NON-LINEAR SYSTEM AND MODAL ANALYSIS

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Abstract

The modal analysis can only be applied for the analysis of linear systems with constant coefficients. Several objects could be practically described by a linear multidegree of freedom mechanical system if there would not be present local, frequently strong nonlinearities. Is it possible to describe the system in the reduced modal subspace if it contains nonlinear elements and if yes how is it?

Keywords: nonlinear system, modal analysis, modal reduction, base system, impact study, loading machine.

1. Introduction

The modal analysis means the investigation of dynamic systems with the aid of modal co-ordinates. These co-ordinates form the so-called modal subspace. In this space the set of the equations which describes the motion of the system is a set of linearly independent equations. It means that the investigated mechanical object can be assembled from subsystems, i.e. can be synthesised (modal synthesis). The advantage describing the system with the aid of modal co-ordinates is just in this! The motion of a multidegree of freedom system can be approached within a prescribed tolerance with the aid of some well-selected modal co-ordinates (modal reduction).

However, the modal analysis can only be applied for the analysis of linear systems with constant coefficients (time invariant systems). The linearity is necessary for the applicability of the principle of superposition, while time invariancy is the condition for the interpretability of the eigenproblem.

On the field of transportation we can find several objects which could be practically described by a linear multidegree of freedom mechanical system if there would not be present local, frequently strong nonlinearities. In this way, e.g. at the investigation of a loading machine there is the strongly nonlinear buffer (*Fig. 2*). In the case of a vehicle moving on a poor road to take into account the jumping of the wheel requires the investigation of a nonlinear system. At the same time the shock absorbers are modeled with their nonlinearities (*Fig. 1*).

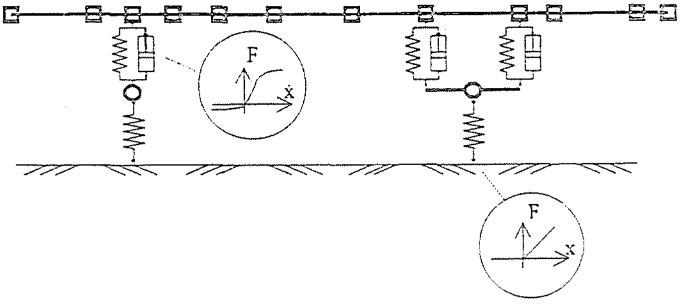

Fig. 1. Vehicle model with elastic body

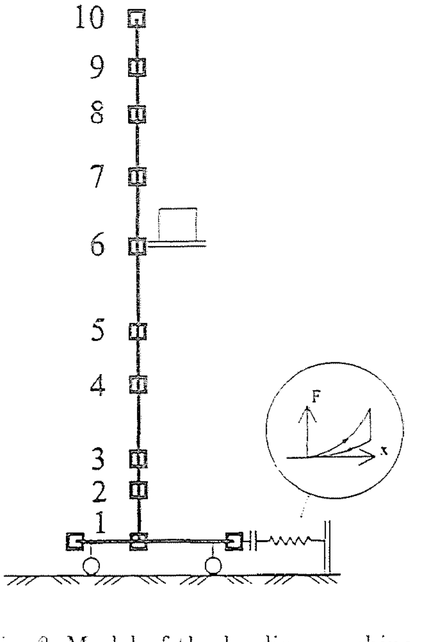

Fig. 2. Model of the loading machine

It asks for a vote of confidence: is it possible to describe the system in the reduced modal subspace if it contains nonlinear elements and if yes how is it? As an example we shall investigate the impact of a loading machine.

2. Theoretical Considerations

The response function of a nonlinear system is practically always produced by numerical integration. The solver subroutines for differential equations approach the values at the time $T + \Delta t$ on the basis of finite differences. It means that at the time T we have a differential equation with constant coefficients. For this equation the modal condensation is applicable.

It is enough to know the modal co-ordinates of only one linear system as a base system. This base system is possible to be arbitrary, but by the experiences it is advantageous to choose a quasi-equivalent linear system to the nonlinear system. We transform the mathematical model of the object that the nonlinear force functions contain only the nonlinear differences with respect to the base system and they present on the right hand side of the equation as force excitation.

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{x}} &= \mathbf{F}(\mathbf{x}, \dot{\mathbf{x}}, t), \\ \mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\dot{\mathbf{x}} + \mathbf{S}\mathbf{x} &= \mathbf{F}_b(\mathbf{x}, \dot{\mathbf{x}}, t)_{t=T}. \end{aligned} \tag{1}$$

At the numerical solution of the *Eq.* (1) for the time Δt the excitation force is constant. By this reason it is possible to transform the equation of motion into the modal subspace of the base system.

$$\mathbf{E}\ddot{\mathbf{q}} + \Delta\dot{\mathbf{q}} + \Lambda\mathbf{q} = \mathbf{T}^{*}\mathbf{F}_b(\mathbf{x}, \dot{\mathbf{x}}, t)_{t=T} = \Phi_{t=T}. \tag{2}$$

And for the *Eq.* (2) the modal condensation is applicable! The values of the n elements of the displacement field at the time $T + \Delta t$ can be estimate approach by the back transformation of the values at time $T + \Delta t$ of some subspace co-ordinates. We can estimate the number of the subspace co-ordinates that we need to take into account on the basis of the type of the object, the investigated response sign, of the trend of the excitation.

3. Investigation of the Impact of the Loading Machine

The finite element model of the loading machine is simplified for the dynamic calculations. We take into consideration only some specific nodes on the column (static condensation). The dynamic model contains only the horizontal displacements and the number of the degrees is 10. The internal damping of the system is modeled proportional to the stiffness. By this way the model is a linear, time invariant system. But the force at the impact is nonlinear, because

- it is proportional to the square of the relative shortening;
- it depends on the direction of the change on the compressive range (hysteresis)

- it can apply an effect on the structure only in a restricted range (separation)

At the analysis of the impact process out of the direct solution of the equation of motion two 2 degree of freedom modal systems with different base systems were investigated. In one case the base system is the model of the loading machine moving freely on the rails. In the second case there is a flexible support with linear spring.

The 'time photo' of the impact is recorded with a time window of $\Delta t = 0.01$ [s] (*Fig. 3*). The record starts when the loading machine as a rigid body with constant speed touches the buffer moving from the left to the right.

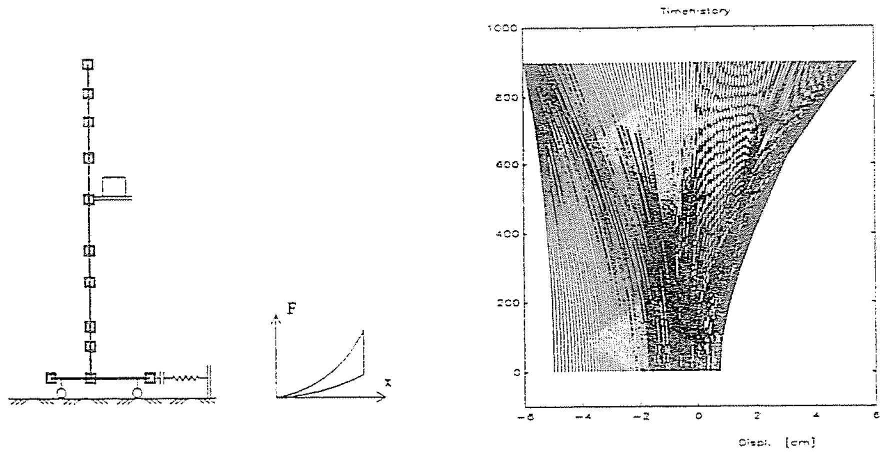

Fig. 3. Loading machine impact study (displacement of the beam, time photo)

The *Fig. 4* presents together the impact force and the displacement of the particle in contact to the buffer.

It can be concluded that

- the change of the force is practically independent of the structure of the model
- the motion of the particle only at the fixed base system differs to practically identical characteristics of the other two systems only over the separation.

The reliability of the applied synthesis is characterised by the deviation functions of the displacements with respect to the direct response signs.

If we choose a free system for base system then the motion of the loading machine can be approached within 1 mm precision (1.5% relative precision) taking into consideration the first two bending modes.

The fixed base system shows a bigger difference.

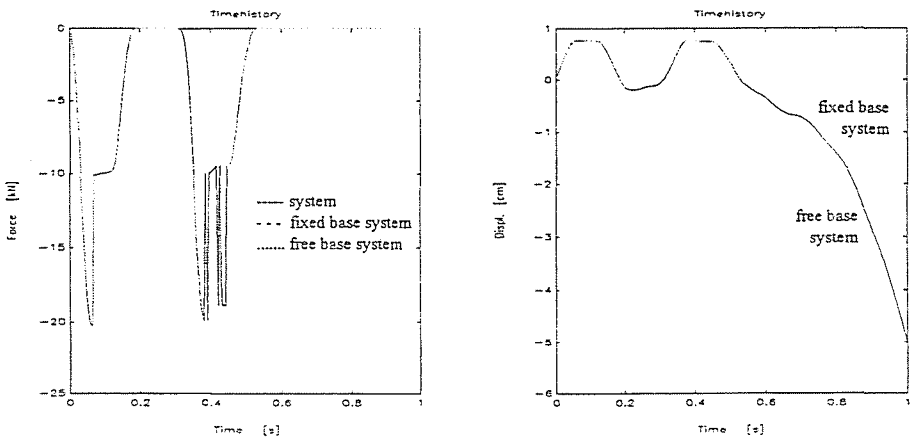

Fig. 4. Impact force and displacement of the particle in contact to the buffer

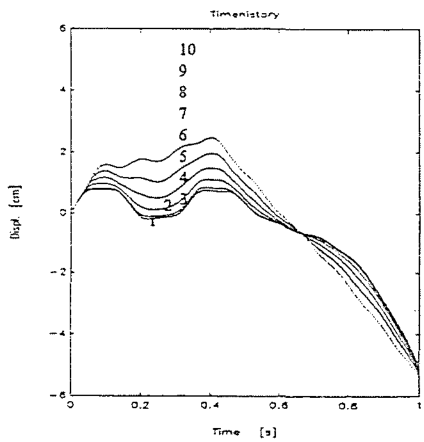

Fig. 5. Displacements of the loading machine under impact (direct response signs)

At the analysis we use linear functions at the buffer spring, too, instead of the parabolic one (*Fig. 7*).

In this case we can only repeat the results of the comparative test.

Finally we can state that

- using a free system as a base system model of the loading machine with two modal co-ordinates it is applicable for the investigation of the impact. It can replace the multidegree of freedom model of the direct method and
- verify the property of the applied method.

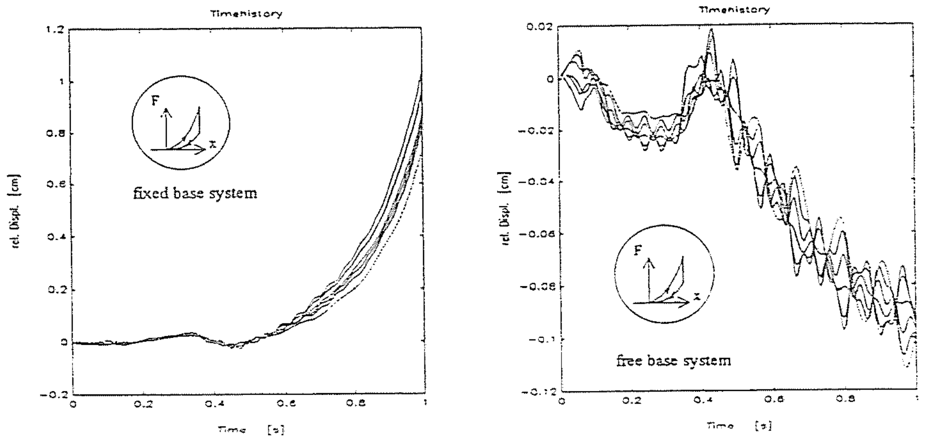

Fig. 6. Deviation (quasi error) functions of the time histories

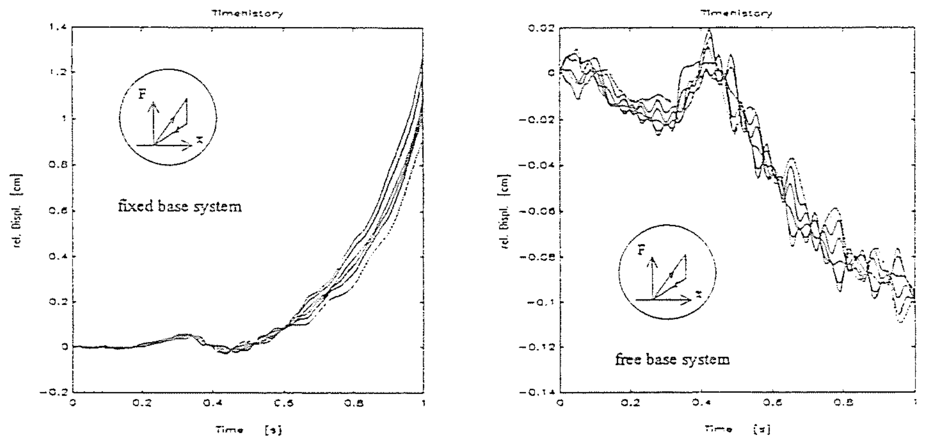

Fig. 7. Deviation (quasi error) functions of the time histories

4. Concluding Remarks

On the basis of the theoretical investigations and numerical simulations, the following conclusion can be drawn:

- the applied method can be used, but further object specific investigations are required (e.g. analysis of vehicles).

References

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