simply assumed to be equivalent to the contact diameter of the wheel load.

When examining a stress impulse, an approximate relationship between the velocity V of the moving wheel and f load frequency of the dynamic modulus can be derived from the RAMBO material model. Based on the asphalt mixtures studied in this study, a wheel load moving at 100 km/h can be equated to a dynamic stress frequency of 10 Hz. However, for a straining impulse, a much lower frequency of 3–4 Hz is obtained for the same speed. It is important to note that the literature suggests that the load frequency for moving vehicles and for impulse based load weighing (FWD) equipment should be calculated differently and therefore the two load cases should be dealt with separately. In the future, field studies should be continued rather than theoretical calculations. Using FWD or Curviametro equipment, the deflections on experimental road sections as well as the straining on the bottom plane of the asphalt layer (using sensors installed in the structures) should be recorded. By laboratory testing of the asphalt layers of the experimental road structures, the time-frequency equivalence factor β can be accurately determined (see RAMBO model). Determining the parameter k of Eq. (45) presented in this paper can be a major step forward in fully answering this question. Data that can be used for this purpose are the stress or strain impulses in the structure and the known wheel load characteristics.

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