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CLOSED LOOP IDENTIFICATION SCHEMES FOR ACTIVE SUSPENSION DESIGN

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Abstract

This paper presents an overview of the most important closed loop identification methods, i.e. of the traditional direct/indirect, of the two stage and of the coprime factorization methods. The role of these algorithms in closed loop design is highlighted. These approaches are illustrated and compared with each other through a vehicle dynamical example taking the active suspension design problem.

Keywords: closed loop identification, identification for control, active suspension design.

1. Controller Design Based on Identified Model

The controller methods assume the knowledge of the actual plant. But in reality the transfer function of the plant is not known exactly, it is only known partially, and it has uncertainties and stochastic features. The model of the actual plant can be determined using measured input and output signals. The more exactly the model is known the more difficult it is to design and to implement the controller. So reduced complexity model is applied for the purpose of design a controller, which satisfies the stability and performance requirements and it can be implemented easily. The controller design based on identified model leads to iterative identification and controller design, where the model identification is performed in closed loop (GEVERS, 1991).

During the control design process the designer selects a controller so that it has to satisfy not only the stability but also some performance criteria. Let $P(q)$ be the transfer function of the actual plant, and let $C(q)$ mean the transfer function of the controller as it can be seen in *Fig. 1*.

The actual plant is modelled using the input and output signals, and the controller is designed on the basis of the identified model. The designed feedback loop consists of the identified model $\hat{P}(q,\theta)$ and the designed controller $C(q)$ as it can be seen in *Fig. 2*.

In reality the controller works together with the actual plant in the achieved closed loop system. So the controller has to be designed in a way that it has to be suitable not only for the identified model but for the actual

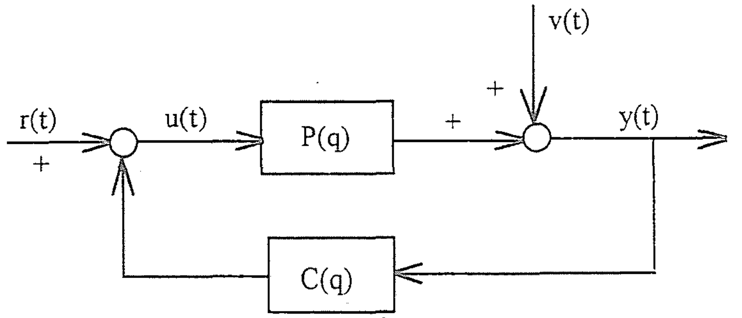

Fig. 1. Actual feedback loop

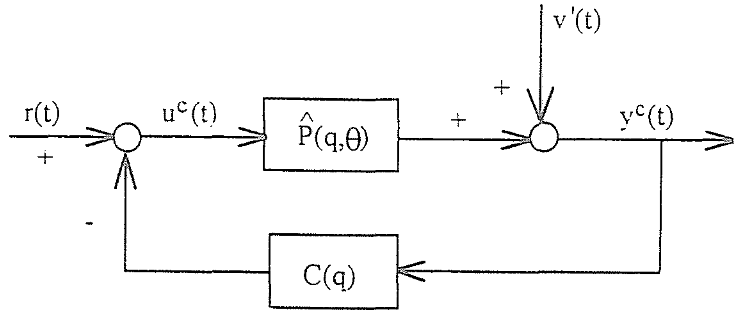

Fig. 2. Designed feedback loop

plant as well. This means that the controller has to ensure the stability of the achieved closed loop system in spite of the uncertainties of the actual plant, which is called as robust stability. On the other hand, the controller has to satisfy some prescribed performance criteria of the achieved closed loop system, which is called as robust performance.

This paper first highlights the theory of closed loop identification. Chapter 3 summarizes the traditional methods, while Chapter 4 describes the modern theories such as the two stage method and the coprime factorization method. Moreover Chapter 5 demonstrates an academic example for closed loop identification approaches.

2. Identification of Systems Operating in Closed Loop

If the input $u(t)$ is not a function of the output $y(t)$ then the plant is said to be operating in open loop, and the output does not feedback and does not affect the input. Unfortunately, practical system identification is usually performed while the plant is operating in a stable closed loop. Here the input is a function of the output and possible of an external loop input

$r(t)$. There are many reasons for identifying $\hat{P}(q, \theta)$ while it is operating in closed loop. If the plant is not stable in open loop operation, the closed loop control is required to maintain the stability for obvious reasons. If the plant is non-linear and the purpose of the identification is to develop a linear model about some nominal operating points, then the closed loop control has to maintain the plant near that nominal operating point.

The connections among the command $r(t)$, disturbance $v(t)$, input $u(t)$, and output $y(t)$ signals of the actual closed loop can be formalized as follows.

$$y(t) = [I + P(q)C(q)]^{-1}P(q)C(q)r(t) + [I + P(q)C(q)]^{-1}v(t) , \quad (1)$$
$$u(t) = C(q)[I + P(q)C(q)]^{-1}r(t) - C(q)[I + P(q)C(q)]^{-1}v(t) . \quad (2)$$

The identification cost function based on the predication error, $\varepsilon(t, \theta)$ which is as follows:

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t, \theta) , \quad (3)$$

where $\hat{y}(t, \theta) = \hat{P}(q, \theta)u(t)$. Then the prediction error cost function is as follows:

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}[\varepsilon(t, \theta)]^2 . \quad (4)$$

The formulas of the prediction error, where $\hat{y}(t, \theta)$ is expressed, in open loop case (5) and in closed loop case (6) are significantly different because of the feedback effect (LJUNG, 1987).

$$\varepsilon(t, \theta) = [P(q) - \hat{P}(q, \theta)]u(t) + v(t) , \quad (5)$$
$$\varepsilon(t, \theta) = [I + P(q)C(q)]^{-1}[P(q) - \hat{P}(q, \theta)]C(q)r(t) + [I - \hat{P}(q, \theta)C(q)][I + P(q)C(q)]^{-1}v(t) . \quad (6)$$

In the 70-s the direct and the indirect identification methods have been developed for closed loop identification. But the new control design approaches involve the development of modern closed loop identification methods, such as the two-stage method and the coprime factorization method.

3. Traditional Closed Loop Identification Methods

The direct identification neglects the effect of the feedback and the identification is performed using the input signal $u(t)$ and the output signal $y(t)$ in open loop way

$$y(t) = \hat{P}(q, \theta)u(t) + v(t) , \quad (7)$$

where $v(t)$ is an estimate of the noise signal. It can be proved that if the command signal $r(t)$ is persistently exciting of a sufficiently high order, then the estimate of $\hat{P}(q,\theta)$ is consistent. As a result a unique and consistent model is obtained despite of the presence of feedback. Moreover, it is sufficient that the external and persistently exciting signal is present (SÖDERSTRÖM, et al., 1976). The direct identification method generally results in a very complicated model.

As an alternative to the direct identification approach, the method of indirect identification avoids to take measured data from the closed loop. It assumes the knowledge of the controller and the measurability of the command signal $r(t)$. The indirect method consists of two steps. In the first step it computes the transfer function of the whole closed loop system $\hat{G}_c(q,\theta_c)$ using the command signal $r(t)$ and the output signal $y(t)$ in open loop way (see *Fig. 2*)

$$y(t) = \hat{G}_c(q,\theta_c)r(t) + w(t)\ , \tag{8}$$

where $w(t)$ is the noise signal. Then in the second step it determines the model applying matrix manipulations using the knowledge of the controller.

$$\hat{G}_c(q,\theta_c) = [1 + \hat{P}(q,\theta)C(q)]^{-1}\hat{P}(q,\theta)C(q)\ . \tag{9}$$

It can be proved that the conditions for a unique and consistent identification result are obtained as in the case of the direct identification with a persistently exciting external signal (SÖDERSTRÖM et al., 1976).

Conditions for consistency of the direct and of the indirect approaches are the same, but this does not mean that both methods give the same result in the finite sample case, or that they are equally easy to apply. Moreover, an advantage of the direct approach is that only one step is required. For the indirect approach it is not obvious how the equations in second step (9) should be solved.

4. Modern Closed Loop Identification Methods

4.1. The Two-Stage Method

The two-stage method is introduced in VAN DEN HOF and SCHRAMA, 1992. This method avoids complicatedly parametrized model sets, as are required in the direct method, and does not need apriori knowledge of the controller. It consists of two identification steps, which can be performed with open loop methods. If we investigate a single input, single output system, then let us define the $S(q)$ factor, as the common factor of the input and the output equations (1) and(2).

$$S(q) = C(q)[I + P(q)C(q)]^{-1}\ . \tag{10}$$

Since $r(t)$ and $v(t)$ are uncorrelated signals, moreover $u(t)$ and $r(t)$ are available from measurements, it follows that we can identify the $S(q)$ function in open loop way from the following equation:

$$u(t) = \hat{S}(q, \theta) r(t) + w(t) , \tag{11}$$

where $w(t)$ is an estimate of the noise signal. From the results in open loop identification $\hat{S}(q, \theta)$ can be identified consistently.

In the second step of the procedure the output relation is employed. It reconstructs the input signal $u^r(t)$ and identifies the close loop model using the reconstructed input and the measured output signal. Since $u^r(t)$ is available from the measurements through the command signal $r(t)$:

$$u^r(t) = \hat{S}(q, \theta) r(t) \tag{12}$$

and $\hat{P}(q, \theta)$ can be estimated in open loop way

$$y(t) = \hat{P}(q, \theta) u^r(t) + \hat{Q}(q, \theta) v(t) , \tag{13}$$

where $v(t)$ is the white noise signal, and $\hat{Q}(q, \theta)$ is an estimate of the transfer function between the noise and the output signal. A block diagram, indicating the recasting of the closed loop problem into two open loop problems, is sketched in *Fig. 3*.

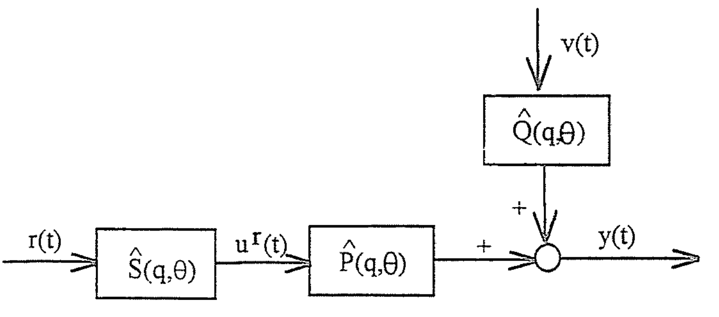

Fig. 3. Block diagram of the two-stage method

4.2. The Coprime Factorization Method

However, a relevant question is, specially under closed loop experimental conditions, how to identify systems that are unstable. The coprime factorization method can be applied for identification of an unstable closed loop system, too. It is introduced in ZHU and STOORVOGEL, 1992, and SCHRAMA, 1991.

Let us define $S(q)$ and $W(q)$ factors that are the common factors of the input ans output equations (1) and (2)

$$S(q) = C(q)[I + P(q)C(q)]^{-1}, \tag{14}$$
$$W(q) = [I + P(q)C(q)]^{-1}. \tag{15}$$

Applying these notations the following two equations can be obtained.

$$u(t) = S(q)r(t) - C(q)W(q)v(t), \tag{16}$$
$$y(t) = P(q)S(q)r(t) + W(q)v(t). \tag{17}$$

The essence of this method is that $S(q)$ can be identified from (16) based on the reference and the input signals independently from $P(q)S(q)$, which can be identified from (18) based on the reference and the output signals. If the reference signal $r(t)$ is uncorrelated with the disturbance $v(t)$, then these are open loop identification problems. The results of these identifications are $\hat{D}(q,\theta)$ and $\hat{N}(q,\theta)$ as it can be seen in (18) and (19). The two transfer functions between $r(t)$ and $u(t)$, $y(t)$ are identified according to the scheme of *Fig. 4*

$$u(t) = \hat{D}(q,\theta)r(t) + \hat{R}(q,\theta)v(t), \tag{18}$$
$$y(t) = \hat{N}(q,\theta)r(t) + \hat{Q}(q,\theta)v(t). \tag{19}$$

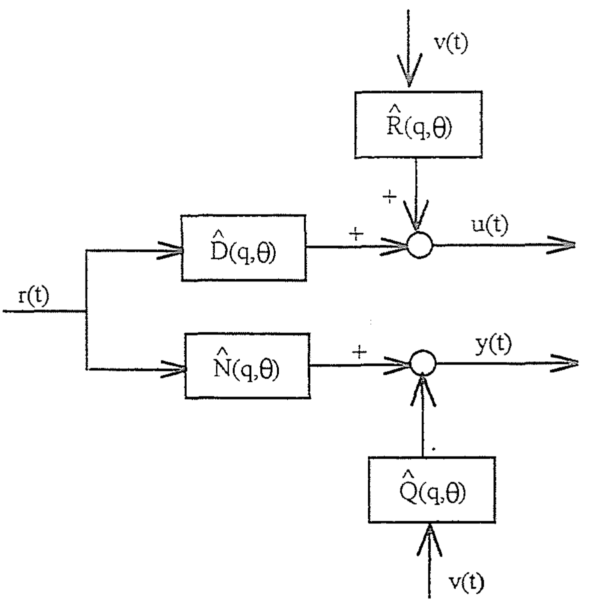

Fig. 4. Block diagram of the coprime factorization method

The pair $P(q)S(q)$ and $S(q)$ can be considered as a factorization of $P(q)$, since $P(q)S(q)[S(q)]^{-1} = P(q)$ assuming that $S(q)$ is non-singular, so the identified model can be obtained applying the identified $\hat{D}(q,\theta)$ and $\hat{N}(q,\theta)$

$$\hat{P}(q,\theta) = \hat{N}(q,\theta)[\hat{D}(q,\theta)]^{-1} \,. \tag{20}$$

Since the closed loop system is stable, the two separate factors composing this factorization are also stable, and they can be identified from data measured in the closed loop. If the estimates of both factors are obtained from independent data sequences, the $\hat{P}(q,\theta)$ estimate will be consistent in the case that both estimators $\hat{D}(q,\theta)$ and $\hat{N}(q,\theta)$ are consistent.

5. Case Study for Active Suspension System

Conventional passive suspensions are typically composed of coil springs and hydraulic dampers. To improve ride comfort, these passive elements must be set on the soft side, but to improve handling, springs and dampers must be made stiffer to damp wheel vibration and to reduce body rolling and pitching motions. These contrary purposes from the classic suspension design compromise. In order to obtain both excellent ride comfort and handling, the suspension characteristics must be changed dynamically according to the demands of the situation. In other words, active control of the suspension is required. The structure of a quarter car model active suspension has been investigated by HROVAT, 1990.

In order to investigate the benefit of active suspension systems, the following two-degree-of-freedom quarter-car model is applied. Let vehicle sprung and unsprung mass be denoted by m_f and m_t, tire stiffness and damping of sprung mass be denoted by s_f and k_f and tire stiffness of unsprung mass be denoted by s_t as it can be seen in the *Fig. 5*. Let $\ddot{z}_t$ be the acceleration of the m_t, $\ddot{z}_f$ the acceleration of the m_f, d stands for the road disturbance.

The differential equations of the model in *Fig. 5* can be formulated as

$$\begin{array}{l} m_f\ddot{z}_f + k_f(\dot{z}_f - \dot{z}_t) + s_f(z_f - z_t) + u = 0 \,, \\ m_t\ddot{z}_t + s_t(z_t - d) - s_f(z_f - z_t) - k_f(\dot{z}_f - \dot{z}_t) - u = 0 \,, \end{array} \tag{21}$$

whose state space representation can be considered as

$$\begin{array}{rcl} \dot{x} & = & Ax + Bu + Gd \,, \\ y & = & Cx + Du \,, \end{array} \tag{22}$$

where $x = (\, z_t \quad z_f \quad \dot{z}_t \quad \dot{z}_f \,)^T$ is the state vector, $y = \ddot{z}_f$ is the output, while

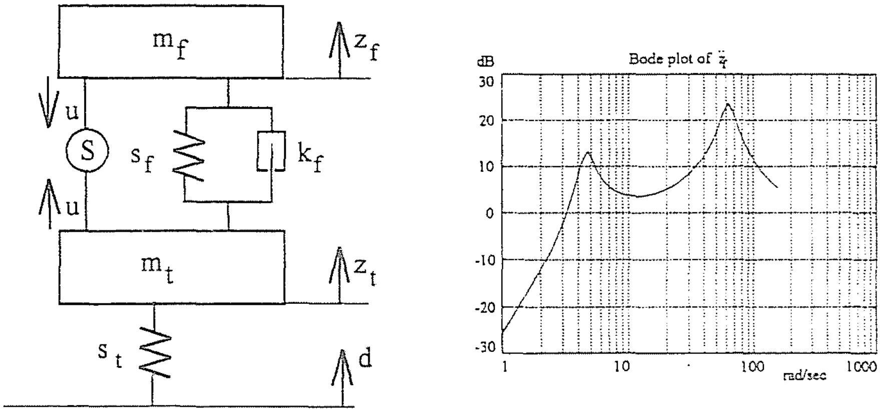

Fig. 5. Scheme of a quarter car model and Bode plot of $\ddot{z}_f$

the constant matrices are

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{s_f + s_t}{m_t} & \dfrac{s_f}{m_t} & -\dfrac{k_f}{m_t} & \frac{k_f}{m_t} \\ \dfrac{s_f}{m_f} & -\dfrac{s_f}{m_f} & \dfrac{k_f}{m_f} & -\frac{k_f}{m_f} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ \dfrac{1}{m_t} \\ -\dfrac{1}{m_f} \end{pmatrix}, \quad G = \begin{pmatrix} 0 \\ 0 \\ \dfrac{s_t}{m_t} \\ 0 \end{pmatrix},$$

$$C = \begin{pmatrix} -\dfrac{s_f + s_t}{m_t} & \dfrac{s_f}{m_t} & -\dfrac{k_f}{m_t} & \dfrac{k_f}{m_t} \end{pmatrix}, \quad D = \begin{pmatrix} \dfrac{1}{m_t} \end{pmatrix}.$$

Let the simulation parameters be $m_f = 400$ kg, $m_t = 33$ kg, $s_t = 9000$ N/m, $s_t = 120000$ N/m, $k_f = 500$ Ns/m. The controller is designed based on the identified model, and this controller is applied for the input u. The Bode plot of the open loop plant, i.e. of the passive suspension system, and one of the actual closed loop system, i.e. of the actual active suspension system can be seen in *Fig. 6*. The system has two peaks, the first is in 4.7 rad/sec, while the other is in 62.8 rad/sec.

The investigation of closed loop identification has been performed based on simulated data, which are demonstrated in *Fig. 7*. In the following the direct method, the two-stage method, and the coprime factorization method are compared. It can be performed since none of them assumes the knowledge of the controller, but they apply the measured signals.

Applying the **direct method** we have investigated, how the different order identified models approach the behavior of the actual plant. The left hand side of *Fig. 8* shows the estimation of the 4.7 rad/sec peak. It can be seen that only the higher models approach this peak. Moreover, the right hand side of *Fig. 8* shows the estimation of the 62.8 rad/sec peak with the same structures. It can be seen that all of them well fit to the peak. This is

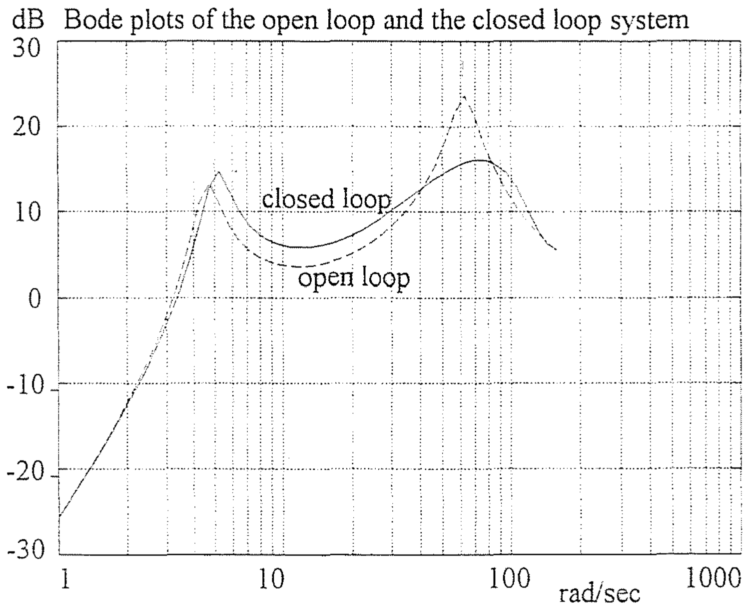

Fig. 6. Bode plots of the open loop plant and the actual closed loop system

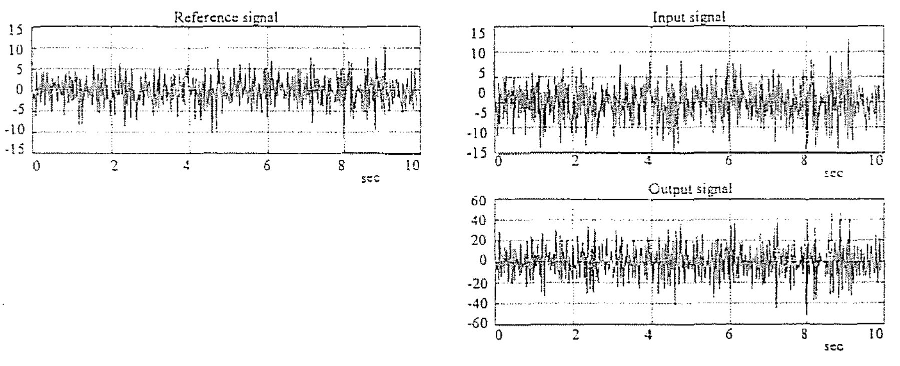

Fig. 7. Sampled time series based on closed loop system

why we select the structure where the autoregressive (AR) order is 30, and the input (INP) order is 30 as well, i.e. the model is ARX(30,30).

On the left hand side of the *Fig. 9* the AR and the INP parameters of the identified model are shown based on direct method as a function of lag. The frequency domain representation of the identified model approaches well the plot of the original systems as it can be seen on the right hand side of the *Fig. 9.*

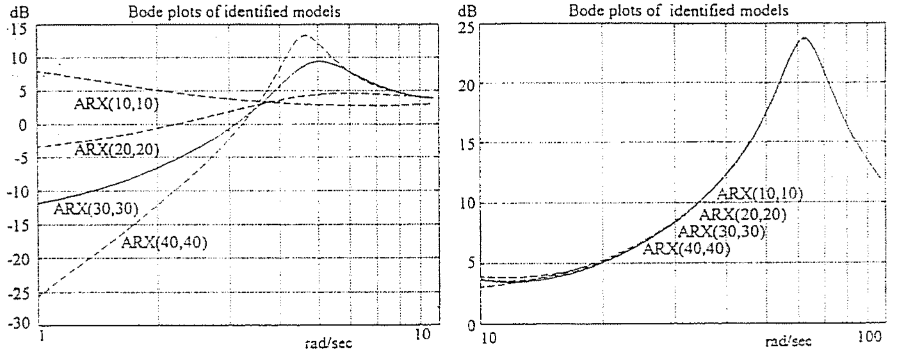

Fig. 8. Original plant and the identified model in the frequency domain

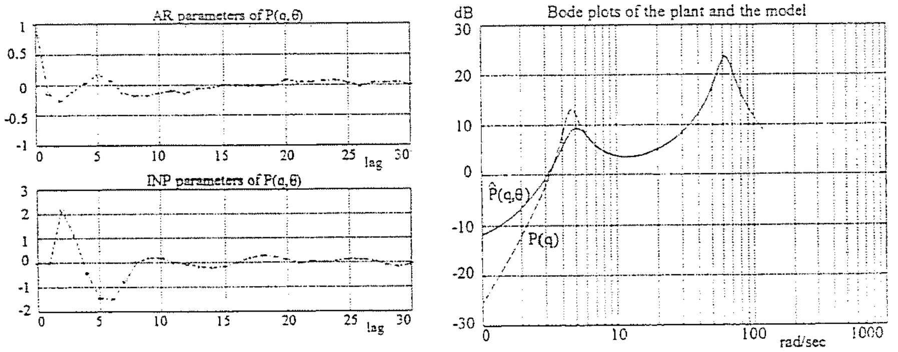

Fig. 9. Model parameters and Bode plot of the identified model based on direct method

The aim of the first step of the **two-stage method** is to reconstruct the input signal using the identified $\hat{S}(q,\theta)$ transfer function between $r(t)$ and $u(t)$ signals. The estimated parameters of the $\hat{S}(q,\theta)$ and its frequency domain representation can be seen in *Fig. 10.* The reconstructed input signal can be seen in *Fig. 11.*

In the second step of the two-stage method, $\hat{P}(q,\theta)$ model can be estimated in open loop way between the reconstructed input and the measure output signals. The order of the identified model is ARX(30,30). *Fig. 12* shows the estimated parameters as a function of lag, and illustrates its frequency domain representation.

Figs. 13 and *14* show the AR and the INP parameters and the frequency domain representation of the $\hat{D}(q,\theta)$ and the $\hat{N}(q,\theta)$ models in the **coprime factorization method**. Based on the $\hat{D}(q,\theta)$ and the $\hat{N}(q,\theta)$ the obtained $\hat{P}(q,\theta)$ model is ARX(59,59). The frequency domain representation of the calculated model can be seen in *Fig. 15.*

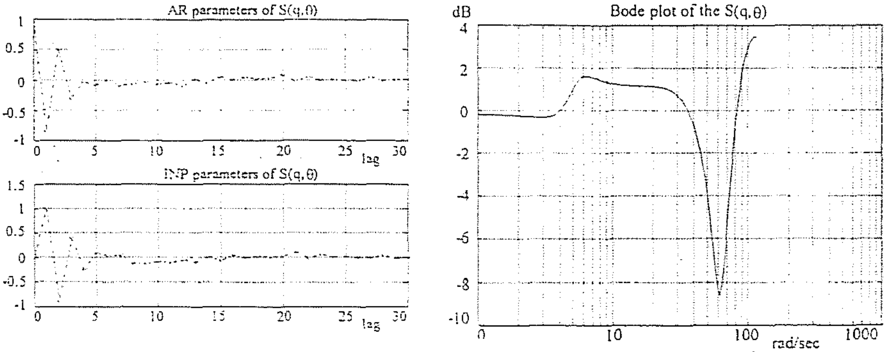

Fig. 10. Model parameters and Bode plot of the identified $\hat{S}(q, \theta)$

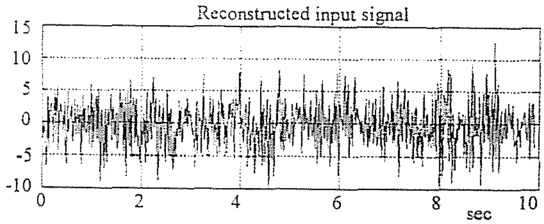

Fig. 11. The reconstructed input signal

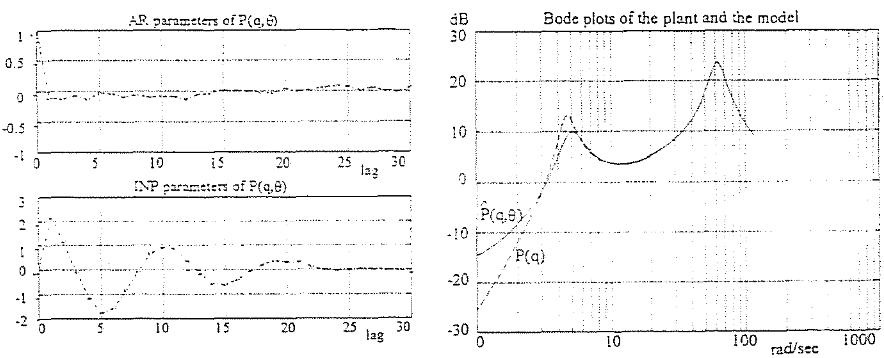

Fig. 12. Model parameters and Bode plot of the model based on two-stage method

In this example we have experienced that the classical direct method supplies the least dimension model in comparison with the other two approaches. The two-stage method has given higher order dimension in this example, but the order of the estimated model can be reduced by improving the reconstruction of the input signal. This method also performs in open loop way, but it takes into account the feedback effect. The coprime factorization method has resulted the highest order model, that can be computed

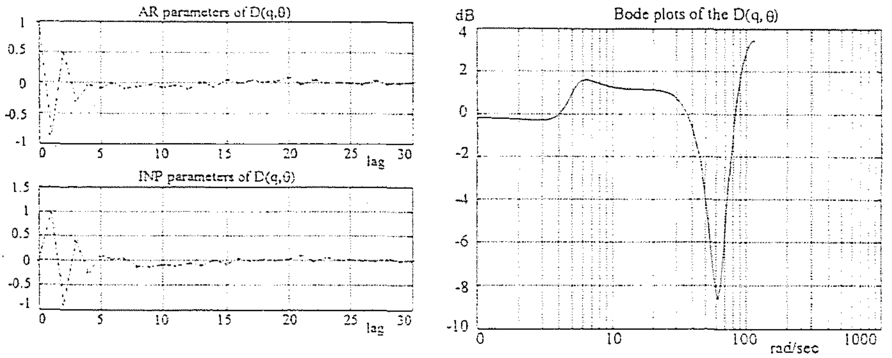

Fig. 13. Model parameters and Bode plot of the identified $\hat{D}(q,\theta)$

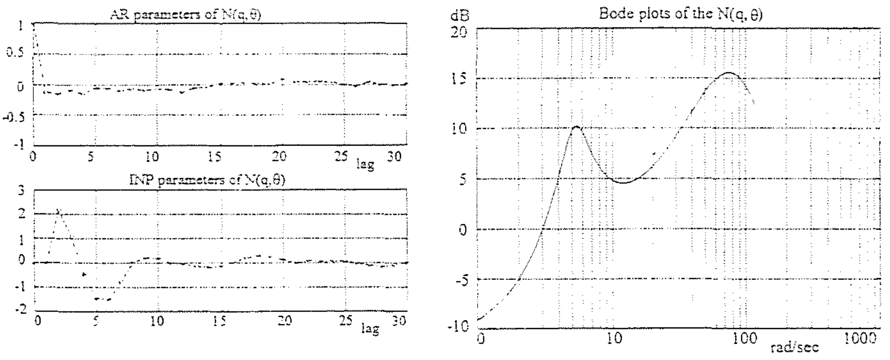

Fig. 14. Model parameters and Bode plot of the identified $\hat{N}(q,\theta)$

from the identified $N(q)$ and $D(q)$ factors. We can get lower order structure of the identified model if $N(q)$ and $D(q)$ have lower degree. The last method is able to handle unstable closed loop systems, too, which is impossible for the two other methods.

6. Conclusion

In this paper we have highlighted that closed loop identification has to be performed for closed loop design. We have summarized the most important closed loop identification methods (traditional direct/indirect, the two-stages and the coprime factorization methods) with respect to the active suspension design example. We have concluded that these methods are well applicable and can be easily used.

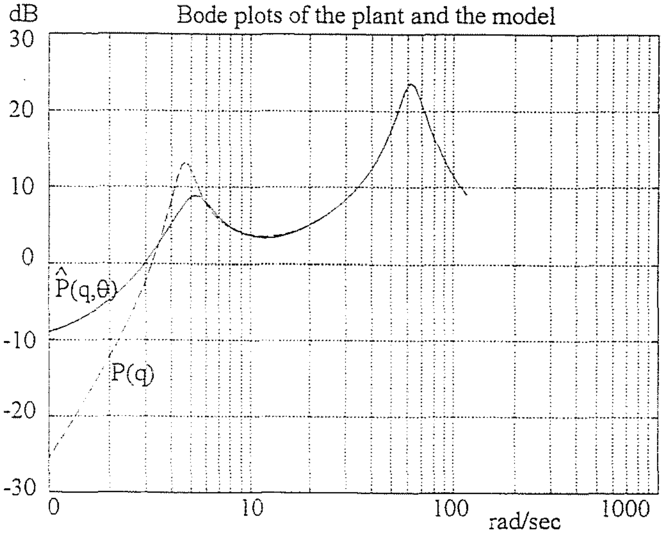

Fig. 15. Bode plot of the identified model based on the coprime factorization method

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