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OPTIMAL CONTROL – OPTIMAL IDENTIFICATION (NEW PARADIGMS – NEW SOLUTIONS)

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Abstract

A new *generic optimal controller* structure and regulator design method are introduced avoiding the solution of polynomial equations. The model sensitivity properties of some combined identification and control schemes are investigated. It is shown that a new structure is superior to the others. An applicable strategy for iterative control refinement based on the *generic scheme* is presented and illustrated by simulation examples. A worst-case optimal input design algorithm is also introduced to increase the robustness of the closed-loop control in the relevant medium frequency range by generating a 'maximum-variance' reference signal. The adaptive version of the control refinement strategy is also shown with a special 'triple-control' extension for recursive optimal input design.

1. Introduction

The need to design high performance control systems has not lost the importance inspite of the thousands of methods and algorithms published in the past decades. The huge number of papers indicates that no unique or best method was found. The solution depends on the model, criterion, uncertainties, disturbances, constraints, etc. (sometimes even on the designer's taste).

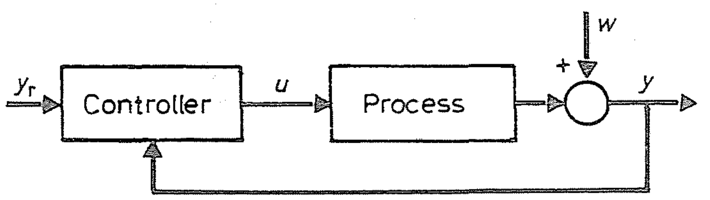

Fig. 1. A general closed-loop control system

A general closed-loop system is shown in *Fig. 1*, where y_r, u, y and w are the reference, input, output and disturbance signals, respectively. Here discrete-time representations are considered for computer controlled systems.

The argument k of variables means the integer value discrete time (integer multiple of the sampling period) and z^{-1} means the backward shift operator $(z^{-1}y(k) = y(k-1))$.

Many experts believe that the essence of all control problems can be led back for the simple problem shown in *Fig. 2*, i.e., how to choose a serial compensator transfer function X to S ensuring a unity dynamic transfer. The trivial solution $X = S^{-1}$ is not always applicable because S is not invertible. This is mostly the case if the control is discrete time and based on sampled linear dynamic systems. In a general case the system $S = S_+S_-$ is factorable for inverse stable S_+ and inverse unstable S_- components. Because S_- cannot be eliminated by simple cancellation mechanism it is sometimes called invariant system component. A heuristic but widely applicable solution is to choose $X = S_+^{-1}$, when the inverse stable part is cancelled, however, the invariant inverse unstable factor is untouched. Several controller design schemes are based on this method, inspite of there is no optimality connected. Unfortunately, the remaining invariant factor can sometimes cause not tolerable dynamics, so its effect must be attenuated. This can be done, if we use a criterion for this purpose. It is known from the classical Wiener framework of optimal stochastic systems, that the solution of the minimum mean square error (H_2) problem

$$X = \arg\min_X \left\{E\left\{\varepsilon^2(k)\right\}\right\} = \arg\min_X \left\{E\left\{[(1 - XS_+S_-)\, n(k)]^2\right\}\right\} \quad (1)$$

can be obtained if

$$X = \tilde{S}_-^{-1} S_+^{-1}, \quad (2)$$

where $\tilde{S}_-$ is obtained by reflecting the zeros (they are unstable!) of S_- through the unit circle and providing

$$\tilde{S}_-^{-1}(1) S_-(1) = 1. \quad (3)$$

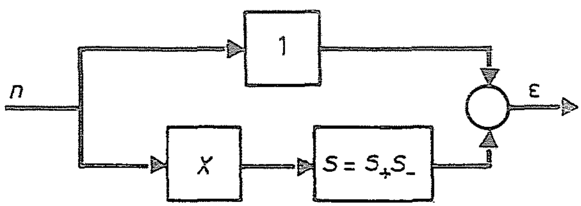

Fig. 2. The simple problem of control systems

Note that the solution depends on the applied input excitation $n(k)$. Here $n(k)$ is assumed as a white noise sequence.

A characteristic approach of optimal controller schemes is called *pole-placement technique* (ÅSTRÖM – WITTENMARK (1984); LANDAU (1990)) targeting to provide prescribed transient properties for the servo and disturbance rejection paradigm of closed-loop controller design. The standard technique representing a two degrees of freedom so-called $\mathcal{R}-\mathcal{S}-\mathcal{T}$ controller assumes basically the structure shown in *Fig. 3*. Here $\mathcal{R}$, $\mathcal{S}$ and $\mathcal{T}$ are polynomials. The advantage of this scheme is that the implementation of a so-called direct adaptive regulator method is very easy, because it is easy to construct a predictor equation linear in the parameters of these polynomials. The disadvantage of this scheme is that it hides the internal operation of an optimal system and special considerations are necessary in a recursive parameter estimation algorithm because the $\mathcal{R}$, $\mathcal{S}$ and $\mathcal{T}$ are redundant, having more parameters than minimally necessary, furthermore the solution of a Diophantine equation is necessary to obtain the regulator polynomials.

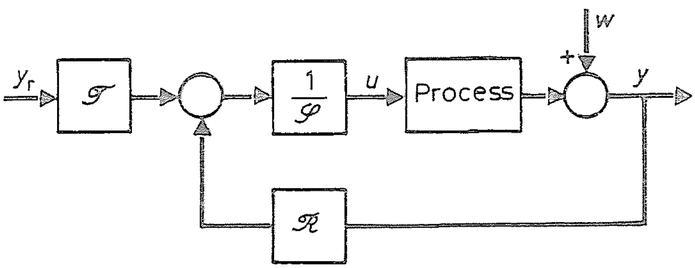

Fig. 3. A classical pole-placement controller

Another well-known classical scheme of optimal control systems is called *internal model principle.* The name originates from the system model applied in the controller. This scheme has a much less known form if we want to use the same principle for inverse unstable factors. This *modified internal model principle* is shown in *Fig. 4*, if only inverse stable factors are cancelled. Note that the whole system should be taken into consideration in the internal model, because realizability problem arises only using the inverted model. Here P_r and P_w are the desired overall tracking and disturbance rejection transfer functions (or reference models) for the design requirements. They can also be interpreted as predictors for the reference and output disturbance signals. In this case, e.g., $\hat{w}$ is the estimated (or predicted) disturbance. (An ideal case was assumed above when the true process S is known to ease the understanding of the basic schemes. This assumption is good to explain the operation of the system, however, only a process model M is available in most practical cases as it will be discussed later.) If we want to attenuate the invariant system component then $\widetilde{S}_-^{-1}S_+^{-1}$ must be used instead of S_+^{-1} in the *partially inverse model*, according to the optimality of *Eq.* (2).

The advantage of this optimal control scheme is that the principal operation of the regulator is very easy to follow and the computations of the regulator polynomials are easy and obvious. The disadvantage of this

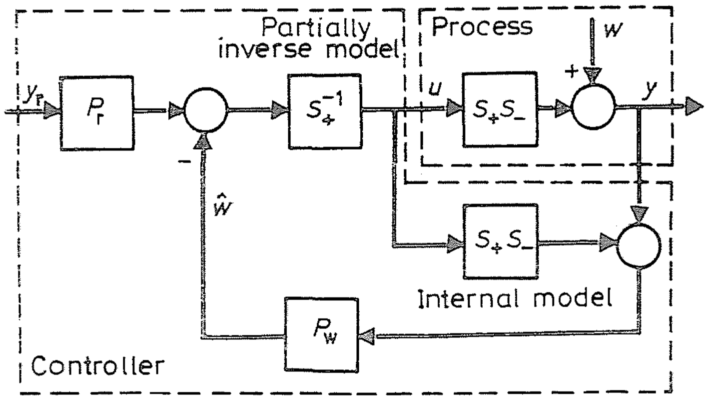

Fig. 4. Modified internal model principle for inverse unstable factor

scheme is that the identification (parameter estimation) method indicated by the *internal model* in the closed loop arises several difficulties.

Both above approaches have the general problems of the known optimal controller structures that the *identification and control errors are different.* Therefore these schemes are not the best ones for developing and analyzing simultaneous identification and (adaptive) control algorithms.

In this paper a new *generic structure* (KEVICZKY – BÁNYÁSZ (1994)) is discussed which allows a very simple procedure to design optimal control systems when the identification and control errors are identical. Their relationships to the previously discussed schemes are also presented.

2. A New Controller Structure

Let us introduce another new structure shown in *Fig. 5* to design optimal controllers. Here S is the system, R is the regulator, $\widetilde{P}_r$ is a precompensator transfer function. The system output

$$y(k) = \widetilde{P}_r S y_r(k) + \frac{1}{1+RS} w(k) = y_t(k) + y_d(k) \tag{4}$$

is very special, because in spite of the closed-loop the tracking behaviour $\left(y_t(k) = \widetilde{P}_r S y_r(k)\right)$ is independent of the regulator R. This structure practically opens the loop for the command signal and the selected feedforward compensator (or observer)

$$\widetilde{P}_r = P_r S^{-1} \tag{5}$$

provides a desired

$$y_t(k) = P_r y_r(k) \tag{6}$$

tracking (servo) transfer function by P_r. Observe that selecting a regulator

$$R = \frac{P_w}{1 - P_w} S^{-1} = CS^{-1} \tag{7}$$

a desired regulating (or disturbance rejection) behavior

$$y_d(k) = (1 - P_w)\, w(k) \tag{8}$$

can be reached by P_w. The P_r and P_w transfer functions contain the desired poles to be placed, so they can be called as reference models. Note that arbitrary zeros can also be placed, however, the calculation of the precompensator $\tilde{P}_r$ and regulator R requires the inverse of the process S. (Here also the ideal case was assumed when the true process S is known. If P_w is the best LS predictor of w, then the obtained regulator is the minimum variance (MV) regulator.) It is easy to prove that this scheme can be rearranged to the scheme in *Fig. 2* by straightforward block manipulations. Assuming that

$$S = \frac{B}{A}, \tag{9}$$

$$P_r = \frac{B_r}{A_r} \quad \text{and} \quad P_w = \frac{B_w}{A_w} \tag{10}$$

the classical pole-placement regulator polynomials can also be easily computed according to the previous formulas and they are given by

$$\mathcal{T} = AA_wB_r, \tag{11}$$

$$\mathcal{S} = A_rB\,(A_w - B_w)\,, \tag{12}$$

$$\mathcal{R} = AA_rB_w. \tag{13}$$

(Observe the common factors that make the parametrization for a direct self-tuning technique redundant and the recursive estimation difficult.

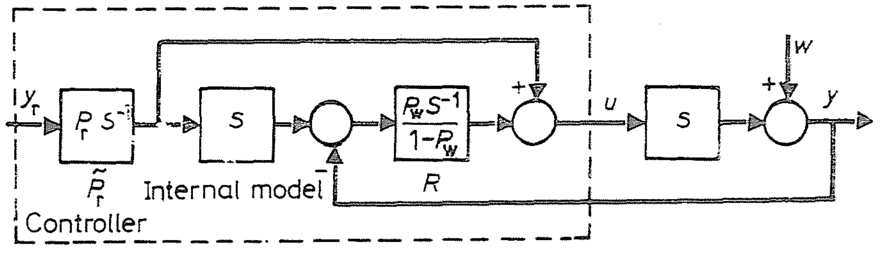

Fig. 5. The new control system structure

If we use the above precompensator and regulator the controller shown in *Fig. 5* can be further simplified according to *Fig. 6*. The advantage of this

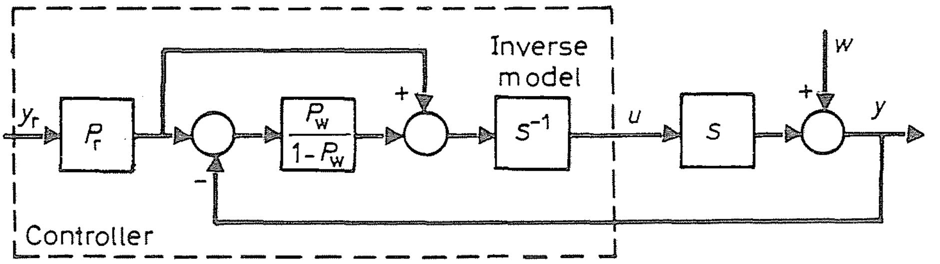

Fig. 6. Equivalent form of the new controller

approach is that the three transfer functions of the controller can directly be computed without solving the usual polynomial equation. A further and very important advantage is that the identification and control errors are identical, so this scheme is especially good for combined identification and control problems. (The very limited usability for inverse stable systems will be eliminated in the next section.) It is easy to show that *Fig. 5* can be rearranged to the schemes either in *Fig. 3* or in *Fig. 4* by straightforward block manipulations. So the *classical pole-placement regulator scheme and the internal model principle scheme are identical* to each other and to the new scheme if the appropriate transfer functions are selected.

3. A Generic Scheme for Optimal Pole-Placement Controllers

Because the above method is based on full pole and zero cancellation the extended applicability can be reached by partial cancellation of those system (model) components only, which are inverse stable. Assume that

$$S = S_+ S_- = S_+ \overline{S}_- z^{-d}, \tag{14}$$

where S_+ means the inverse stable factor. Here $S_- = \overline{S}_- z^{-d}$ is the non-invertible part where the discrete time delay z^{-d} is also a factor whose inverse z^d is not realizable. In case of a partial cancellation we should use the precompensator

$$\widetilde{P}_r = P_r S_+^{-1} \tag{15}$$

instead of (5). This results in

$$y_t(k) = P_r S_+^{-1} S_+ \overline{S}_- z^{-d} y_r k = P_r \overline{S}_- z^{-d} y_r k. \tag{16}$$

Selecting the regulator as

$$R = \frac{P_w S_+^{-1}}{1 - P_w \overline{S}_- z^{-d}} = C S_+^{-1} \tag{17}$$

one can obtain the regulatory transfer as

$$y_d(k) = \left(1 - P_w \overline{S}_- z^{-d}\right) w(k). \tag{18}$$

Eqs (16) and (18) show that in case of partial cancellation, which is the general case, when we have inverse unstable system factors (e.g. nonminimum phase systems) or time delay we cannot reach the ideal servo (P_r) and disturbance rejection ($1 - P_w$) transfer functions only their modified $\left(P_r \overline{S}_- z^{-d}\right)$ and $\left(1 - P_w \overline{S}_- z^{-d}\right)$ forms. Note that the modifications do not depend on us, instead they depend on the system itself only. Therefore $\overline{S}_-$ is sometimes called invariant process factor (mostly zeros). (Because reasonably $P_r(1) = 1$ and $P_w(1) = 1$ are selected, it is also reasonable to choose $\overline{S}_-(1) = 1$ in the factorization *Eq.* (14)). *Fig. 7* shows the transfer functions in the new controller scheme, which can even be called a *generic* optimal pole-placement controller in this case. This scheme can be further simplified according to *Fig. 8*. It is easy to show that *Fig. 7* can be rearranged to the schemes either in *Fig. 3* or in *Fig. 4* by straightforward block manipulations. So the *classical pole-placement regulator scheme* and the *modified internal model principle scheme* are identical to each other and to the *new generic scheme* if the appropriate transfer functions are selected.

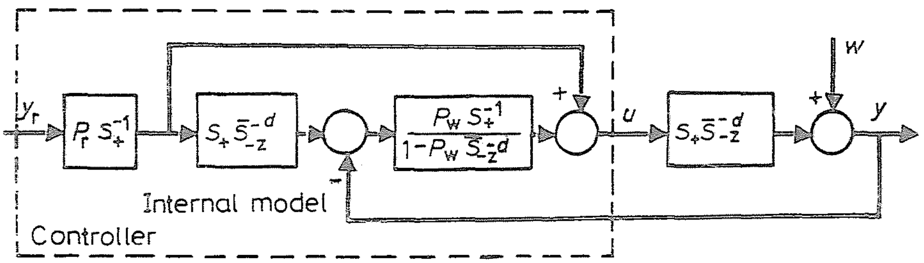

Fig. 7. The *generic pole-placement controller*

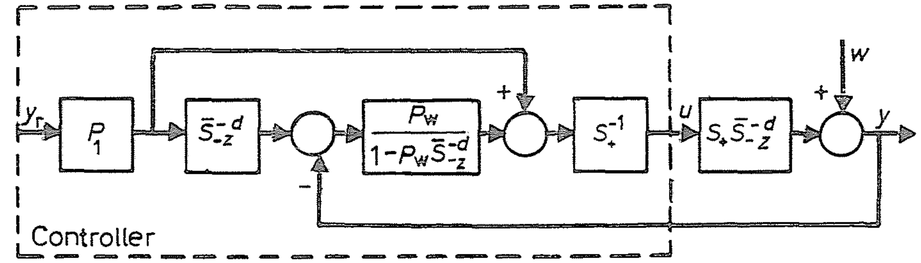

Fig. 8. The equivalent form of the *generic controller*

Assuming that instead of (9) the system transfer function corresponding to (14) is given by

$$S = \frac{B_+ B_-}{A} z^{-d} \tag{19}$$

only the polynomial $\mathcal{S}$ will be changed

$$\mathcal{S} = A_r B_+ \left(A_w - B_w B_- z^{-d} \right) \tag{20}$$

and the computation of $\mathcal{T}$ and $\mathcal{R}$ remains according to (11) and (13).

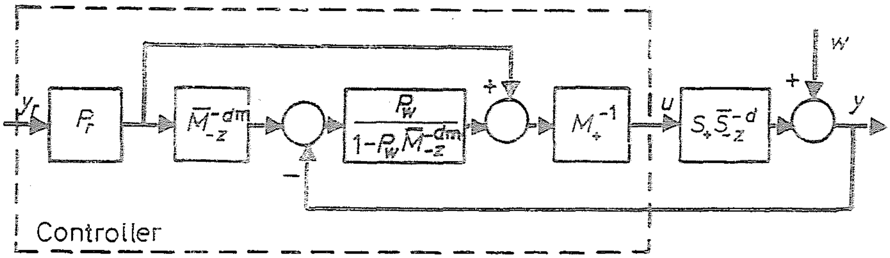

Fig. 9. The *generic* model based pole-placement controller

The final optimal controller is already very general because it covers the most critical processes where the design is not trivial. In the practice we should rather use the model M instead of the system S in the internal model, so the design procedure should apply

$$M = M_+ \overline{M}_- z^{-d_m} = \left(\frac{\widehat{B}_+}{\widehat{A}} \right) \left(\widehat{B}_- \right) z^{-d_m} = \frac{\widehat{B}_+ \widehat{B}_- z^{-d_m}}{\widehat{A}} = \frac{\widehat{B}}{\widehat{A}} z^{-d_m} \tag{21}$$

instead of (14). Here M_+, $\overline{M}_-$ and z^{-d_m} are the inverse stable, inverse unstable factors and the delay time of the model, respectively. So the optimal controller will change according to *Fig. 9.* This controller is very easy to implement in a computer controlled system and it keeps the advantage of the original idea of the new controller structure, i.e., it does not require the solution of a polynomial (Diophantine) equation to obtain the controller transfer functions or polynomials, instead (11), (13) and (20) should be applied now, where $\widehat{B}_+ \widehat{B}_- z^{-d}$ and $\widehat{A}$ are the numerator and denominator polynomials of the model M instead of the system S, as *Fig. 10* shows.

It was already mentioned that the not cancellable factors of the system will be factors of the modified reference models, so they are invariant to any control strategy. However, their influence on the original reference models can be minimized, if we use a criterion for this purpose. Using the Wiener design principle shown in the introduction the precompensator

$$\widetilde{P}_r = P_r \widetilde{\overline{S}}_-^{-1} S_+^{-1} \tag{22}$$

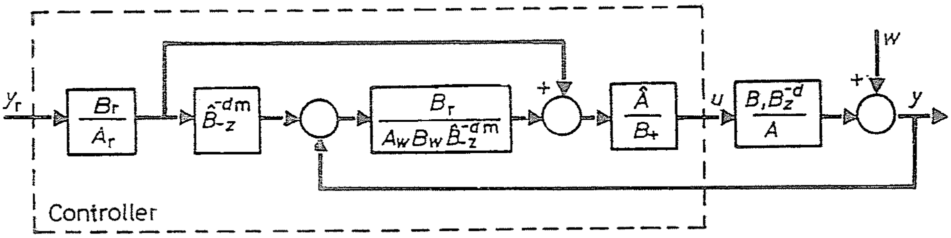

Fig. 10. The polynomial design of the *generic* pole-placement controller

and the regulator

$$R = \frac{P_w \tilde{\overline{S}}_-^{-1} S_+^{-1}}{1 - P_w \tilde{\overline{S}}_-^{-1} \overline{S}_- z^{-d}} = C S_+^{-1} \tag{23}$$

must be used and the overall system equation

$$y(k) = P_r \tilde{\overline{S}}_-^{-1} \overline{S}_- z^{-d} y_r(k) + \left(1 - P_w \tilde{\overline{S}}_-^{-1} \overline{S}_- z^{-d}\right) w(k) \tag{24}$$

is obtained. *Fig. 11* shows the practical realization of this strategy if the model of the system is used.

Note that the application of the compensator $\tilde{\overline{S}}_-^{-1}$ on $\overline{S}_-$ is optimal for a given excitation only, in this case for white noise (or approximately for wide bandwidth) disturbance. Therefore, the optimal pole-placement controller shown in *Fig. 8* is suggested for most practical applications.

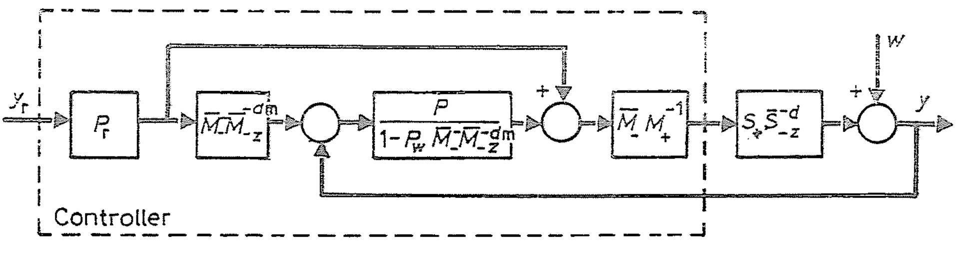

Fig. 11. Attenuating the invariant factors by Wiener design

4. Combined Identification and Control Schemes

Since the beginning the key paradigm of designing control systems is how to handle uncertainties associated with the plant. One of the main techniques is adaptive control intending to learn parameter and disturbance uncertainties in varying circumstances. Another important approach is to implement robustness features at simultaneous identification and control procedures.

In the traditional approach to analysis and design of an adaptive control system the unknown plant is represented by a model, which is almost known except a few parameters assumed time varying. Having the estimated parameters the controller is updated according to the certainty equivalence principle. The unstructured uncertainties are mostly ignored in these cases, therefore these adaptive regulators are not robust. To tell the truth it is not easy at all to consider these uncertainties at the classical adaptive systems and to guarantee proper transients during the learning adaptation phase or abrupt parameter changes.

In a classical robust control approach the regulator is designed on the basis of a nominal model of the plant associated with the associated parametric and unstructured model uncertainties explicitly taking into account. (Unfortunately, mostly analytical forms are required which are very rarely available at practical applications, except a few special real cases or the examples 'god given' apriori information). Here stability robustness is guaranteed and performance robustness is achieved sometimes. The weakness of this approach is that it considers only the apriori information on the model and neglects that the characteristics of the plant could be learnt even when it is controlled. Therefore classical robust control approaches mostly result in a conservative design in terms of performance.

(In Section 8 a new approach will be introduced, when the uncertainty of the model coming from the parameter estimation will be minimized in the relevant medium frequency range around the cross-over frequency.)

4.1. Open-loop Identification and Closed-loop Control

The simplest strategy to combine modelling and control if the identification is performed in an open-loop experiment to obtain an optimal model M^*, selected from a model class $\mathcal{M}$, by minimizing an identification criterion $Q_{i0}(\varepsilon_0)$ function of the open-loop identification error ε_0, i.e.,

$$M^* = \arg\min_{M\in\mathcal{M}} Q_{i0}(\varepsilon_0) = \arg\min_{M\in\mathcal{M}} Q_{i0}(M, \mathbf{x}, \Delta) = M_0^*(\mathcal{M}, \mathbf{x}, S), \quad (25)$$

where $\mathbf{x} = \{x(k); k = 1, \ldots N\}$ is the applied input excitation series and

$$\Delta = S - M \quad (26)$$

is the additive model uncertainty between the system S and model M (*Fig. 12*). The optimal regulator R^*, selected from a regulator class $\mathcal{R}$, is obtained by minimizing a control criterion $Q_c(e_c)$ function of the closed-loop control error e_c (*Fig. 12*), i.e.,

$$R = \arg\min_{R\in\mathcal{R}} Q_c(e_c) = \arg\min_{R\in\mathcal{R}} Q_c(R, S) = R^*(\mathcal{R}, S). \quad (27)$$

(Note that the open-loop input excitation x is different from the closed-loop input u acting on the process.) Because only the model of the system is known in a practical case, therefore the most widely applicable strategy is to substitute S by M in (27). This strategy is called the *certainty equivalence principle* and realized by

$$R = \arg\min_{R\in\mathcal{R}} \widehat{Q}_c\left(\widehat{e}_c\right) = \arg\min_{R\in\mathcal{R}} \widehat{Q}_c(R, M) = \widehat{R}^*(\mathcal{R}, M). \tag{28}$$

In this case the optimal regulator does not reach the theoretical optimum, because the model M is used instead of the system S. However, it is possible to form an iterative control refinement procedure improving the model and regulator step by step:

1. Identify the model using the modelling step
$$M_i = M_0^*\left(\mathcal{M}, \mathrm{x}_{i-1}, S\right). \tag{29}$$
2. Calculate the optimal regulator
$$R_i = \widehat{R}^*\left(\mathcal{R}, M_i\right). \tag{30}$$
3. Determine an optimal input excitation for the open-loop identification
$$\mathrm{x}_i = \widehat{\mathrm{x}}_0^*\left(\mathcal{H}, M_i, R_i\right) = \mathcal{D}_{x\in\mathcal{H}}\left(\mathcal{H}, \mathrm{x}, M_i, R_i\right). \tag{31}$$
Here $\mathcal{H}$ is the (mostly amplitude or energy) constrained input signal domain. This step is sometimes called optimal input design and the operation is denoted by $\mathcal{D}\left(\ldots\right)$.
4. Once M_i and R_i are found we can continue to increase the closed-loop bandwidth repeating the procedure. The iterative process is continued from step 1, while a stop condition is not fulfilled (until the ultimate control objective is achieved or it is terminated because of reaching some vital constraints).

For comparison it is interesting to see that in the above case the identification and control errors are

$$\varepsilon_0 = (S - M)x = m\left(\frac{\Delta}{M}\right)x = H_0\left(\frac{\Delta}{M}\right)x \tag{32}$$

and

$$\epsilon_c = \frac{1}{1+RS}\widehat{y}_r + \frac{1}{1+RS}w; \qquad \widehat{y}_r = P_r y_r, \tag{33}$$

where P_r is the reference model, $\widehat{y}_r = P_r y_r$ is the desired process output (model output or predicted reference signal) and w is the output disturbance.

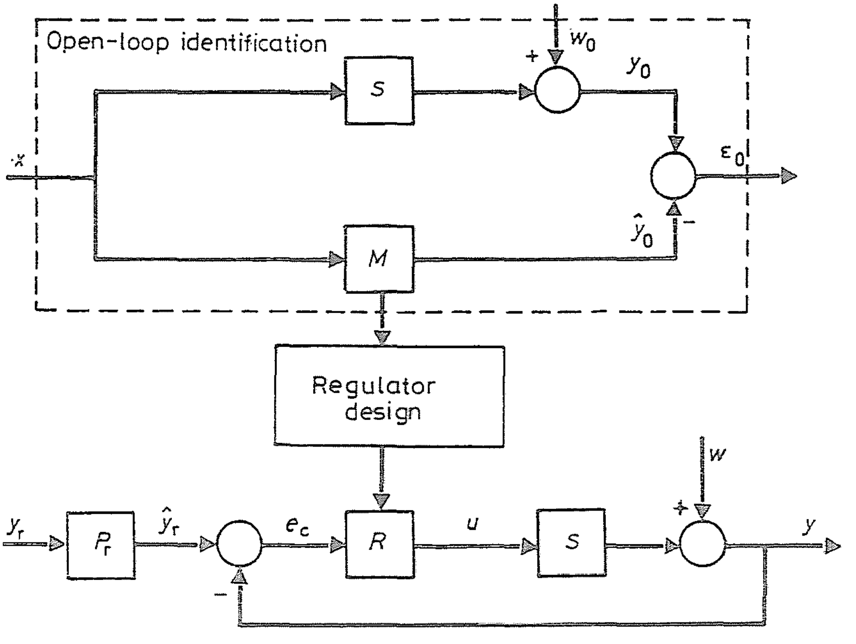

Fig. 12. Combination of *open-loop identification* and *closed-loop control*

4.2. Closed-loop Identification and Closed-loop Control

The formal description of the procedure how to combine modelling and control if the identification is performed in a closed-loop experiment is very similar to the previous section, however, the identification criterion $Q_{ic}(\varepsilon_c)$ is now the function of the closed-loop identification error ε_c, i.e.

$$M^* = \arg\min_{M\in\mathcal{M}} Q_{ic}(\varepsilon_c) = \arg\min_{M\in\mathcal{M}} Q_{ic}(M, R, y_r, \Delta) = M_c^*(\mathcal{M}, y_r, S), \tag{34}$$

and the regulator is designed again by *Eq.* (28). It is possible to form different combined schemes depending on the structure of the optimal control and the combination of the sequential identification and control steps. In this sequential procedure it is a general observation that

1. The human first learns to control over a limited bandwidth, and learning pushes out the bandwidth over which an accurate model of the plant is known.
2. The human first implements a low gain controller, and learning allows the loop to be tightened.

On the basis of these observations an adaptive robust control philosophy, the *windsurfer approach*, was proposed by ANDERSON – KOSUT (1991). They use a *parallel closed-loop optimal controller scheme*, which is very widely used in the analysis of control relevant identification and iterative control refinement procedures. Note that the classical so-called direct adaptive control algorithms generally use a somewhat different scheme which is called as *parallel in-loop optimal controller scheme*. We will analyze these schemes and the new *generic optimal controller scheme* also in the sequel. For the sake of simplicity this comparison will be discussed here for inverse stable processes first.

The generic optimal controller scheme. The *generic optimal controller scheme* (KEVICZKY – BÁNYÁSZ (1994)) was shown in Section 2.

The structure of the optimal controller gives a special insight to understand the operation of a feedback loop for the servo and disturbance rejection paradigm. It is easy to see the role of the knowledge of the model of the system and the role and appearance of the factors of the system that are invariant to any control strategy. A little bit modified form of the *generic scheme* of *Fig. 5* will be used here as *Fig. 13* shows.

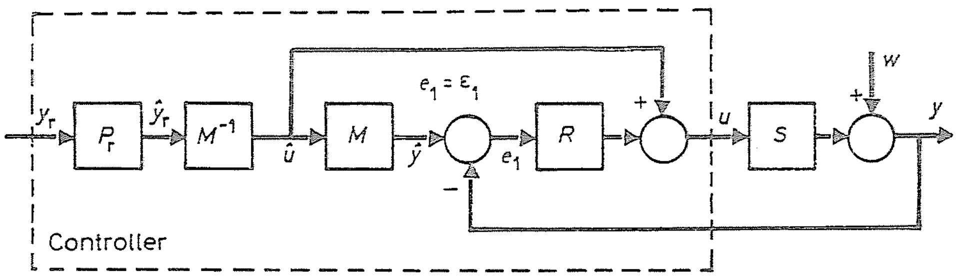

Fig. 13. The *generic optimal controller scheme*

This structure has further advantages in handling system uncertainties and a new canonic sensitivity scheme can also be obtained. This *generic scheme* opens a new way to analyze combined modelling and control issues. *Fig. 13* is the long searched ideal scheme for the combined identification and control problem, because in this case *the control and modelling errors are identical.* So the new scheme provides an excellent possibility also to study robust identification for control.

The common identification and control errors are

$$e_1 = \varepsilon_1 = \frac{1}{1+RS}\left(\frac{\Delta}{M}\right)\hat{y}_r - \frac{1}{1+RS}w = -\left[H_1\left(\frac{\Delta}{M}\right)\hat{y}_r + E_1 w\right]. \quad (35)$$

5. On the Generic Optimal Controller Scheme

Inverse stable processes. It was presented above that the *generic optimal controller scheme* has certain advantages comparing to others shown. It is also interesting to show that the common modelling and control error $e_1 = \varepsilon_1$ given by (35) can be also expressed as

$$e_1 = \varepsilon_1 = -\left[H_1\left(\frac{\Delta}{M}\right)\widehat{y}_r + E_1 w\right] = -[(Su+w) - M\widehat{u}] = -(y-\widehat{y}). \quad (36)$$

This form and *Fig. 13* shows an obvious way how to perform the identification step in a combined identification and control scheme, i.e., we should use a regular identification algorithm based on the auxiliary variable $\widehat{u}$ and the measured controlled variable y as *Fig. 14* shows. Note that $\widehat{u}$ and y must be obtained from the closed-loop operated by the generic optimal controller structure.

Because $\widehat{u}$ depends on the model M only an iterative control refinement procedure can be performed. Its simplest – so-called relaxation type – iteration can be formed in the following way for an off-line case using N samples (i-th iteration is shown):

1. Calculate the auxiliary variable $\widehat{u}_i$ based on the available model M_{i-1}, the reference model P_r and the applied reference signal series $y_r^i = \{y_r^i(k);\ k=1,\ \ldots, N\}$

$$\widehat{u}_i(k) = M_{i-1}^{-1} P_r y_r(k); \qquad k=1,\ \ldots, N. \quad (37)$$

   Here i denotes the index of the iteration and note that y_r^i does not necessarily change by iteration.
2. Identify a model between $\widehat{\mathbf{u}}_i$ and $\mathbf{y}_i$ using the modelling step

$$\begin{aligned} M_i &= \arg\min_{M\in\mathcal{M}} Q_{ic}(\mathcal{M}, \Delta, \widehat{\mathbf{u}}_i, R_{i-1}, P_w) = \\ &= M_c^*(\mathcal{M}, M_{i-1}, R_{i-1}, \mathbf{y}_r, S, P_r, P_w) \end{aligned} \quad (38)$$

$$\left(\widehat{\mathbf{u}}_i = \{\widehat{u}_i(k);\ k=1,\ \ldots,\ N\}; \qquad \mathbf{y}_i = \{y_i(k);\ k=1,\ \ldots,\ N\}\right).$$

3. Calculate the optimal regulator based on (7)

$$R_i = CM_i^{-1} = \frac{P_w}{1-P_w} M_i^{-1} \quad (39)$$

   the compute the process input u_i as

$$u_i(k) = P_r\left(R_i + M_i^{-1}\right) y_r^i(k) - R_i y_i(k); \qquad k=1,\ \ldots,\ N \quad (40)$$

   and apply to the process.
4. (There is a possibility to optimize the applied reference signal series in this step by a proper input design procedure.)

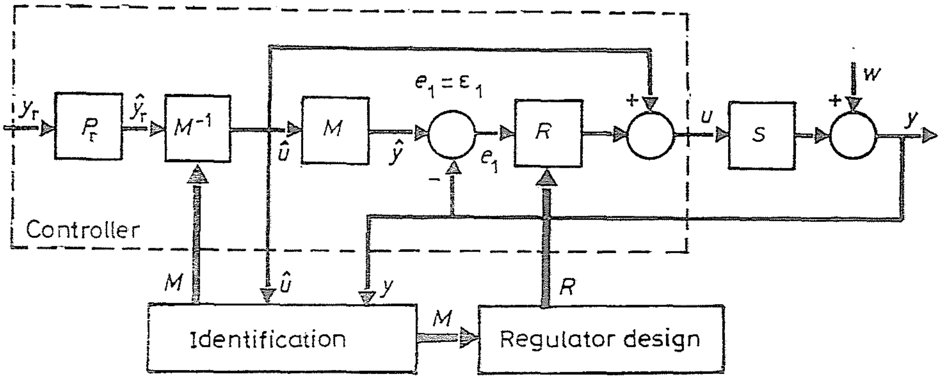

Fig. 14. Identification and regulator design at the *generic optimal controller scheme*

5. Once M_i and R_i are found we can continue to increase the closed-loop bandwidth repeating the procedure. The iterative process is continued from step 1, while a stop condition is not fulfilled (until the ultimate control objective is achieved or it is terminated because of reaching some vital constraints).

Inverse unstable processes. Note that both $\hat{u}$ and R need the inverse of M. Since this method is based on full pole and zero cancellation the extended applicability can be reached by partial cancellation of those system (model) components only, which are inverse stable. This *extended generic optimal controller scheme* is shown in *Fig. 15* which corresponds to *Fig. 7.* The identification and control errors are identical at this scheme, too, i.e.,

$$\begin{aligned} e_1 &= \frac{-\Delta P_r M_-}{(1+RS)M_+M_-} y_r - \frac{1}{1+RS} w = \\ &= \frac{-1}{1+RS}\left(\frac{\Delta}{M_+}\right)\hat{y}_r - \frac{1}{1+RS} w = \\ &= -\left[H_1\left(\frac{\Delta}{M_+}\right)\hat{y}_r + E_1 w\right] = \varepsilon_1 . \end{aligned} \tag{41}$$

The off-line iterative control refinement procedure described by *Eqs* (37), (38), (39) and (40) and steps 1 – 5 can also be applied here, if instead of these equations the corresponding

$$\hat{u}_i(k) = \left(M_+^{i-1}\right)^{-1} P_r y_r(k); \qquad k = 1, \ldots, N, \tag{42}$$

$$R_i = \frac{P_w}{1-P_w}\left(M_+^i\right)^{-1} = C\left(M_+^i\right)^{-1}, \tag{43}$$

$$u_i(k) = P_r\left(R_i M_-^i + \left(M_+^i\right)^{-1}\right) y_r^i(k) - R_i y_i(k); \qquad k = 1, \ldots, N \quad (44)$$

formulas are used.

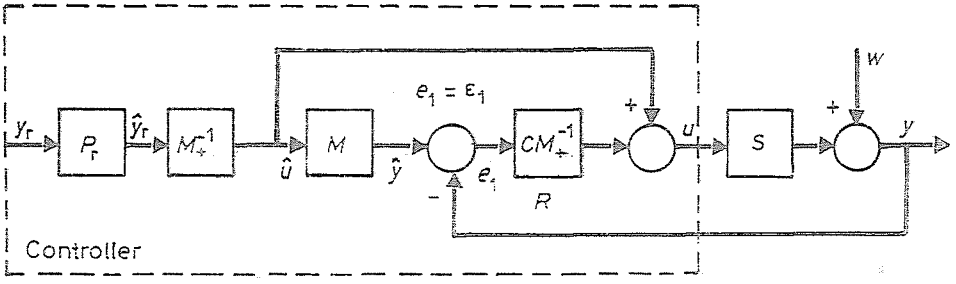

Fig. 15. The *extended generic optimal controller scheme* for inverse unstable processes

6. Examples for Off-line Iterative Regulator Refinement

Example 6.1 Let the process be given by

$$S = \frac{0.007869}{1 - 0.606531z^{-1}} z^{-4} \qquad (45)$$

which is a sampled-time (sampling time is $h = 0.05$ s and $d = 4$) first order approximation of a helicopter 'stick-input/roll-rate-output' model. Apply the unity gain tracking and disturbance rejection reference models

$$P_r = \frac{0.5z^{-1}}{1 - 0.5z^{-1}} \qquad \text{and} \qquad P_w = \frac{0.2z^{-1}}{1 - 0.8z^{-1}}, \qquad (46)$$

and start the iterative control refinement by the model

$$M_0 = \frac{0.01}{1 - 0.4z^{-1}} z^{-4}, \qquad (47)$$

i.e., $d_m = d = 4$. *Fig. 16* shows the control and identification loss functions (variances) by the iteration. It can be well seen that the iteration is quite fast reaching the optimal values after 4 steps.

A unity amplitude square wave reference input signal with periodic time 40 samples was applied and the off-line procedure used $N = 100$ samples. In the simulation runs an additive white noise was used as output disturbance with a standard deviation $\lambda = 0.01$. We used a simple off-line LS method for parameter estimation only to demonstrate the operation of

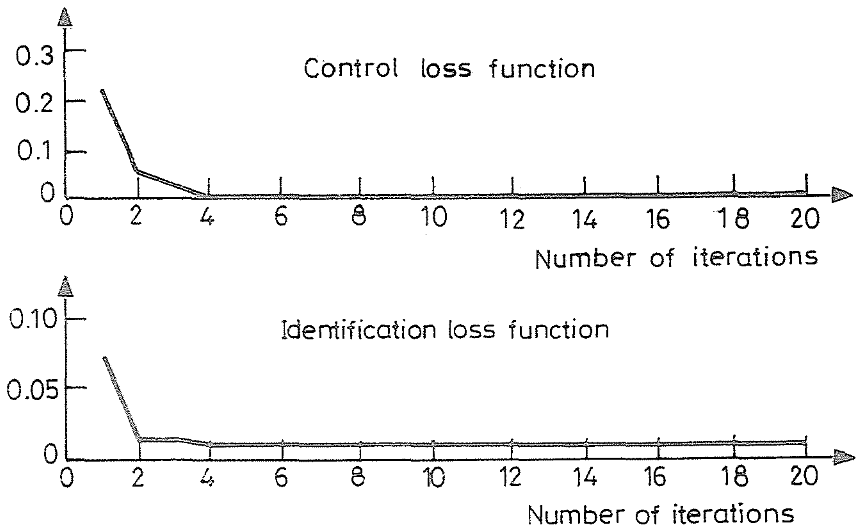

Fig. 16. Loss functions in an iterative control refinement procedure (first order example)

the iterative algorithm. (Our experience showed that this method works very fine in this scheme while the noise level is low. However, in a noisy case the proper parameter estimation method (ELS, ML, etc.) should be applied corresponding to the given process and noise model structure.) The outputs of the tracking reference model (continuous) and the controlled process (dashed) are shown in *Fig. 17* before and after the iterative control refinement.

Example 6.2 Let the process (GEVERS (1991)) be given by

$$S = \frac{0.0364z^{-1}\left(1 + 1.2z^{-1}\right)}{1 - 1.6z^{-1} + 0.68z^{-2}}, \tag{48}$$

where the same reference input and output noise was used as in the previous example with the initial model

$$M_0 = \frac{0.04z^{-1}\left(1 + 1.0z^{-1}\right)}{1 - 1.4z^{-1} + 0.5z^{-2}}. \tag{49}$$

Fig. 18 shows the control and identification loss functions (variances) by the iteration. The outputs of the reference model (continuous) and the controlled process (dashed) are shown in *Fig. 19* before and after the iterative refinement performed by off-line LS parameter estimation. Outputs of the controlled process (continuous) and identified model (dashed) before and after the iteration are shown in *Fig. 20.* This figure is a nice example

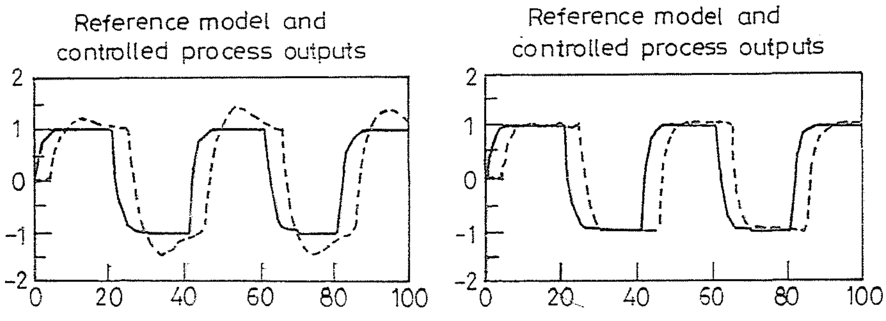

Fig. 17. Outputs of the reference model and the controlled process before and after the iteration (LS)

to explain the necessity for iterative control refinement. One can see that the identified model output fits very nicely to the process output before and after the iteration, too, so the model error is small. However, the control error is very bad before the iteration shown in *Fig. 19.*

7. Adaptive Solution for the On-line Iterative Regulator Refinement

The previous examples demonstrated the nice operation of the off-line iterative control refinement procedure based on the *generic scheme*. Inspite of the good convergence properties the necessary measurements are remarkable. In many applications this is a costly and long procedure to design the optimal regulator. However, it is easy to construct the adaptive control refinement procedure based on the iterative scheme. Following the same steps and properly changing the iteration i to sampling time k the following formulas are obtained (not discussing here the details):

$$\widehat{u}(k) = \left(M_{+}^{k-1}\right)^{-1} P_r y_r(k), \tag{50}$$

$$f\left(k-d_m\right) = \left[\widehat{u}\left(k-d_m\right), \widehat{u}\left(k-d_m-1\right), \ldots, -y(k-1), -y(k-2), \ldots\right]^T, \tag{51}$$

$$\mathbf{K}_k = \frac{1}{\rho^2}\left\{\mathbf{K}_{k-1} + \frac{\mathbf{K}_{k-1}\mathbf{f}\left(k-d_m\right)\mathbf{f}^T\left(k-d_m\right)\mathbf{K}_{k-1}}{\rho^2 + \mathbf{f}^T\left(k-d_m\right)\mathbf{K}_{k-1}\mathbf{f}\left(k-d_m\right)}\right\}, \tag{52}$$

$$\widehat{\mathbf{p}}_k = \widehat{\mathbf{p}}_{k-1} + \mathbf{K}_k\mathbf{f}\left(k-d_m\right)\left[y(k) - \mathbf{f}^T\left(k-d_m\right)\widehat{\mathbf{p}}_{k-1}\right],$$

$$\widehat{\mathbf{p}}_k = \left[\widehat{b}_0^k, \widehat{b}_1^k, \ldots, \widehat{a}_1^k, \widehat{a}_2^k, \ldots\right]^T, \tag{53}$$

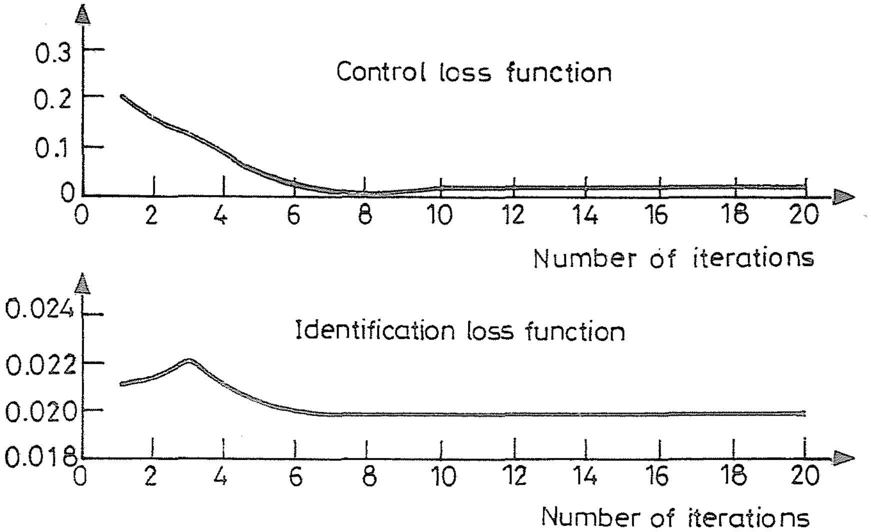

Fig. 18. Loss functions in an iterative control refinement procedure (second order example)

$$R_k = \frac{P_w}{1-P_w}\left(M_+^k\right)^{-1} = C\left(M_+^k\right)^{-1}, \tag{54}$$

$$u(k) = P_r\left[R_k M_-^k + \left(M_+^k\right)^{-1}\right] y_r(k) - R_k y(k), \tag{55}$$

where ρ is an exponential forgetting factor and *Eqs* (50) – (53) represent a recursive *LS* method (*RLS*) in the simplest so-called naive programming form. Several other recursive parameter estimations can be applied instead of the above *RLS* algorithm.

Note that after having the estimated parameter vector $\widehat{\mathbf{p}}_k$ corresponding to (21) obtained M_+ and M_- should be computed by factorization. The above procedure is an on-line strategy performing all refinement steps (50) – (55) in one sampling instance. Here we assumed that the signal $y_r(k)$ itself or its generation rule is given apriori.

8. Adaptive Examples

Example 8.1 Let the process be given by

$$S = \frac{0.001\,(1.1 + 1z^{-1})}{(1 - 1.6693z^{-1} + 0.7788z^{-2})} z^{-2}, \tag{56}$$

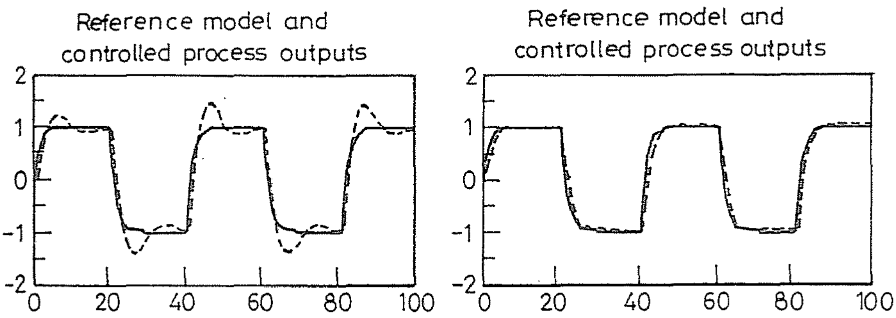

Fig. 19. Outputs of the reference model and the controlled process before and after the iteration (LS)

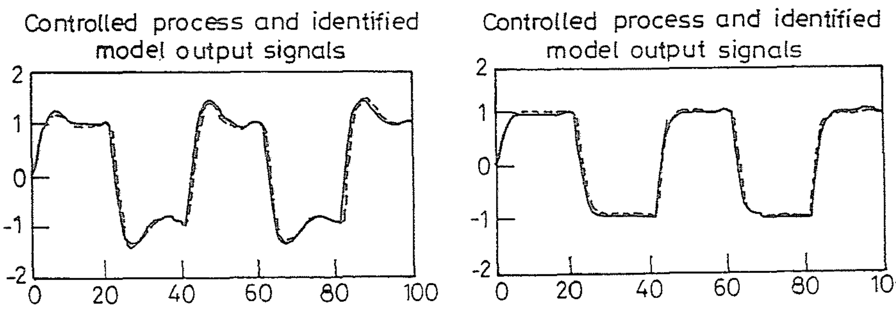

Fig. 20. Outputs of the controlled process and identified model before and after the iteration (LS)

which is a sampled-time (sampling time is $h = 0.05$ s and $d = 2$) second order approximation of a helicopter 'stick input/roll rate output' model. Here the same P_r, P_w, λ and square wave y_r excitation was used as in Example 6.1. The initial model was

$$M_0 = \frac{0.001\,(20 + 0.5z^{-1})}{(1 - 1.5z^{-1} + 0.8z^{-2})} z^{-2}, \tag{57}$$

i.e., $d_m = d = 2$. The outputs of the reference model (continuous) and the controlled process (dashed) furthermore the control (continuous) and modelling (dashed) error signals are shown in *Fig. 21* for $N = 200$ samples using the adaptive control refinement strategy (50) – (55) with $\mathbf{T}_0 = 100\mathbf{I}$ and $\rho^2 = 0.95$.

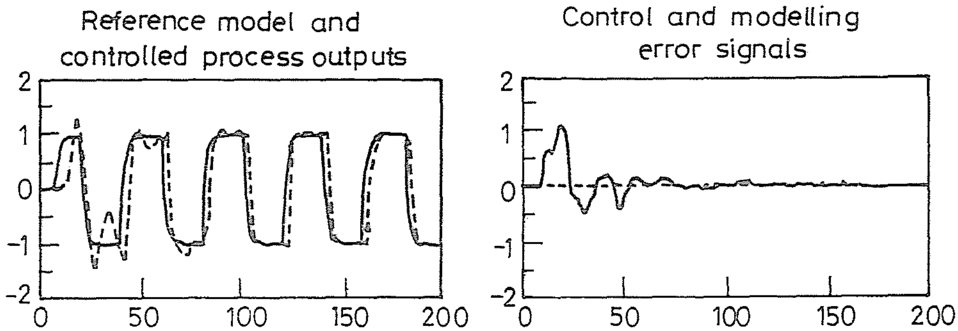

Fig. 21. Operation of the adaptive control refinement

9. Conclusions

The paper introduced a new structure to design optimal pole-placement controllers. This new scheme allows to avoid the explicit solution of a polynomial equation obtaining the transfer function elements of the optimal controller directly. The new design principle is quite general and applicable for nonminimum phase (inverse unstable) and delay time systems, too. The controller is easy to be implemented in computer controlled systems. The structure of the optimal controller gives a special insight to understand the operation of a feedback loop for the servo and disturbance rejection paradigm. It is easy to see the role of the knowledge of the system model and the role and appearance of the system factors that are invariant to any control strategy.

This structure has further advantages in handling system uncertainties and new canonic sensitivity schemes can also be obtained. The new *generic optimal controller scheme* seems to be the best among the investigated methods. It behaves also well because the identification and control errors are the same and they can vanish only at the same time. This scheme is easy to be implemented because general identification techniques can be applied between a calculated auxiliary input and the measured closed-loop output variables.

An applicable strategy for iterative control refinement based on the *generic scheme* was presented and illustrated by simulation examples. The adaptive version of the control refinement strategy was also shown and demonstrated by an example.

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