

AN INVERTED PENDULUM TOOL FOR TEACHING LINEAR OPTIMAL AND MODEL BASED CONTROL

Alexandros SOUMELIDIS*, Péter GÁSPÁR** and József BOKOR**

*Computer and Automation Research Institute
Hungarian Academy of Sciences
H-1111 Budapest, Kende u. 13-17, Hungary
Fax: +36 1 166-7503
Phone: +36 1 366-7483

** Department of Control and Transport Automation
Technical University of Budapest
H-1521 Budapest, Hungary
Fax: +36 1 463-3087
Phone: + 36 1 463-1013

Abstract

The use of the inverted pendulum experiment in teaching control theory and practice is now widely accepted in the area of control education. This paper describes an environment built upon an operating inverted pendulum suitable to design and test several linear controllers by using the well known MATLAB and its toolboxes. The environment has been designed to maximally fit to MATLAB procedures and interfaces. A significant part of the environment is a support for identification and model based control, which provides a MATLAB compatible mode to acquire data of the relevant variables and to generate disturbances with predesigned characteristics. Some didactic examples – mainly from the field of model based control – are described. As the examples present, the inverted pendulum experiment is suitable to use it in teaching – beyond the conventional ones – the more contemporary topics of the control science.

Keywords: education, optimal control, linear identification.

1. Introduction

Teaching control theory and practice has got great significance in the recent years in several areas of the undergraduate and graduate level technical education. For many years the researchers of the System and Control Laboratory of the Computer and Automation Research Institute are involved in teaching control science within the educational program of the Department of Control and Transport Automation of the Technical University of Budapest. As the background which the transportation engineering students possess differs from that of e.g. control engineering students, a significant point in teaching control science must be the good interpretation of theoretical results. Computer simulations are not sufficient, really operating

devices are preferable for the purpose of understanding the objectives, requirements, means and methods of control actions. Therefore the attention has been oriented to some real experiments which represent the control actions in descriptive enough manner. The inverted pendulum experiment has found to be optimal for the purposes stated:

- The effect of the control actions is obvious – without control, the rod of the inverted pendulum falls down.
- As being a basically mechanical system, it fits to the way of thinking of students of mechanical or transportation engineering.
- The structure of the inverted pendulum device is quite simple, it can easily be modeled – by applying the most fundamental rules of physics.
- The quality requirements to set up for the controlled system can easily be expressed on the basis of common sense considerations, e.g. small deviations of the rod, fast return to the original position at a disturbance, indifference to disturbances, etc.

The System and Control Laboratory in collaboration with the Department of Control and Transport Automation of the Technical University of Budapest has built multiple inverted pendulum devices for classroom usage. The main design criterion has been the simplicity, therefore moving the cart has been solved with a DC servo motor driven by a power amplifier, cart position and rod angle measurements have been implemented by using potmeters (see *Fig. 1*), i.e. the inverted pendulum is totally analog device. Direct digital control can be realized by means of a computer complemented with analog to digital and digital to analog converters. IBM compatible personal computers can be used based upon Intel processors, preferably 486 or Pentium ones. Several types of AD/DA cards can be used including e.g. data acquisition products of National Instruments.

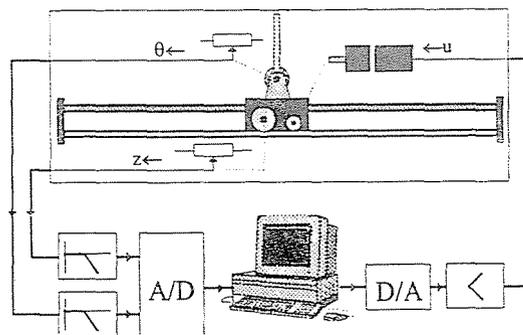


Fig. 1. Schematic diagram of inverted pendulum

The control program – it can be considered as a control environment – has been developed by the Systems and Control Laboratory. It has been written on the basis of LabWindows/CVI for WindowsTM environment,

product of the National Instruments (Austin, TX), which has provided excellent user interface and means of graphical representation.

The control environment will be described in the Section 2. Then a brief outline of the modeling issues and the design of a nominal controller will be given Section 3. Section 4 contains some didactic examples from the field of the model based control design, providing evidence for the strength of the system realized.

2. The Control Environment

The inverted pendulum control environment developed by the authors has been realized as a Microsoft WindowsTM application. The program assumes some type of AD/DA card – several installable versions are available for different platforms.

The control environment represents not only a control algorithm, but consists of several features which supports a higher level control design such as iterative identification based design schemes.

The basic features of the environment are enumerated as follows.

The control environment provides a discrete real-time direct control facility with configurable timing characteristics. The control algorithm can be downloaded to the environment. The downloadable form is a MATLAB compatible file (namely MAT), which contains the state-space form of any discrete linear controller, i.e. a series of four matrices, $A_{n \times n}$, $B_{n \times n}$, $C_{2 \times n}$ and $D_{2 \times 1}$, respectively, where n is a user selectable order of the controller. This form enables the user to design any linear controller he/she wants, including optimal and robust controllers.

In regular undergraduate education the principles and methods of the classical control theory are applied, hence the method of pole-placement and simplified optimal LQ design has been introduced as laboratory experiments. In these schemes the connection between the performance properties of the designed closed loop system and the methods or design parameters (values of weighting factors, location of poles, etc.) can be indirectly observable. At graduate level courses the linear quadratic and Gaussian (LQG) controller design, the Kalman filtering based observer design (ANDERSON & MOORE, 1989). The higher level courses can deal with the model based control design, which encloses the identification of the structure and of the model parameters, as well as the estimation of the plant uncertainties. In this way students can comprehend the fundamental principles of the robust (postmodern) control theory (MACIEJOWSKY, 1989). Later in this paper some examples for model based control design from the graduate course will be illustrated.

Besides the real-time control the environment contains an inverted pendulum simulator which is aimed to serve as a preliminary test of the new

algorithms to avoid damages of the hardware. There is a built-in nonlinear simulator as well as possibility to download any type of linear simulator of MATLAB form.

The control environment offers a menu-driven mode to configure the system for a given inverted pendulum hardware, to test and calibrate the configuration, as well as gives possibility to look graphically the time-functions of the significant system variables in history-like manner with configurable extent. A sample screen of the environment with an operating simulation can be seen in *Fig. 2*.

Beyond the basic ones the control environment provides several advanced features, these are outlined as follows.

The control environment support identification and iterative control design schemes. There are two built-in facilities to support them: data acquisition on the most significant system variables and application of disturbances on them.

Data acquisition is based upon the history mentioned above. The user has the possibility to store a selected part of the history (of course after a successful operation of the inverted pendulum control) in a MATLAB compatible form. These records can be used in any identification procedure implemented in MATLAB.

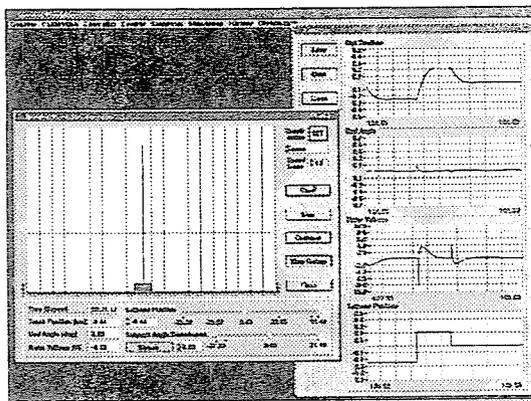


Fig. 2. A sample screen of the environment

The disturbance facility is also based upon MATLAB. The user can design several disturbances (deterministic or stochastic signals, e.g. sine wave, Gaussian noise process, etc.) to be applied on system variables in additive manner. The disturbances can be downloaded to the control environment, and assign them to several system variables as well as to switch on or off depending upon the test experiment to be performed. The environment also has local capabilities to generate simple disturbances. The use of iden-

tification and disturbances will be introduced in detail in section 4 of the paper.

3. Modeling and Control

First step of the control design must be the construction of a nominal model of the inverted pendulum. In the stated framework this can be a linear state-space model. The starting point of the model construction can be the Newtonian laws of mechanics.

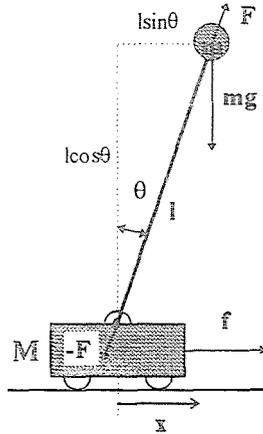


Fig. 3. Modeling the inverted pendulum

By applying them on the schematic view of system seen in Fig. 3, we obtain the nonlinear equations as follows (see for detailed derivation e.g. (OGATA, 1990)):

$$\begin{aligned} (M + m)\ddot{x} - ml\dot{\theta}^2 \sin \theta + ml\ddot{\theta} \cos \theta &= f, \\ m\ddot{x} \cos \theta + ml\ddot{\theta} &= mg \sin \theta. \end{aligned}$$

After applying $\sin \theta \cong \theta$, $\cos \theta \cong 1$ for small angles and $\dot{\theta}^2 \cong 0$ for the uppermost position of the mass m , we obtain linear differential equations, which with the $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \vartheta$, $x_4 = \dot{\vartheta}$ selection of the state-variables can be expressed in the state-space form as follows:

$$\begin{aligned} \dot{x}_1 &= x_2, & y_1 &= x_1, \\ \dot{x}_2 &= -\frac{m}{M}gx_3 + \frac{1}{M}f, & y_2 &= x_3, \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \frac{M + m}{Ml}gx_3 - \frac{1}{Ml}f. \end{aligned}$$

The input of the model is force f influencing the cart which must be substituted with the input voltage of the DC servo motor (see *Fig. 1*). A modified form of the equations can be derived by applying the torque/revolution and torque/current characteristics of the DC servo motor (see e.g. the product catalogue of MAXON motors (MAXON, 1992, p. 123)) supposed to be linear ones, as well as the mechanical relations between the rotational and linear motion. The modified state-space equations can be seen as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{R_m}{M} \left(\frac{K_m K_g}{R_m r} \right)^2 & -\frac{m}{M} g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{R_m}{Ml} \left(\frac{K_m K_g}{R_m r} \right)^2 & \frac{M+m}{Ml} g & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \frac{K_m K_g}{R_m r} \\ 0 \\ -\frac{1}{Ml} \frac{K_m K_g}{R_m r} \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

where u is the input voltage of the DC servo, R_m is the armature resistance, K_m is the torque constant, K_g is the mechanical attenuation of the rotational motion, r is the radius of the wheel transforming the rotation into linear motion. The dynamics of the motor armature, the mechanical uncertainty of the coupled mechanical elements, as well as any friction is neglected in this simplified description.

The system represented by the model – as it can easily be shown – is unstable, however, it is controllable. A bit difficulty occurs in the control design by the fact that it is not a minimum phase system.

A nominal controller, which can be served as a starting point of applying any iterative scheme for model based control, can simply be designed on the basis of optimal linear quadratic control. As the state of the system is not fully measured a state observer must be established. The design of an optimal linear quadratic controller can be realized with the procedures of the Control Toolbox of MATLAB. The output of the MATLAB procedure is the discretized state-space form of the controller, which can directly be loaded into the inverted pendulum control environment.

The transfer characteristics of the nominal controller designed on the basis of the nominal model can be seen in *Fig. 4* (step response of the closed loop). An LQ optimal full state observer has been realized with covariance matrices with diagonal elements $\langle 0.01, 0.1, 0.05, 0.5 \rangle$ for process noise and $\langle 0.01, 0.01 \rangle$ for measurements noise, respectively. The weighting matrix used for LQ feedback gain computation has also been diagonal with elements $\langle 0.25, 0.01, 4.0, 0.01 \rangle$ and input weighting has been 0.0003. The design procedure has resulted in quite good closed loop characteristics, sufficiently fast settling and small deviations around the desired state.

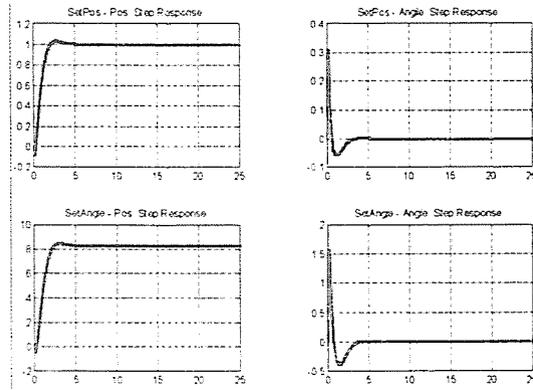


Fig. 4. Step response characteristics of control

4. Didactic Examples

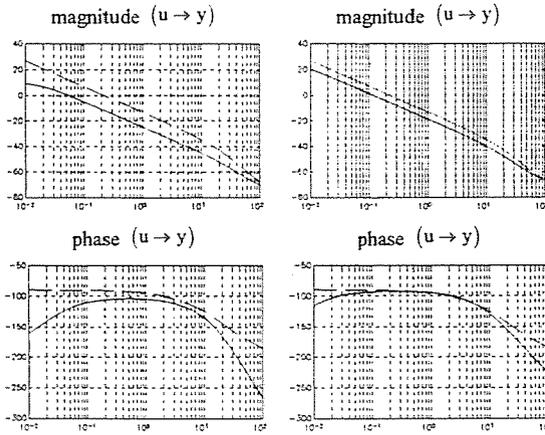
The inverted pendulum experiment seems to be an excellent means to illustrate and test principles and methods of the postmodern control theory, and can be served as a good basis for the use in teaching. In this section some examples will be presented – selected from the field of model based control – to illustrate the strength of this approach.

While the inverted pendulum is an unstable system so the identification has to be performed in closed loop, namely applying data collected in closed loop on the plant while an initialized and known controller operates. The closed loop identification can be performed on the one hand in classical way using direct or indirect method (SÖDERSTRÖM & STOICA, 1989) and on the other hand applying modern methods, which take the feedback connection among signals into account, e.g. two stage method, coprime factorization method or dual Youla parametrization method (VAN DEN HOF & SCHRAMA, 1994).

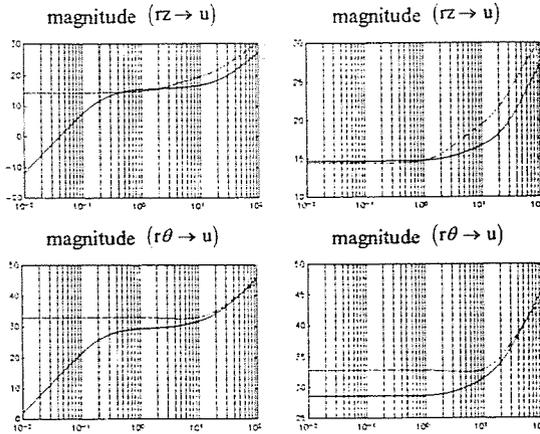
In Fig. 5 the magnitude and phase frequency response of two kinds of identified transfer function between the motor input voltage and the cart position are demonstrated. Figures of the left hand side show the result of the direct method, while figures of the right hand side illustrate the result of two stage method. The orders of the autoregressive and input operators are selected as 4. The solid lines show the plots of the identified model, while the dashed lines show the theoretical plots.

Although this example illustrates an identification of a single-input single-output (SISO) system in the laboratory experiments there is the possibility to investigate the identification of the multi-input multi-output (MIMO) system.

Based on the identified model a state feedback gain matrix and an estimator (or filter) gain matrix are designed, then the combined linear



Direct method *Two stage method*
 Fig. 5. Bode plots of the identified models



Controller *Reduced controller*
 Fig. 6. Magnitude plots of the controller

quadratic and Gaussian (LQG) controller is formed. On the left hand side of Fig. 6 the solid lines represent the magnitude plots of the designed controller and the dashed lines show the plots of the optimal controller based on the theoretical model. In the laboratorial experiments there is the possibility to examine the effect of changing the weighting factors and to compare the different designed controller in the sense of performance characteristic of the closed loop system. In this example the dimension of the designed controller is 12, which can be reduced as follows.

The well known model reduction methods can be applied for controller reduction, e.g. balanced realization or minimal realization, but those meth-

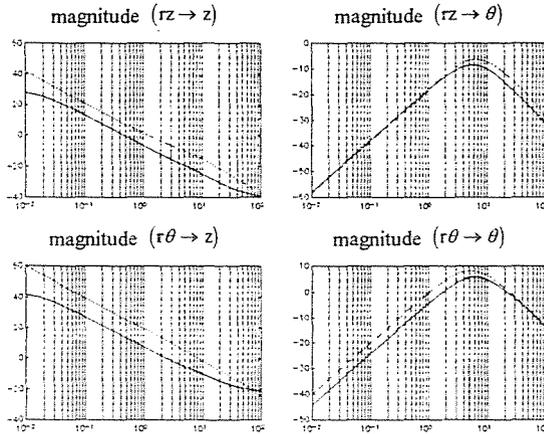


Fig. 7. Bode plots of the open loop system

ods can be also used which were developed for controller reduction purpose, e.g. closed loop balanced reduction, coprime factorization reduction, etc., (LIU & ANDERSON, 1990). These latter methods perform the dimension reduction taking the performance criterion of the closed loop into account. In this example the controller reduction has been performed by frequency weighted balanced truncation method. The order of the reduced controller is 8. The magnitude plots of the reduced controller can be found on the right hand side of Fig. 6.

The magnitude plots of the open loop can be seen in Fig. 7. The solid line represents the designed open loop based on the identified model, while the dashed line illustrates the nominal open loop. It can be seen, that the designed frequency functions are close to the nominal functions in the whole frequency domain.

The following two examples illustrate the behavior of the designed controller in the actual loop. The displacement and the angle of the pendulum measured in 30 sec time domain can be seen in Fig. 8. In this case additive extra disturbances of displacement and of the angle are not introduced to the system. In the last example a reference signal has been introduced

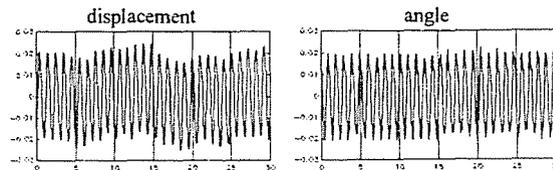


Fig. 8. Examination of LQG controller

for the displacement while the angle is disturbed by a random signal. In

Fig. 9 it can be seen that the output displacement signal slowly follows the reference signal. Of course, it is possible to investigate other solutions for reference signal tracking, e.g. the augmentation of the LQG controller by an integrator term or the solution of a proper prefilter design.

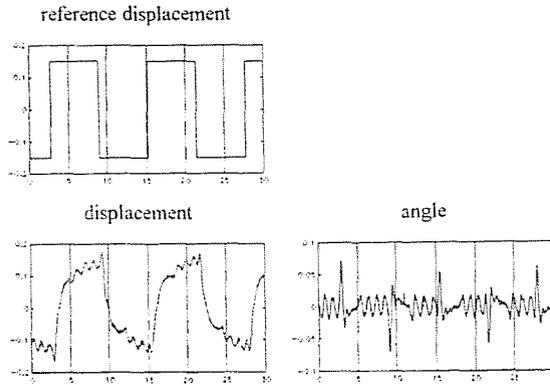


Fig. 9. Examination of reference signal tracking

5. Conclusion

As the didactic examples presented, the inverted pendulum experiment is suitable to use in teaching – beyond the conventional ones – the more contemporary topics of the control science, e.g. robust and model based control. Laboratory experiments are based upon MATLAB environment and its toolboxes, which are well known by most of the undergraduate and graduate students. The control environment developed by the authors results in a compact solution to the whole experiment, including hardware handling, real-time control, identification, and design of control algorithms.

Acknowledgements

The authors would like to express their thanks to Károly Kurutz and Sándor Sárközi, Professors of the Department of Control and Transport Automation of the Technical University of Budapest, for their cooperation in producing inverted pendulum devices and introducing this field in the educational program. This project has been supported by the Hungarian National Science Foundation through grant. no. T-016418.

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