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TRACK QUALIFICATION METHOD AND ITS REALISATION BASED ON SYSTEM DYNAMICS[1]

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Abstract

The reliable qualification of railway tracks is of great importance especially in the era of spreading higher velocity passenger traffic. It is obvious that the simple geometrical track measurement methods will be supplemented through the determination of the inhomogeneities in the elastic and dissipative parameters of the rail-supporting components along the permanent way. On the basis of Dr. M. Destek's fundamental idea about an acceleration measuring wheelset of constant vertical load, the Department of Railway Vehicles at the TU of Budapest elaborated the plans of a Track Qualifying Vehicle and a system dynamics based evaluation method belonging to it. The measured signals of the axle-box accelerations of the measuring wheelset and the complex non-linear dynamical model of the measuring vehicle–track system make possible to identify the variation of the track stiffness and damping vs. track arc-length functions. These non-constant stiffness and damping functions are the sources of the parametric excitation of the 'track–vehicle' dynamical system. Certain fraction of track irregularities measured by traditional inspection cars can be traced back to the inhomogeneities in track stiffness and damping. The knowledge of inhomogeneous properties of the track identified by using the proposed method leads to a more realistic cognition of the actual technical state of the track and ensures an exact basis for modelling and simulation of the dynamical processes, as well as to a more reliable prediction of the loading conditions realising both on the railway tracks and vehicles.

Keywords: track–vehicle system, measuring vehicle, qualification method.

1. Introduction

In this paper a system-dynamics based track qualification method and the contours of a measuring system belonging to it are elaborated. The measuring system consists of a two-axle railway vehicle with usual suspension and a measuring wheelset with special suspension which practically discouples the measuring wheelset from the vehicle body. The basis of the measuring method is the vertical and lateral acceleration measurement on the two axle boxes of the measuring wheelset. The goal of our present investigation is to

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identify the vertical elastic and dissipative parameters along the track and to qualify the latter on the basis of the identified parameter inhomogeneities.

2. Dynamical Model of System 'Measuring Vehicle – Track'

The dynamical model of the 'measuring vehicle – track' system consists of an in-space dynamical model of a two-axle railway vehicle equipped with a measuring wheelset of special layout and a part of the track supporting the measuring vehicle. The measuring wheelset has a special suspension which ensures a practically steady vertical force transfer between the vehicle body and the measuring wheelset, while the longitudinal axle-box guidance of the measuring wheelset is relatively stiff. The lateral connection between the measuring wheelset and the vehicle body is extremely soft.

The track model consists of two continuous (or discretised) beams for the rails connected to the discrete masses modelling sleepers by linear springs and dampers modelling the rail fastenings and pads. The ballast support is modelled also by springs and dampers. Thus, the sleepers are connected both to the beams (or beam elements) and to the stationary basic plane.

The measured signals are the vertical accelerations arising on the axle-boxes of the measuring wheelset. In *Fig. 1* the in-plane version of the dynamical system model is shown.

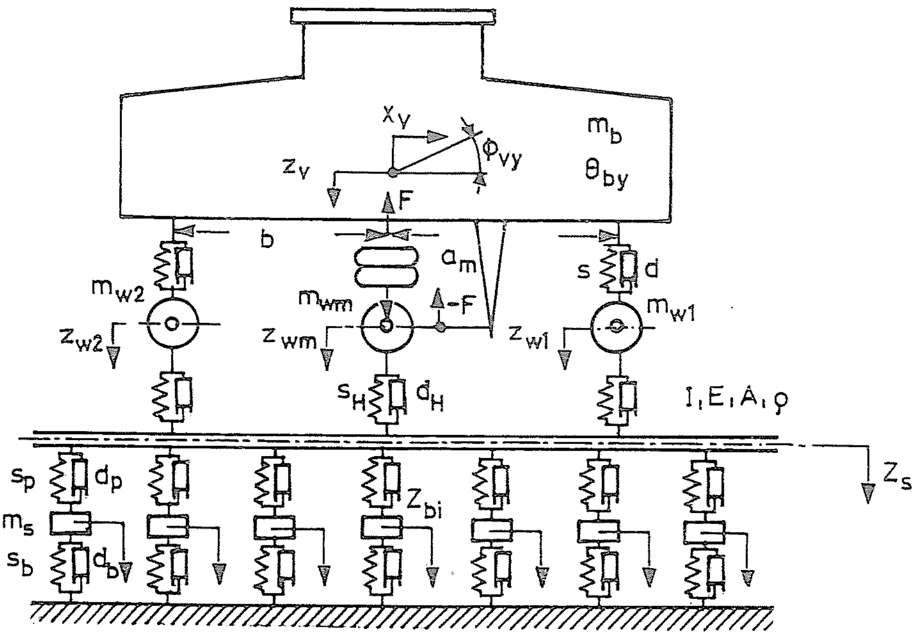

Fig. 1. In-plane dynamical model of 'measuring vehicle – track' system

3. Principles of the Elaborated Qualification Method

As it has been mentioned, the quantities on which the track qualification is based are the right and left vertical axle-box accelerations a_{mr} and a_{ml} measured on the measuring wheelset. So, vector function $[a_{mr}, a_{ml}]^T = f\left([x_r, x_l]^T\right)$ plays a basic role where x_r and x_l are the track directional arc-length co-ordinates of the right and left rails of the track. These accelerations are influenced by the following track characteristics:

- vertical geometry of the unloaded track;
- dynamics of the 'measuring wheelset – track' system;
- inhomogeneities in the elastic and dissipative parameters of the supporting components (rails, fastenings, pads and ballast) along the permanent way.

To identify the longitudinally inhomogeneous vertical track parameters the following train of thoughts can be used for example in the framework of an in-plane model for the case of vertical track stiffness $s(x)$ or track damping $d(x)$.

The inhomogeneities in the elastic and dissipative parameters of the supporting components along the permanent way are taken into consideration as a sum of the mean value and the linear combination of functions generated by shift, expansion and amplifying from an appropriately smooth basic function $s_w(x)$.

The Main Steps of the Identification Method

1. Let $s_w(x)$ be an appropriately selected basic function of the vertical track-stiffness inhomogeneities to describe the elastic properties of the supporting components. In *Fig. 2* a version of $s_w(x)$ is shown, which proved to be advantageous to generate the vertical track-stiffness inhomogeneity function.
2. Let $\mathbf{J}_\infty$ be an operator representing the mapping realised by the moving measuring system, inasmuch as it transfers the vertical track stiffness inhomogeneity function $s(x)$ into the vertical axle-box acceleration function $a_m(x)$ measured on the measuring wheelset. It is clear that $\mathbf{J}_\infty$ is determined by the structural parameters of the 'measuring vehicle – track' system. In formal description:

$$\xrightarrow{s(x)} \boxed{\begin{array}{c} SYSTEM \\ \text{'}\mathbf{J}'_\infty \end{array}} \xrightarrow{a_m(x)} . \qquad a_m(x) = \mathbf{J}_\infty s(x) \tag{1}$$

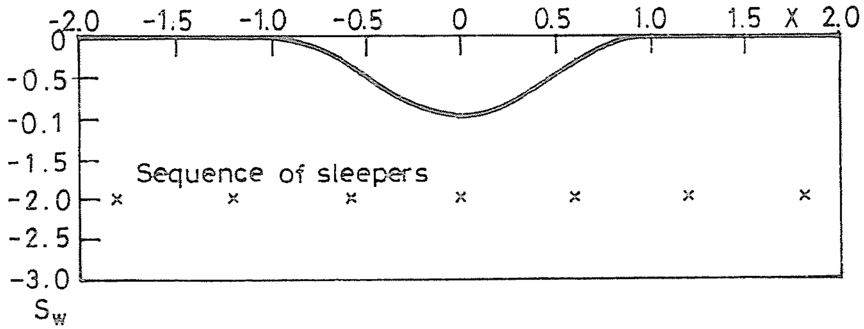

Fig. 2. Basic function to generate vertical stiffness inhomogeneities of the supporting track components

3. Taking into account that the track deformation practically vanishes far enough from the measuring vehicle, the moving 'measuring vehicle – track' system can be successfully approximated by a N degree of freedom dynamical model containing only a finite section of the track, so the number of sleepers being in the scope of the measuring vehicle (the so called effective track zone) is always finite. If the vertical axle-box acceleration function $a_m(x)$ of the measuring wheelset is known from measurements, it can be used to identify the vertical stiffnesses and dampings of the track. The most decisive stiffness and damping inhomogeneities are connected with the ballast bed of the permanent way. In the following stiffness function $s(x)$ will stand for the vertical stiffness of the ballast, i.e. $s(x_i)$ means the stiffness of the ballast supporting the sleeper located at x_i. For a finite sequence of intersleeper intervals belonging to the actual effective zone the mapping realised by the finitised model can be characterised by approximate equality

$$a_m(x) \approx \mathbf{J}_N s(x) \tag{2}$$

based on operator $\mathbf{J}_N$, where $s(x)$ is the unknown function describing the stiffness of the ballast under the sleepers.
4. Let us suppose that the above unknown stiffness function can be composed in the form

$$s(x) \approx \overline{s} + \sum_{j=1}^{n} b_i \cdot s_w\left(e_j \cdot (x - c_j)\right), \tag{3}$$

where $\overline{s}$ is the unknown mean stiffness, b_j, c_j and e_j are further unknown parameters representing the necessary shifts, expansions and amplifyings applied on the basic function $s_w(x)$, while n is an appropriate integer determined by the required accuracy prescribed for approximating formula (3). With the knowledge of the N degree of freedom linear dynamical model of the track–vehicle system in question a computer simulation can be carried out to determine the vertical axle-box acceleration function $a(x)$ of the measuring wheelset by numerical realisation of mapping:

$$a(x) = \mathbf{J}_N s(x). \tag{4}$$

It is obvious that $a(x)$ is image of function of $s(x)$ received by applying operator $\mathbf{J}_N$ on $s(x)$, thus $a(x)$ should take over the parameter dependence of $s(x)$, namely parameters $\overline{s}$, $\mathbf{b} = [b_1, b_2, \ldots, b_n]^T$, $\mathbf{e} = [e_1, e_2, \ldots, e_n]^T$ and $\mathbf{c} = [c_1, c_2, \ldots, c_n]^T$ appeared in formula (3), so the latter parameters will influence $a(x)$. Accordingly, notation $a = a(x, \overline{s}, \mathbf{b}, \mathbf{e}, \mathbf{c})$ is introduced for the simulated vertical axle-box acceleration function, which is obtained from the solution of the initial value problem

$$\begin{aligned} \dot{\mathbf{x}} &= (\mathbf{A}_{2N} + \mathbf{B}_{2N}(\overline{s}, \mathbf{b}, \mathbf{e}, \mathbf{c}) + \mathbf{C}_{2N}(t))\, \mathbf{x}, \\ \mathbf{x}(t_0) &= \mathbf{x}_0, \end{aligned} \tag{5}$$

where $\mathbf{x}$ is the state–vector of the 'measuring vehicle – track' system model, $\mathbf{A}_{2N}$ is the constant component of the system matrix of it, $\mathbf{B}_{2N}$ is the track-parameter-dependent component matrix and $\mathbf{C}_{2N}$ is the time-dependent component matrix describing the contact conditions of the considered track model and the measuring wheelset, whilst $\mathbf{x}_0$ is the initial state vector of $2N$ dimension of the 'measuring vehicle – track' system and t stands for the time. Operator $\mathbf{J}_N$ represents the existing relation between the components of the time-derivative of the state vector $\mathbf{x}$ and the stiffness of the ballast under the measuring wheelset.

5. On the basis of the simulated and measured acceleration functions $a(x, \overline{s}, \mathbf{b}, \mathbf{e}, \mathbf{c})$ and $a_m(x)$, respectively, the identification of the unknown parameters $\overline{s}$, $\mathbf{b}$, $\mathbf{e}$ and $\mathbf{c}$ can be carried out by using the least-square method, according to the following objective function:

$$\int_{X_1} [a(x, \overline{s}, \mathbf{b}, \mathbf{e}, \mathbf{c}) - a_m(x)]^2 \, dx = \psi(\overline{s}, \mathbf{b}, \mathbf{e}, \mathbf{c}) = \min!, \tag{6}$$

where the integration should be carried out over the considered inter-sleeper interval X_1.

6. A more detailed formulation of the above defined objective function can reflect the supposed structure of the simulated vertical axle-box acceleration function of the measuring wheelset, namely it points out that the acceleration function in question appears as a result of application of operator $\mathbf{J}_N$ on the multiparameter function $s(x)$ specified

in (3):

$$\int_{X_1} \left\{ \mathbf{J}_N \left[\overline{s} + \sum_{j=1}^{n} b_j \cdot s_w \left(e_j \cdot (x - c_j) \right) \right] - a_m(x) \right\}^2 \mathrm{d}x =$$

$$\psi(\overline{s}, \mathbf{b}, \mathbf{e}, \mathbf{c}) = \min! \tag{7}$$

7. It is to be emphasised that the formal application of operator $\mathbf{J}_N$ on function $s(x)$ assigns the solution of set of motion equations governing the excited motion of the N degree of freedom dynamical model of the 'measuring vehicle – track' system under given initial conditions. So, formula (7) can be considered as a rather complicated prescription to determine the unknown track parameters. The practical numerical procedure of the parameter identification reflects the above mentioned complicated solution structure: starting from an arbitrarily selected system of parameters $\mathbf{p} = [\overline{s}, \mathbf{b}, \mathbf{e}, \mathbf{c}]^T$ the differential equation should be numerically solved under given initial conditions over track section X_1, to realise the statement assigned by operator $\mathbf{J}_N$. With the knowledge of the numerically simulated axle-box acceleration over interval X_1, the evaluation of the deviation square appearing in (6) can be carried out if the measured axle-box acceleration function is also known over interval X_1. Function $\psi(\overline{s}, \mathbf{b}, \mathbf{e}, \mathbf{c})$ can be minimised by successive repeating of the procedure introduced, if a proper version of the gradient method is included. Let us designate the k^{th} step of the iterative procedure to determine the optimal $\mathbf{p}$ over effective zone X_1. In accordance with the theory of numerical minimisation methods formula

$$\mathbf{p}_{k+1} = \mathbf{p}_k - \frac{\mathrm{grad}\psi(\mathbf{p}_k)}{|\mathrm{grad}\psi(\mathbf{p}_k)|} \cdot \tau \tag{8}$$

can be used, where τ is a given scalar increment. If $|\psi(\mathbf{p}_{k+1}) - (\mathbf{p}_k)| < \varepsilon$, the iteration procedure can be interrupted.

Theoretically, the identified inhomogeneities in track stiffness $s(x)$ carried by vector $\mathbf{p}$ – consisting of elements $\overline{s}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{e}$ – are valid for a finite track section, namely for an effective track zone X covering M number of sleepers and the combined dynamics of the mentioned track zone and the measuring vehicle is characterised by operator $\mathbf{J}_N$. The real track investigations require identification over a long sequence of overlapping effective track zones. Since the parameter optimisation determined by formula (8) treats the intersleeper intervals contained by effective zone X and results in M estimated ballast stiffness values $s(x_j)$, $j = 1, 2, \ldots, M$, belonging to X, it is clear that the dynamical simulation can be repeated over the adjacent intersleeper intervals traversed by the measuring wheelset taking into consideration the simple fact that final values of the state vector of the dynamical system belonging to the endpoint of the former intersleeper interval can be taken as the initial values of the state vector for simulating the motion process of the system over the subsequent intersleeper interval. This train of thoughts means that the scope of the measuring vehicle consisting of M sleepers is moving along the track and to any sleeper positioned at x_j belongs a sequence $s_i(x_j) i = 1, 2, \ldots, M$ of ballast stiffness estimation.

Beyond the effective track zone there is practically no dynamical effect caused by the vehicle on the track. The effective track zone is moving together with the vehicle (*Fig. 3*). So the effective zone is shifted after the measuring wheelset having traversed an intersleeper interval. Since opera-

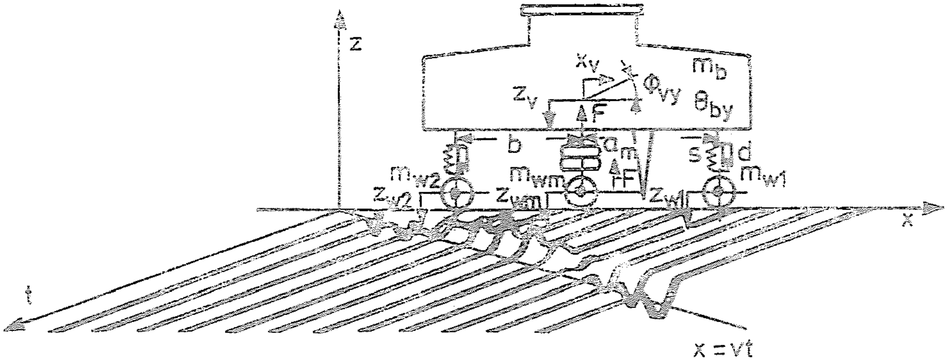

Fig. 3. Vertical displacements of the sleepers under the measuring vehicle

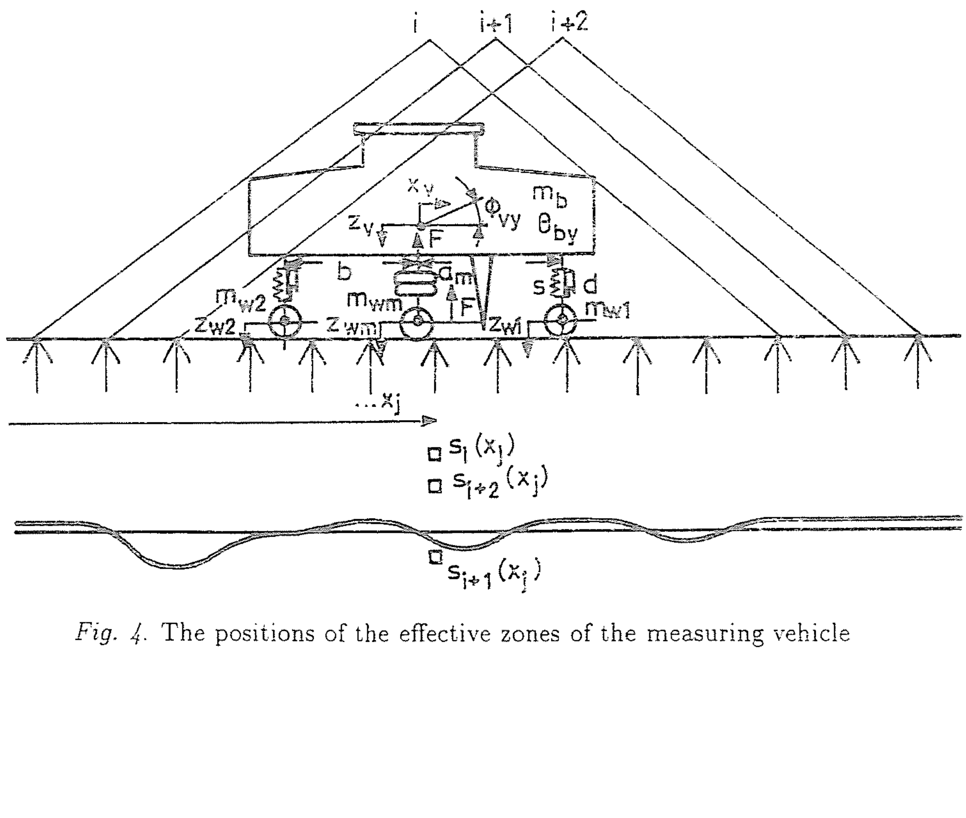

Fig. 4. The positions of the effective zones of the measuring vehicle

tor $\mathbf{J}_N$ depends on the track stiffness belonging to the considered effective zone of the track, we have an operator assembly $\mathbf{J}_{N_i}$, $i = 1, 2, \ldots, m$, where m is the number of the effective zones generated by sequential shifting of the initial zone along the track the step length of each shifting is, i.e. the inter-sleeper interval. Since the identification of the unknown stiffness function $s(x)$ is basicly related to operators $\mathbf{J}_{N_i}$ function $s_i(x)$ belonging to the i-th effective zone. As it has been told we are given a series of the estimations of ballast stiffness $\{s_i(x_j)\}$ at point x_j, see *Fig. 4*.

We can assume that the correct estimated ballast stiffness value $\widehat{s}(x_j)$ can be approximated by the arithmetical average of estimation values $\{s_i(x_j)\}$, so the estimation of the required ballast stiffness function $s(x_j)$ is determined by the point-sequence of the arithmetical averages $\widehat{s}(x_j) = \frac{1}{M} \sum_{i=1}^{M} s_i(x_j)$. Of course all the mentioned relations can be transferred to estimate other parameters of the track, e.g. damping coefficient $d(x)$.

4. Numerical Procedure for the Identification of Track Characteristics

The numerical procedure realising the mentioned identification method can be based on *Eq.* (1) which determines the axle-box acceleration function with the knowledge of ballast stiffness function $s(x)$. The numerical method can be demonstrated by showing an example in which ballast stiffness function $s(x)$ is known. The known track-stiffness inhomogeneity function can be seen in *Fig. 5*.

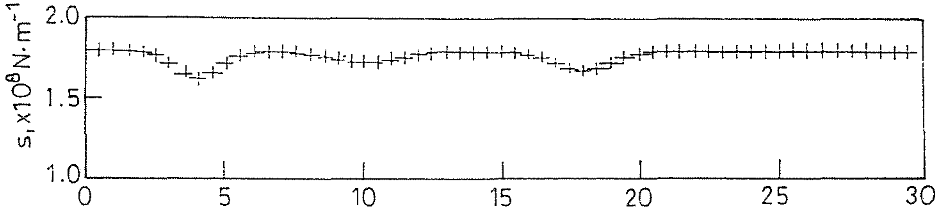

Fig. 5. Stiffness function of the ballast

Operator $\mathbf{J}_N$ belonging to the measuring system and determined by the structure and actual parameters of the track transfers furthermore the ballast stiffness inhomogeneities into the vertical acceleration function of the axle-box of the measuring wheelset. The computed acceleration function $a_i(x)$ is shown in *Fig. 6*. In *Fig. 7* the computed ballast-stiffness estimations are presented. The numerical method is an iteration of gradient method type, described by *Eq.* (7), which represents a practical approximate method to generate the effect of inverse operator $\mathbf{J}_N^{-1}$. The figure shows the

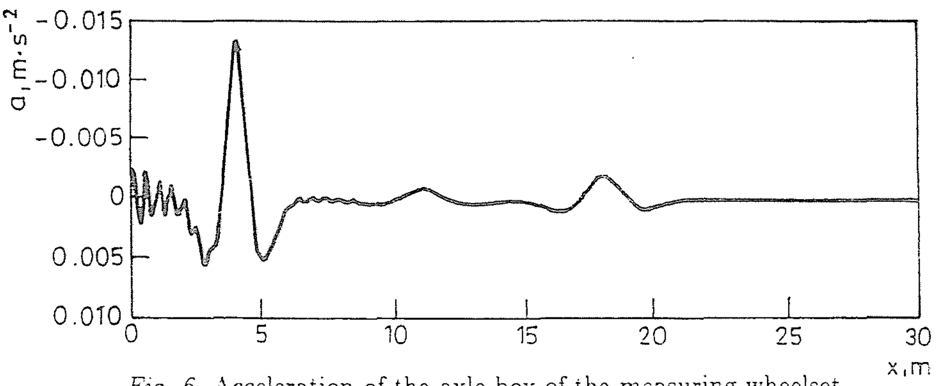

Fig. 6. Acceleration of the axle-box of the measuring wheelset

original ballast stiffness (full line) and the obtained crude ballast stiffness estimations appearing in vertical sequences at each sleeper together with the average function (dashed line). *Fig. 7* shows that the known stiffness

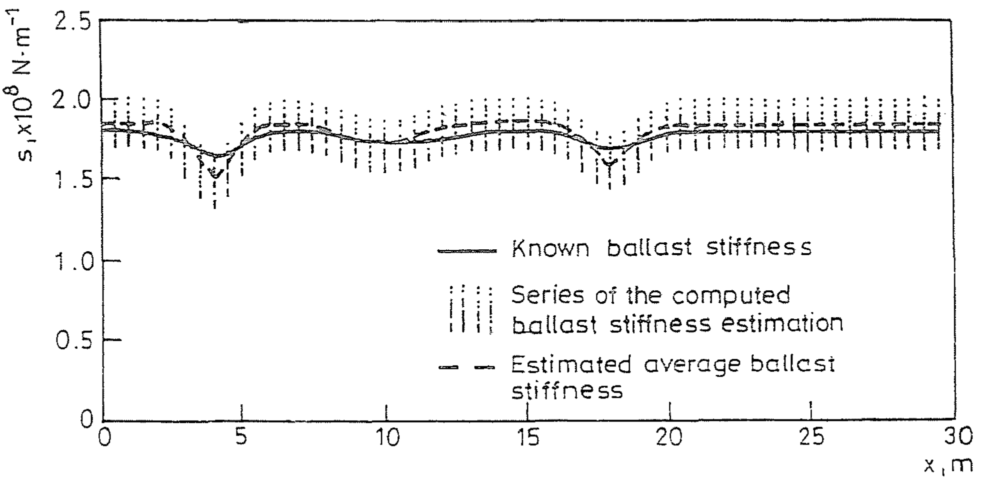

Fig. 7. The known and identified stiffness function of the track

function is fairly well approximated by the average function of the computed ballast stiffness estimations. The shape of the estimated stiffness function is close to the known original stiffness function but it shows greater maximum variations than the original one. The average relative error of the approximation is less than 3%. The permitted ballast stiffness band widths can be specified, on the basis of which the track qualification can be carried out by evaluating the actual variation of the estimated ballast stiffness function, i.e. those track intervals can be determined, over which the ballast stiffness function exceeds the specified band widths mentioned. As *Fig. 7* shows, the maximum difference of the estimated average ballast stiffness $\widehat{s}(x)$ from the exact $s(x)$ in the worst case is less than 8%.

5. Concluding Remarks

- A computation procedure is elaborated for simulating the dynamical processes of the track–vehicle system by determining operator $\mathbf{J}_N$; reflecting the dynamics of the measuring vehicle and the track components belonging to the moving effective zones.
- A computation procedure is elaborated to generate the approximate inverse of operator $\mathbf{J}_N$ by applying a numerical method, in the course of which the accelerations are determined by operator $\mathbf{J}_N$ applied on the known stiffness function. The unknown parameters $(\overline{\mathbf{s}}, \mathbf{b}, \mathbf{c}, \mathbf{e})$ of the computed functions can be estimated by using the method of least squares, composed as a linear combination of the shifted, expanded and amplified versions of the selected specific basic function.
- The obtained approximate stiffness (and damping) parameters can be used to qualify the track, by evaluating the deviations in stiffness (and damping) determining the track intervals over which the variation of the stiffness exceeds certain lines, namely the prescribed band widths.
- Further research is necessary to reduce the range of the ballast stiffness estimation values belonging to the sleepers along the track. In this respect it seems to be feasible on the one hand to increase the length of effective track zone X, and the number of terms taken into consideration in formula (3), on the other.

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