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NON-LINEAR DYNAMIC ANALYSIS OF VEHICLES USING LARGE FINITE ELEMENT MODELS

István KUTI
Department of Vehicle Frame and Lightweight Structures
Technical University of Budapest
H-1521 Budapest, Hungary

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Abstract

In the dynamic analysis and design of vehicles the behaviour of suspensions and tyres beyond the linear range is often interesting. In order to get acceptable responses it is necessary to apply large mechanical models with many degrees of freedom. In this paper a computational method and a computer program, developed for the dynamic analysis of elastic systems containing local non-linearities are presented. The applicability of this method and program is demonstrated by numerical experiments using a 648 degrees of freedom flat-bed truck finite element model.

Keywords: nonlinear analysis, vehicle dynamics, modal analysis, modal condensation, transient vibrations

1. Introduction

Nowadays, in the design or research of mobile machines and dynamically excited structures, besides experiments, the different numerical computational methods provide efficient tools. However, the solution of most large-scale (non-linear) dynamic structural problems is usually extremely time consuming. Therefore, in latest decades, considerable efforts have been made in the application and development of numerical methods, to increase their accuracy and speed of computation. For this type of problems the modal time history analysis, combined with modal condensation, seems to be as a successful way, where the local non-linearities are considered as pseudo forces [1], [2], [3].

The dynamic behaviour of road and off-road vehicles (cars, buses, trucks, cross country cars, agriculture vehicles, etc.) can be mentioned as typical examples for the previously discussed dynamical problems in case of certain driving conditions. For example, passing over road defects (bulge, hole) or driving on roads of wrong quality. In these cases the non-linear properties of vehicles may not be neglected.

With regard to preliminaries, in this paper, a computational procedure and a computer program is presented, developed for non-linear dynamic analysis of large elastic systems with local non-linearities. As it was mentioned above, the pseudo force method is applied to the calculation of internal forces that arise from the effects of local non-linearities [4]. From the point of view of practical applications, the elaboration of large mechanical (usually finite element) models can give rise to significant difficulties. Recently the use of commercial finite element programs seems to be the only reasonable way. In order to utilise this advantage of commercial finite element programs, the developed procedure consists of two phases. In the first phase, the linearized and undamped finite element model of the given structure can be elaborated, using any commercial finite element program. Great advantage of this way is that the required natural modes (natural frequencies and vectors) can be determined by these programs. Then, in the second phase, having considered the local non-linearities (springs, dampers, gaps, etc.) the dynamic analysis can be carried out.

To verify the applicability and efficiency of the developed method and computer program, using a flat-bed truck finite element model (with 648 degrees of freedom), numerical experiments are presented. The accuracy of the elaborated computer program in comparison with COSMOS/M finite element program will be demonstrated.

2. Mathematical Formulation

Assume that the dynamic equilibrium equation of the studied structure is described by n pieces of coupled second order ordinary differential equations, as follows,

$$[\mathrm{M}]\{\ddot{\mathrm{x}}\} + [\mathrm{K}]\{\mathrm{x}\} = \{\mathrm{F}(t)\} + \{\mathrm{N}(\{\mathrm{x}\}, \{\dot{\mathrm{x}}\}, \{\ddot{\mathrm{x}}\})\}, \tag{1}$$

where:

$[\mathrm{M}]$	=	mass matrix,
$[\mathrm{K}]$	=	stiffness matrix of the linear part of the structure,
$\{\mathrm{F}\}$	=	vector of time varying forces and kinematic excitations applied on the structure,
$\{\mathrm{N}\}$	=	pseudoforce vector of non-linear internal forces,
$\{\mathrm{x}(t)\}$	=	generalized displacement vector,
$\{\dot{\mathrm{x}}(t)\}$	=	generalized velocity vector,
$\{\ddot{\mathrm{x}}(t)\}$	=	generalized acceleration vector,
t	=	time.

Mode shapes, for modal time history analysis, are calculated from the left side of *Eq.* (1) in the next form,

$$\left(\lambda_i[\mathrm{I}] - [\mathrm{M}]^{-1}[\mathrm{K}]\right)\{\Phi_i\} = \{\mathbf{0}\}, \tag{2}$$

where:

$[\mathrm{M}]^{-1}$ = inverse of mass matrix,
$[\mathbf{I}]$ = identity matrix,
$\{\Phi_i\}$ = i-th natural vector,
λ_i = i-th natural value,
$\{\mathbf{0}\}$ = zero vector.

If the natural vectors are normalized to mass matrix, then natural values are the square of the corresponding natural frequency. The natural vectors as column vectors can be arranged, according to the ascending order of natural values, into the matrix $[\Phi]$, called the modal matrix. Modal displacement, velocity and acceleration vectors can be defined as,

$$\{\mathrm{x}\} = [\Phi]\{\mathrm{q}\}, \tag{3-a}$$

$$\{\dot{\mathrm{x}}\} = [\Phi]\{\dot{\mathrm{q}}\}, \tag{3-b}$$

$$\{\ddot{\mathrm{x}}\} = [\Phi]\{\ddot{\mathrm{q}}\}, \tag{3-c}$$

where $\{\mathrm{q}\}$, $\{\dot{\mathrm{q}}\}$ and $\{\ddot{\mathrm{q}}\}$ are the modal displacement, velocity and acceleration vectors, respectively. Substituting *Eqs* (3.a – c) into *Eq.* (1), then premultiplying by $[\Phi]^T$, the transpose of $[\Phi]$, we get,

$$[\Phi]^T[\mathrm{M}][\Phi]\{\ddot{\mathrm{q}}\} + [\Phi]^T[\mathrm{K}][\Phi]\{b\dot{\mathrm{q}}\} = [\Phi]^T\left(\{\mathrm{F}\} + \{\mathrm{N}\}\right), \tag{4}$$

where $[\Phi]^T[\mathrm{M}][\Phi]$ is the $n \times n$ identity matrix, and matrix $[\Phi]^T[\mathrm{M}][\Phi]$ is a diagonal one and its diagonal elements are equal to the square of the corresponding natural frequencies. Thus the modal differential equations can be written as,

$$\ddot{\mathrm{q}}_i + \omega_i^2 \mathrm{q}_i = \sum_{i,j} \Phi_{i,j}\left(\mathrm{F}_i + \mathrm{N}_i\right), \qquad i, j, = 1,\ 2,\ \ldots,\ n. \tag{5}$$

where subscripts i and j are the indices of the elements of previously applied matrices and vectors. When only the first m ($m \gg n$) pieces of modal differential equations are applied for dynamic analysis, the amount of computational time can significantly be reduced into an acceptable practical range (modal condensation). Modal and Rayleigh damping can additionally be included into *Eq.* (5).

3. On the Developed Computer Program

On the basis of previous relationships an algorithm has been elaborated and a computer program, called MODANAL, has been coded in MICROSOFT FORTRAN V.03. This program contains approximately *1200* FORTRAN statements.

As it was mentioned above, the complete, modelling and computational, process is divided into two phases. In the first phase the elaboration of the (undamped) linearized finite element model and its mode shape calculation can be carried out by the use of an appropriate commercial finite element program. Thereafter, in the second phase, having utilized data and results from the first phase and having prepared the additional data, concerning to the description of local non-linearities and external excitations, the non-linear dynamic analysis can be performed by the developed computer program. This program uses six input files, three output files and seven temporary files, detailed below.

Input files,

- file to store control parameters, description of non-linear characteristics and the specification of required output,
- file to store natural frequencies and vectors involving into the non-linear analysis (extracted from the output file of the applied commercial finite element program),
- file to store mass data of the whole model to the calculation of inertial forces (extracted from the input data file of the applied commercial finite element program),
- file to store initial displacements and velocities (initial displacements can be calculated, for example, by static analysis of the linearized finite element model, carried out by the applied commercial finite element program),
- file to store kinematic excitations,
- file to store external forces and moments.

Output files,

- file to store required displacements,
- file to store required velocities,
- file to store required accelerations.

To the numerical step by step solution Hamming's predictor-corrector method is applied. The characteristics of local non-linearities are described by piecewise linear functions, as it is usual in finite element programs.

4. Description of the Applied Truck Model

In order to demonstrate the applicability and effectiveness of the discussed computational procedure and the related computer program, a simple flatbed truck model has been elaborated, making use of COSMOS/M commercial finite element program (*Fig. 1*). In the suspension sets leaf springs are applied and their elasticity is considered at the extremities of the leaf springs, while the dampers (in suspensions) are positioned at the midpoint of the rigid spring arms, just above the truck axles. In *Fig. 1,* the damper and springs of the right hand side front suspension are shown up with short thick lines. The elasticity and damping of each tyre is represented by three springs and (viscous) dampers positioned in longitudinal, lateral and vertical directions. Linearized and non-linear suspension spring characteristic are shown in *Fig. 2,* where the non-linear sections of spring characteristic represent the upper and lower bumpers which limit the stroke of suspensions. The non-linear spring characteristic is shifted along the linear one, in accordance with the initial displacements arising from payload and the weight of the truck. Similarly, the linearized and non-linear suspension damper characteristics are represented in *Fig. 3.* The linearization of the non-linear damper is based on manufacturer's data, and both of them absorb the same kinetic energy in case of the prescribed frequency and stroke (1.67 Hz and 100 mm stroke). Anti-roll torsion bar (stabilizer) is built in between the chassis and rear axle.

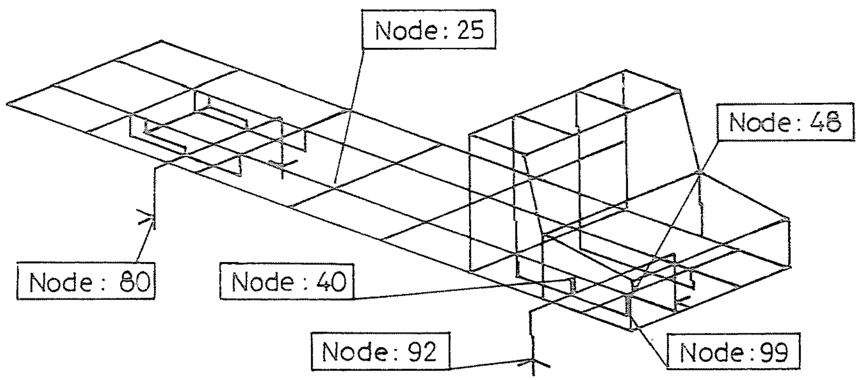

Fig. 1. Skeleton structure of the studied truck

Finite element model data:

Total number of nodes:	125
Number of degrees of freedom:	648
Total number of beam elements:	159
Total number of mass elements:	51
Total number of non-linear spring elements:	12
Total number of linear spring elements:	8
Total number of non-linear damper elements:	4
Total number of linear damper elements:	8
Total mass (payload is included):	13751kg

It is necessary to emphasize that this truck model has been developed for vertical excitation (its velocity is assumed to be constant during the analysis), only to demonstrate the applicability and effectiveness of the computational method and computer program presented here. In this truck finite element

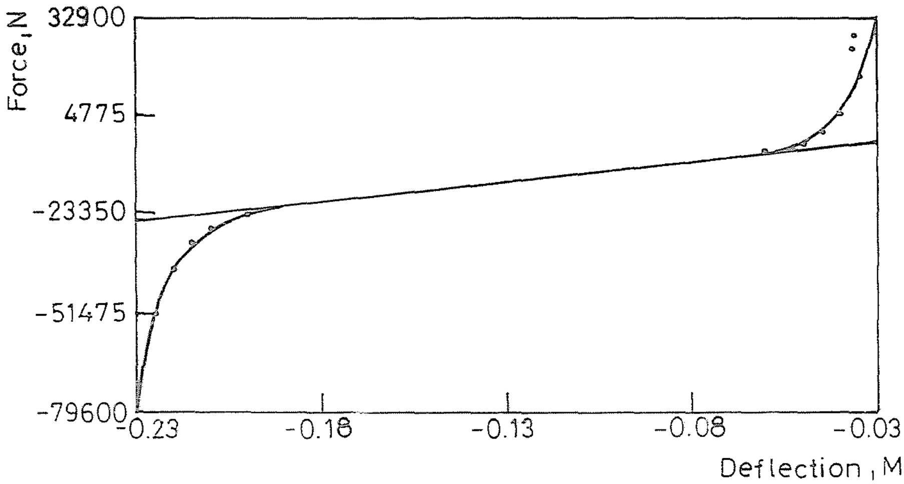

Fig. 2. Spring characteristics (in suspensions)

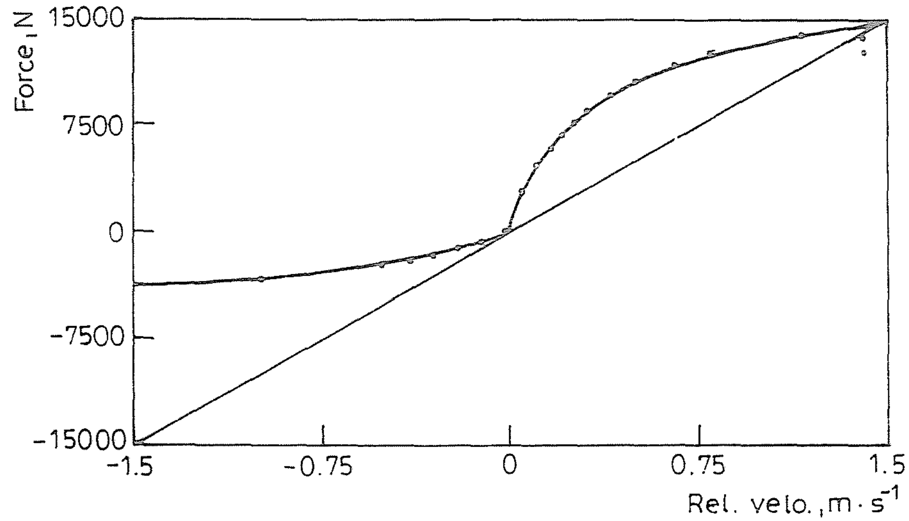

Fig. 3. Damper characteristics (in suspensions)

model, there can be found such kinds of simplifications which practically do not affect disadvantageously on the following demonstrations, however, they may not be applied in most cases in the dynamic analysis of actual structures. For example, the finite element model of the truck is not detailed enough for strength calculation, payload is connected to nodes, structural damping is considered roughly by Rayleigh damping (only to avoid the undesirable fictitious resonance because of the lack of internal damping), etc.

5. Verification of the Applied Method

The accuracy of the developed computer program was comprehensively investigated earlier. Herein a comparison is made between the presented computer program and COSMOS/M. For this purpose steady state harmonic force excitations are applied, in vertical direction, at the extremities of the front axle, in case of linearized springs and damper characteristics. Zero initial conditions, and no weight of the truck and payload, are applied. Amplitude of exciting forces is equal to 50000 N and their frequency is equal to 9.5 Hz. Calculation is carried out between 0 and 12.5 seconds in 5000 steps. There can be seen, in *Fig. 4*, the vertical displacement response of node No. 99 calculated by COSMOS/M. The difference, for this node between the responses calculated by COSMOS/M and MODANAL, is shown in *Fig. 5* It can be seen from *Figs 4 – 5* that the relative difference between the amplitude of these responses is no more than 0.45%, which is an excellent agreement. From computational aspect it is worth to mention that for numerical solution in COSMOS/M Newark's iterative method, while in MODANAL a predictor-corrector method is applied.

Usual principle in vehicle structural dynamics is that it is enough to apply the low natural modes up to 20 Hz. To confirm this principle the same harmonic kinematic excitation is applied on both tyres of front wheels in vertical direction. Its amplitude is 20 mm and its frequency, being proportionate to time, is swept from 0 Hz to 20 Hz. The linearized and non-linear response of node No. 25 (a structural point in chassis), for this sweeping excitation, can be seen in *Figs 6 – 7*, respectively. These figures confirm this reduction of the numbers of natural modes, since the amplitudes of vibration above 6 Hz significantly decrease in the function of frequencies. In correspondence with this principle, in this paper, the first thirty natural modes are applied for dynamic analysis. The magnitude of the highest natural frequency is equal to 21.45 Hz.

6. Numerical Experiments

6.1. Passing through a Bulge

In this example the better damping properties of non-linear dampers, in suspensions, compared to the linearized ones are demonstrated. For this purpose, assume that a small bulge is in the perfectly smooth road surface, positioned in lateral direction and described by a simple cosine function. Its length is 1.5 m and its maximum height, in the middle, is equal to 25 mm. In this case the spring deflections remain in linear range, therefore, the linearized and non-linear dampers can be compared directly. The truck is passing over this bulge with the speed of 72 km/h. Both, left and right,

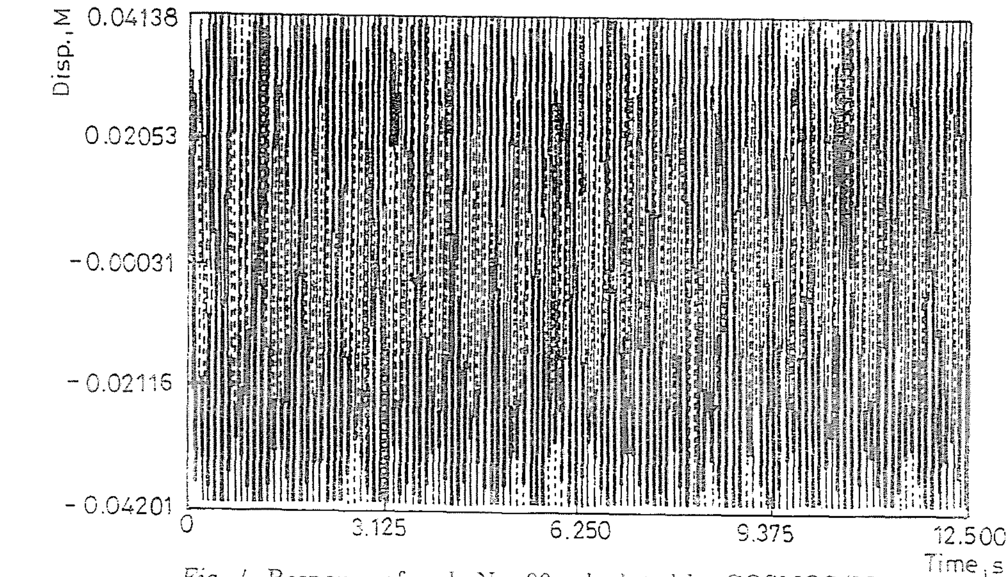

Fig. 4. Response of node No. 99 calculated by COSMOS/M

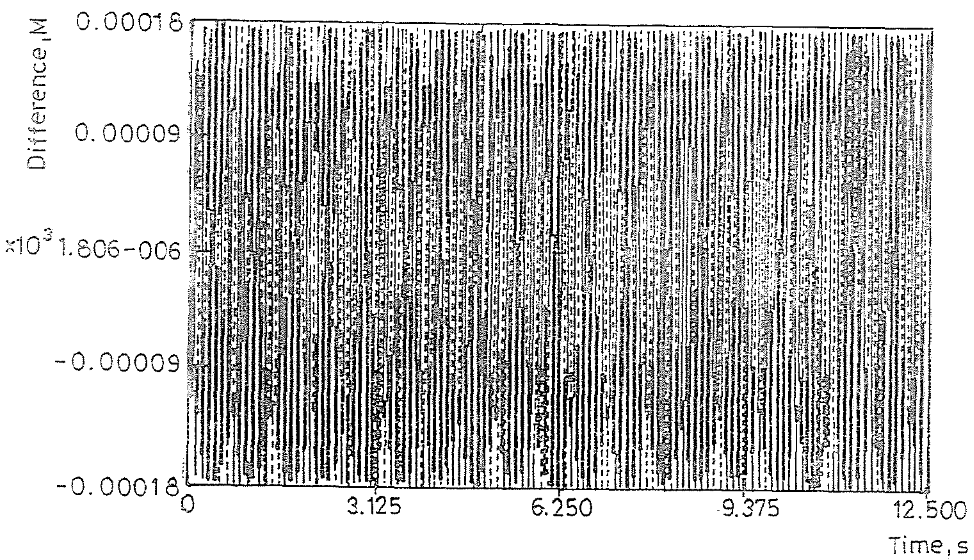

Fig. 5. Difference between responses of COSMOS/M and MODANAL

wheels are passing over it at the same instant. Time delay between the excitations of front and rear wheels is taken into consideration. In relation

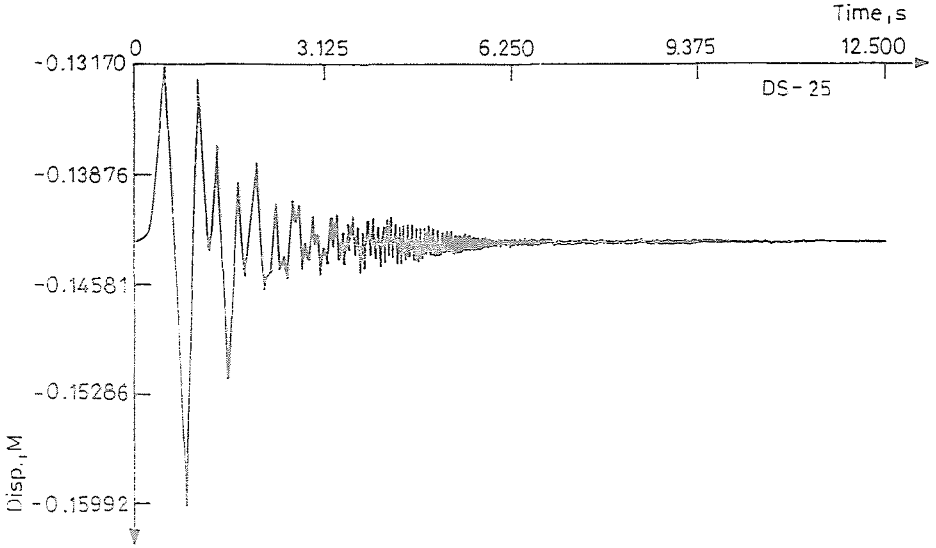

Fig. 6. Linearized response of node No. 25 for sweeping excitation

to this excitation, there can be seen the linearized and non-linear vertical displacement and velocity responses of node No. 40 in *Figs 8 – 9* respectively. Node No. 40 represents the point of attachment of the right hand side front suspension to the chassis (*Fig. 1*).

In *Fig. 8* the thick line shows the linearized, while the thin line shows the nonlinear responses. Similar marking is used for the velocity response in *Fig. 9*. Both figures show that the linearized responses are overestimated and, at the same time, there can be seen the better damping effects of non-linear dampers.

6.2. Driving on a Minor Road of Wrong Quality

In case of actual vehicles the stroke of suspensions is limited, which is taken into consideration here by the highly non-linear parts of spring characteristics (*Fig. 2*). When a vehicle is driving on a road of wrong quality, the undercarriages can collide with the body of vehicle at both extreme positions of suspensions. In *Fig. 10* there can be seen road profile realizations, generated from a two dimensional isotropic power spectral density function of road surface roughness [5], [6] ,[7]. It is assumed that the truck is driving on this road with a speed of 36 km/h. Initial displacements and the total

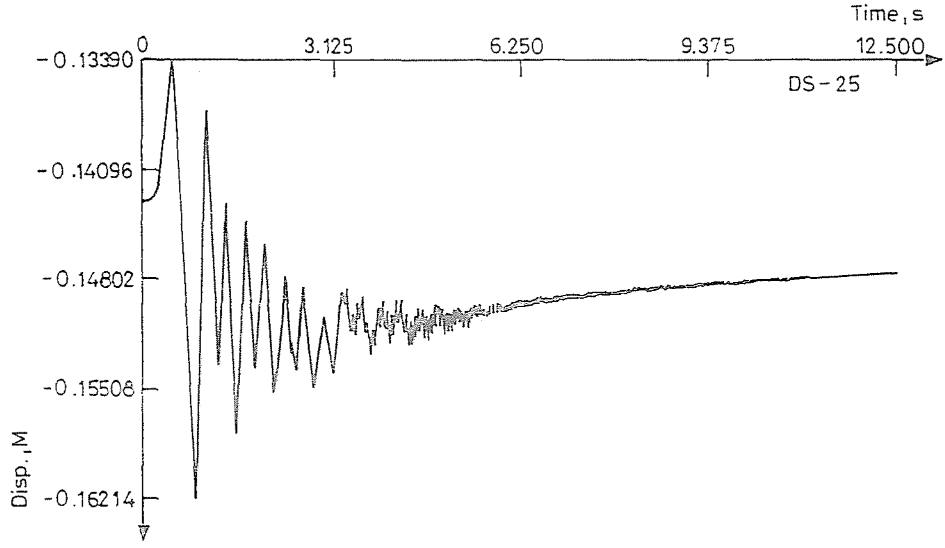

Fig. 7. Non-linear response of node No. 25 for sweeping excitation

weight of the truck are involved into the dynamic analysis. Time delay between the excitations of front and rear wheels is considered. In *Fig. 11* there can be seen the linearized and in *Fig. 12* the non-linear vertical acceleration of node No. 48 under the action of this excitation. Node No. 48 is the point of attachment of one of the front springs in the right hand side suspension above the front axis. The greater values in non-linear response (greater lower and upper peaks) with respect to the linearized one, are arising from the collisions of the front undercarriage to the chassis of the truck at both extreme positions of this suspension. From *Figs 11 - 12* it is clear that, in this case, the linearized response is underestimated.

6.3. *Demonstration of Wheel Bouncing*

In order to illustrate the wheel bouncing, the total mass (sum of the dead mass and payload) of the truck is decreased from 13751 kg into 6696 kg and its velocity is risen to 108 km/h. Total weight of the truck and the initial displacements under the action of total weight are included in the analysis. For the sake of better visualisation the roughness of the road surface is neglected. In *Fig. 13*, there can be seen the applied ramp. Its upward-slope portion is equal to 15 m and its downward-slope portion is equal to 30 m.

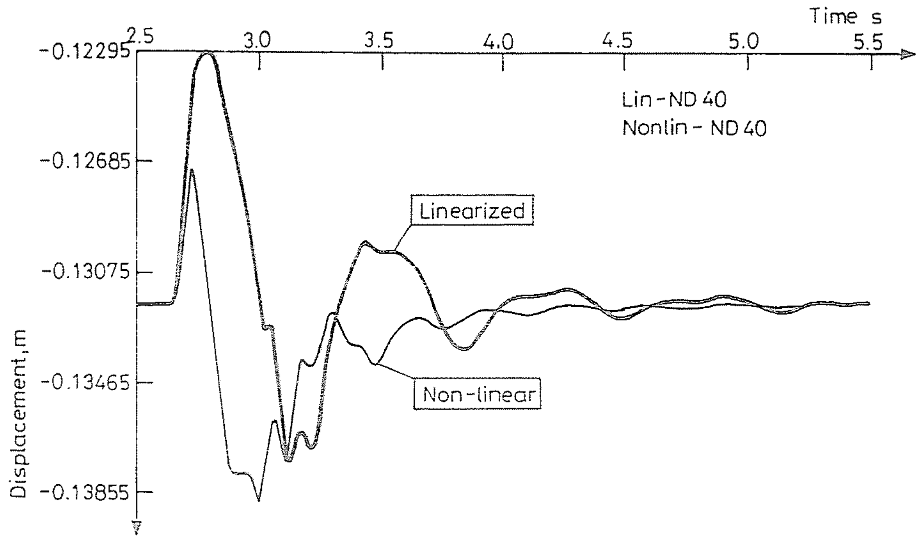

Fig. 8. Linearized and non-linear responses (displacement) of node No. 40

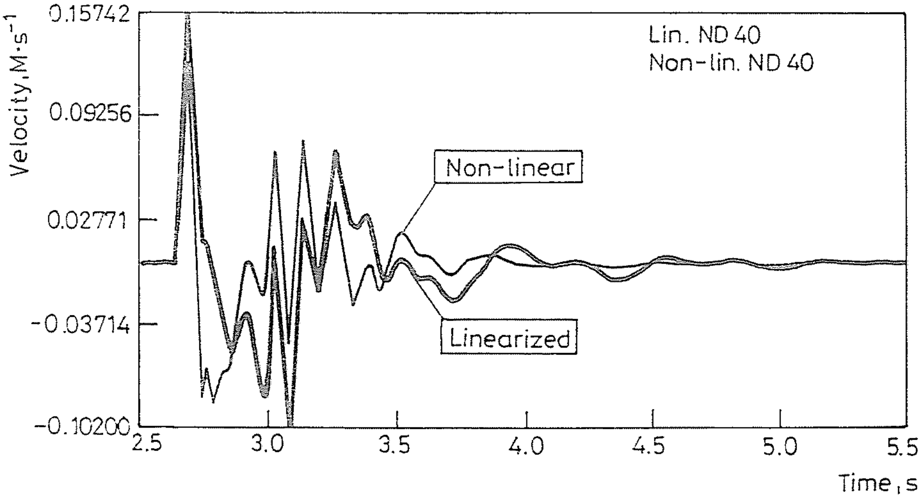

Fig. 9. Linearized and non-linear responses (velocity) of node No. 40.

In the upward-slope portion, the gradient of left track is different from the gradient of the right track. Therefore there are different vertical velocities of the left and right hand side wheels at the top of the ramp, originating a rotation of the truck along its longitudinal axis (rolling motion). *Fig. 14*

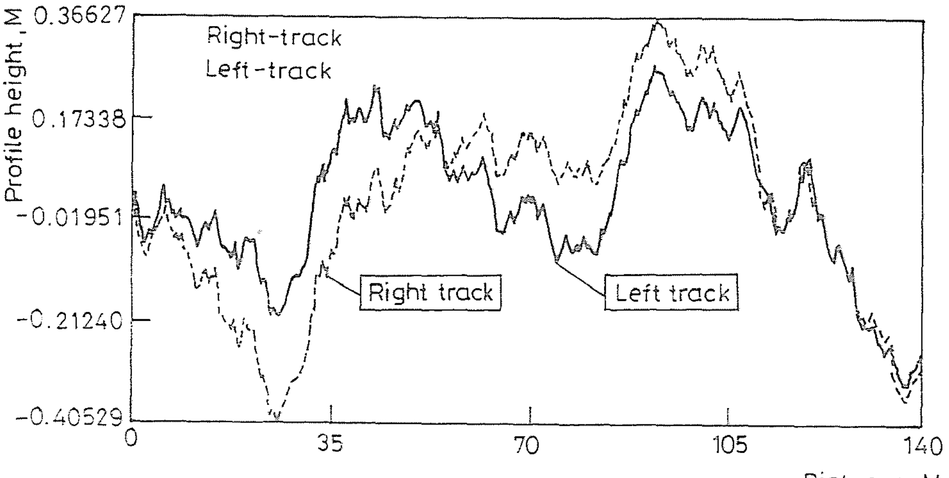

Fig. 10. Simulated road profiles

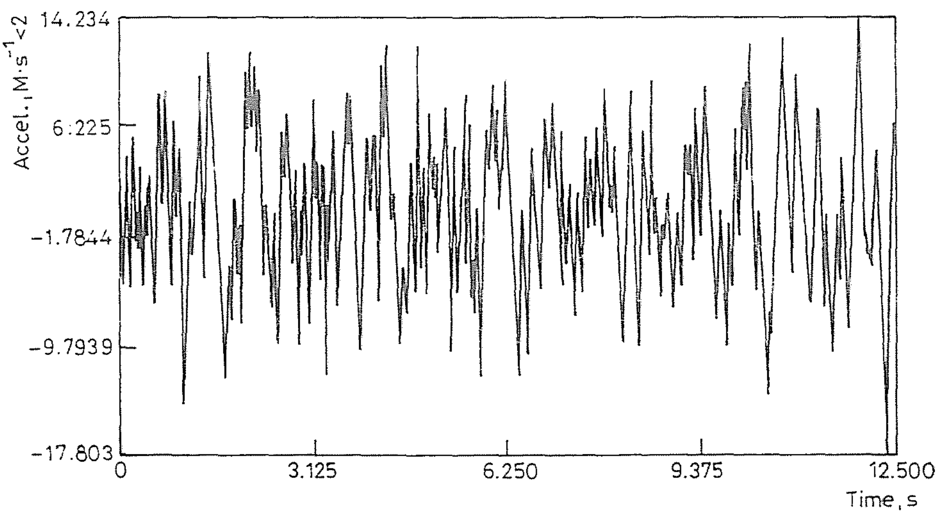

Fig. 11. Linearized response (acceleration) of node No. 48

shows the rotations of front and rear axles in radians. At the same time. because of this rotation, there exists a lateral motion of different elements of the truck. There can be seen in *Fig. 15* the lateral motion of the centres of the right hand side wheels.

In *Fig. 16* the vertical position of node No. 80, connected to the road surface by springs and dampers, is illustrated. Node No. 80 represents the lowest point of the right hand side rear wheel disc. The thick piecewise linear line in *Fig. 16*, symbolizes the right track, while the parallel thin one represents the position of node No. 80 when the tyre is assumed to be

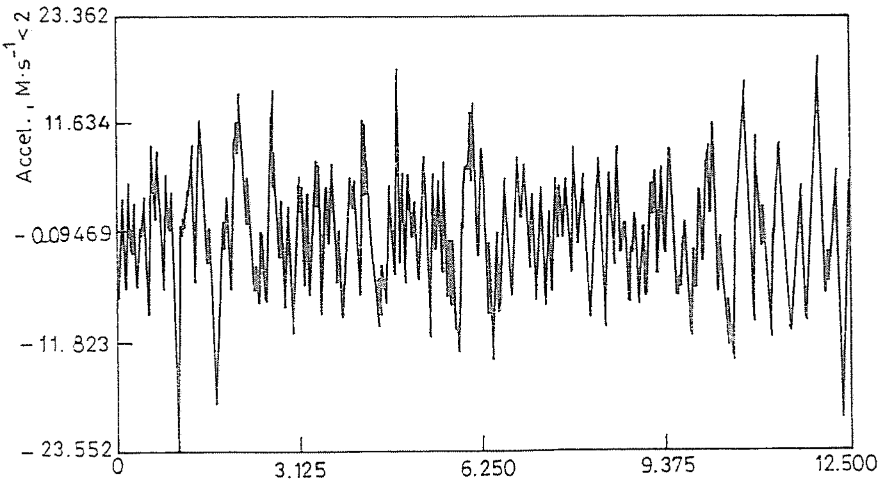

Fig. 12. Non-linear response (acceleration) of node No. 48

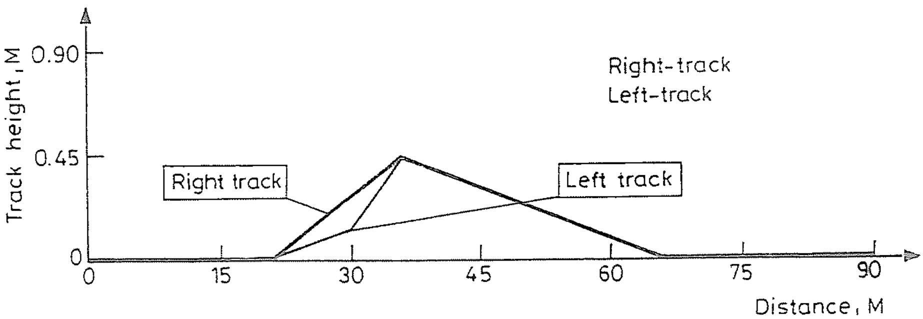

Fig. 13. Ramp to demonstrate wheel bouncing

unloaded. The distance between the two piecewise linear lines illustrates the vertical dimension of the unloaded tyre.

The curved line shows the vertical position of node No. 80 calculated from its excited displacement while the truck is passed over the ramp. When the curved line is below the thin piecewise linear line the wheel is in contact with the surface of the ground (ramp), and when it is above this line the wheel is bouncing. In *Fig. 17*, similarly to *Fig. 16*, the vertical position of the lowest point of the left hand side rear wheel disc is shown.

The magnitude of bouncing of a given wheel can be determined if the unloaded position of the lowest point of wheel disk is subtracted from its excited vertical position at each step. When it is greater than zero the wheel is bouncing. The magnitudes of the bounces of rear and front wheels are demonstrated in *Figs 18 – 19*. In *Fig. 18* there can be seen consecutive bounces of the right hand side rear wheel. An interesting thing can be seen

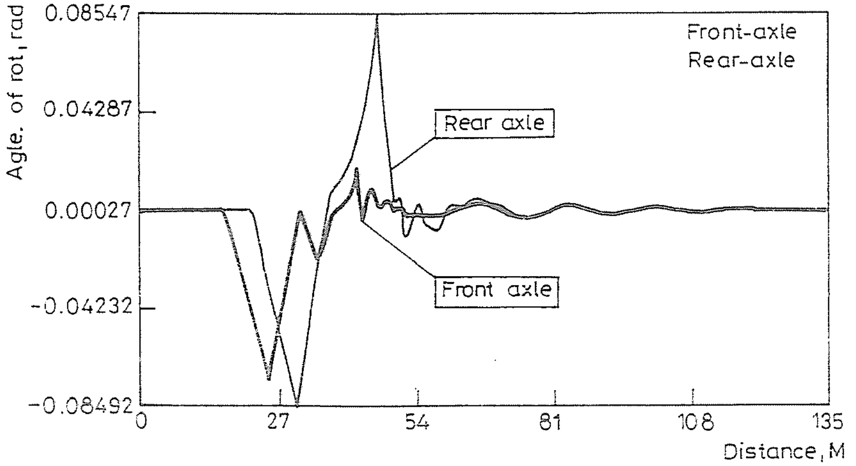

Fig. 14. Rolling motion of front and rear axles

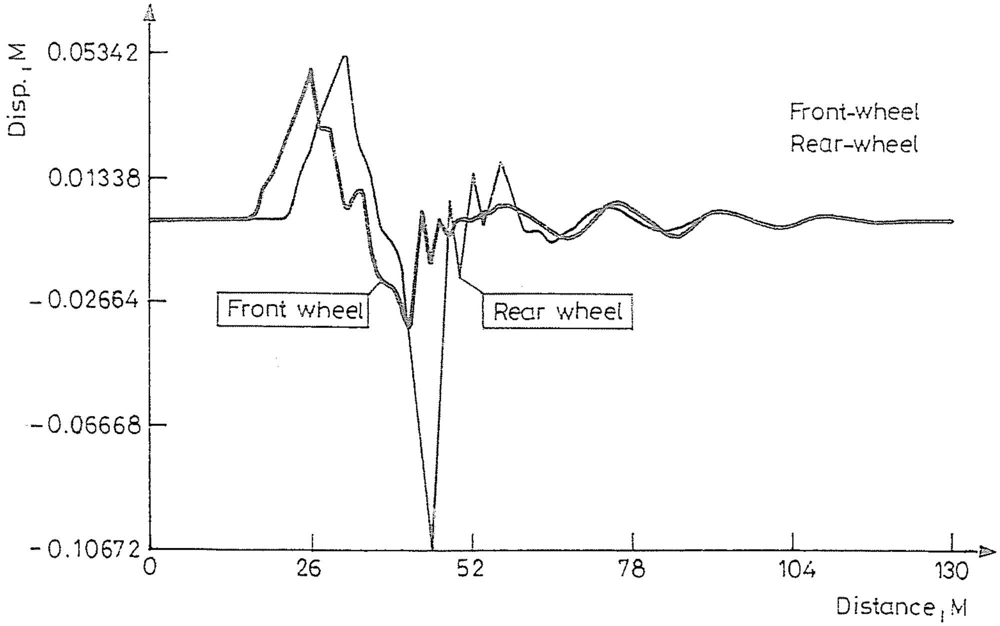

Fig. 15. Lateral motion of the right hand side wheels

in *Figs 18 – 19*, namely each wheel is bouncing at the same time from 1.209 s to 1.386 s, that is the truck is flying over the ramp 5.31 meters.

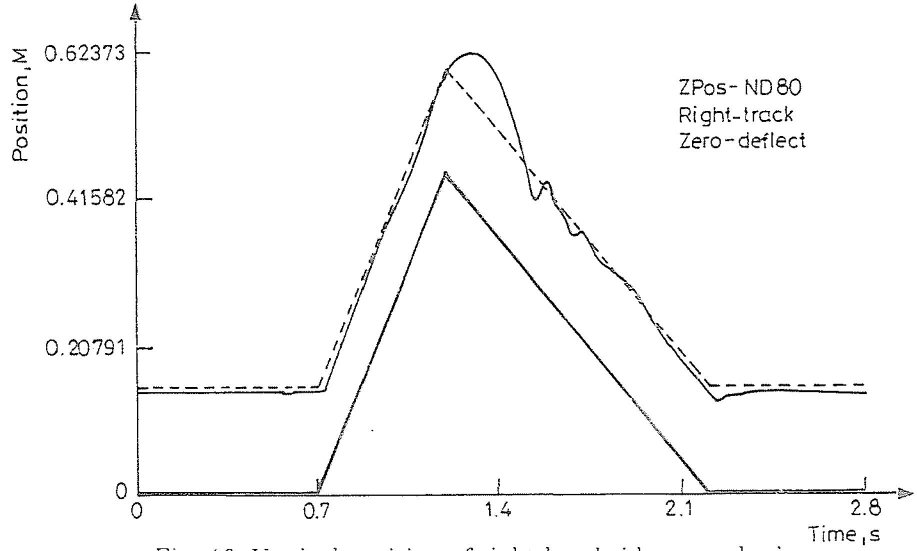

Fig. 16. Vertical position of right hand side rear wheel

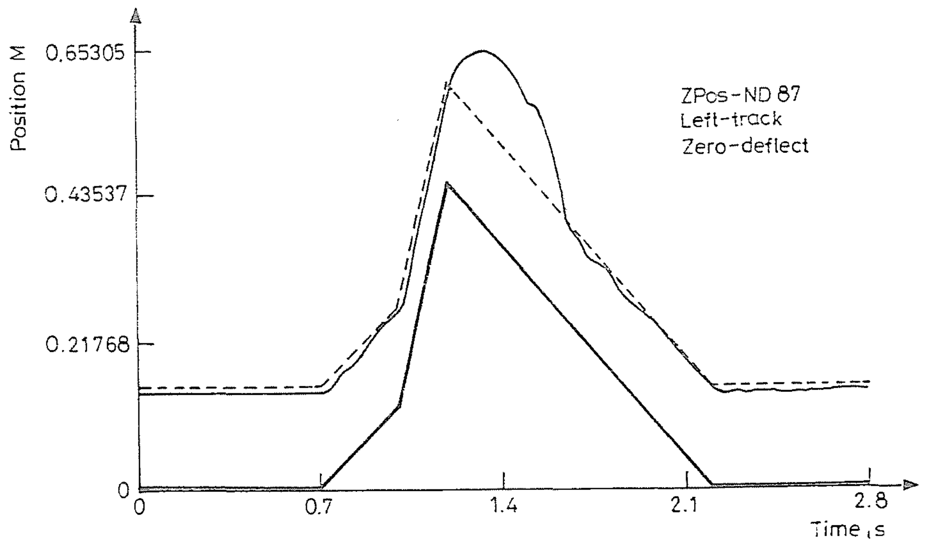

Fig. 17. Vertical position of left hand side rear wheel

7. Conclusions and Closing Remarks

The numerical experiments presented herein indicate the accuracy, effectiveness and applicability of the presented computational procedure and computer program, using in non-linear structural dynamic analysis. It is par-

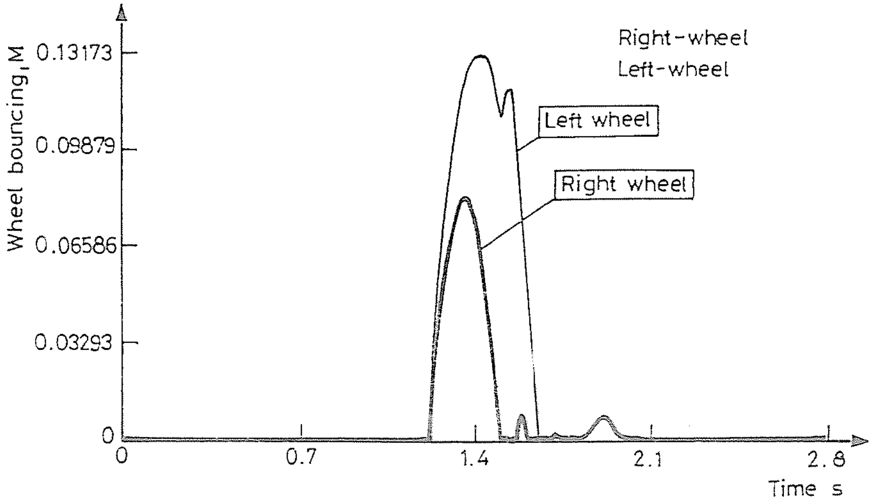

Fig. 18. Magnitude of wheel bouncing of rear wheels

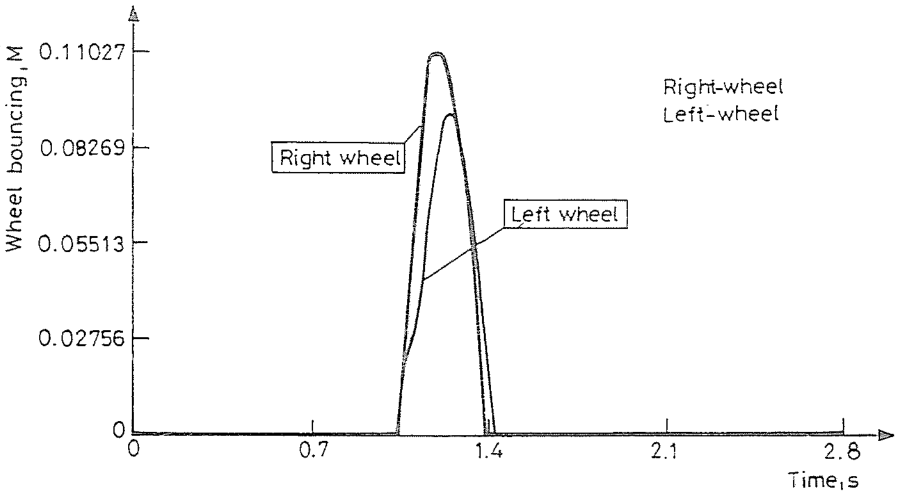

Fig. 19. Magnitude of wheel bouncing of front wheels

ticularly effective when the order of largest stiffness of local non-linearities is not significantly higher than the order of stiffness of the linearized part of the studied structure. Road and off road vehicles (cars, trucks, buses, cross-country cars, agricultural vehicles, etc.) fall within this category. Numerical examples show that the discussed method is especially useful in case of wrong driving conditions, such as, driving vehicles on minor roads of wrong quality, or driving overland cars on terrain containing large irregularities.

The number of degrees of freedom of the applied truck finite element model, in this paper, is equal to 648 and the lower 30 natural modes were involved into the non-linear dynamic analysis, up to 20 Hz. The time period of numerical solution phase, during 5000 steps, was equal to 123 seconds which contain the time for binary output of modal variables in each step, and the total solution time period was equal to 197 seconds. Since natural modes depend on structural properties rather than the number of degrees of freedom of the applied finite element model, the writer assumes that this method, in vehicle dynamics, can be applied for significantly larger finite element models in case of more thousands degrees of freedom, and probably more than ten thousand ones. To support this assumption, the solution time duration of a similar non-linear dynamic analysis of a bus structure, containing 1848 degrees of freedom, was equal to 162 seconds (the total solution time was 281 seconds). In the analysis the lower 50 natural modes were involved, up to 20 Hz, and the calculation was also carried out in 5000 steps [8].

In this paper, in all the numerical examples only vertical excitations are applied, however, the model description and the directions of excitations are not restricted in this method, consequently, it can also be applied for horizontal dynamics of vehicles. Moreover, additional equations and conditions of different mechanical effects can be attached easily to the modal equations, describing the vehicle motion. This possibility significantly enlarges the fields of application of the method presented in this paper, for example, the equations of breaking processes for simulation and to support the design of brake systems (including optimal ABS (Anti-lock Brake System) control strategies). Other important areas are: the simulation, the behaviour and support the design of optimal active suspension systems, application in the identification of vehicle parameters and, at last, numerical stability analysis of vehicles subjected to complex driving and loading conditions.

A program module for the calculation of internal forces and stresses can be built in the developed finite element program.

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