PERIODICA POLYTECHNICA SER. TRANSP. ENG. VOL. 23, NO. 1–2, PP. 19–35 (1995)

ON STOCHASTIC SIMULATION OF THE WHEEL-PROFILE WEAR PROCESS OF A RAILWAY VEHICLE OPERATING ON A SPECIFIED NETWORK

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Received: November 8, 1994

Abstract

The alteration in wheel and rail profiles due to wear involves considerable vehicle and track-maintenance costs, and influences the loading capacity of the rails, as well as the operation safety and riding comfort of the vehicles. In the past five years, a vehicle dynamics-based numerical procedure was elaborated at the TU of Budapest to predict the wear-caused wheel profile alterations and to maximize the mileage performance by selecting the optimum axle-box guidance stiffnesses in case of traditional running-gears operating on a specified railway line [1]. This paper introduces the fundamental principles and conditions decisive in the dynamical and tribological procedure mentioned, and deals with an extension of it. This extension regards the wheel wear analysis and mileage performance maximization of the vehicle in case of stochastic operation on a given railway network. The network is characterized by its graph. The operation process is described in the framework of a semi-Markovian model [12], [13]. The elaborated stochastic simulation and optimization method is visualised for a simple railway network by introducing the two important two-parameter stochastic fields, namely those of the wheel profile and the mileage performance.

Keywords: stochastic simulation, railway wheel-profile wear.

1. Introduction

The wear simulation technique based on a non-linear dynamical track/vehicle model – elaborated by the Department of Railway Vehicles at the TU of Budapest for the numerical investigations into the wheel-profile wear of railway vehicles operating on a specified railway line [1], [10], [11] – is extended by the authors. The goal of the extension is to describe the propagation of the wheel-profile-wear under stochastic operation conditions of the vehicle on a whole railway network as a function of the distance covered by the vehicle. The stochastic operation process taking place on the network is treated on the basis of the theory of semi-Markovian stochastic processes [12], [13]. The statistical characteristics of the bivariate stochastic field describing the wheel profile alterations and also the stochastic field describing the mileage performance defined as a function of the longitudinal and lateral axle-box guidance stiffnesses are analyzed. The expected value function and the standard deviation function of the latter stochastic field makes possible to optimize the axle-box guidance system by maximizing the expected mileage performance between two profile renewals.

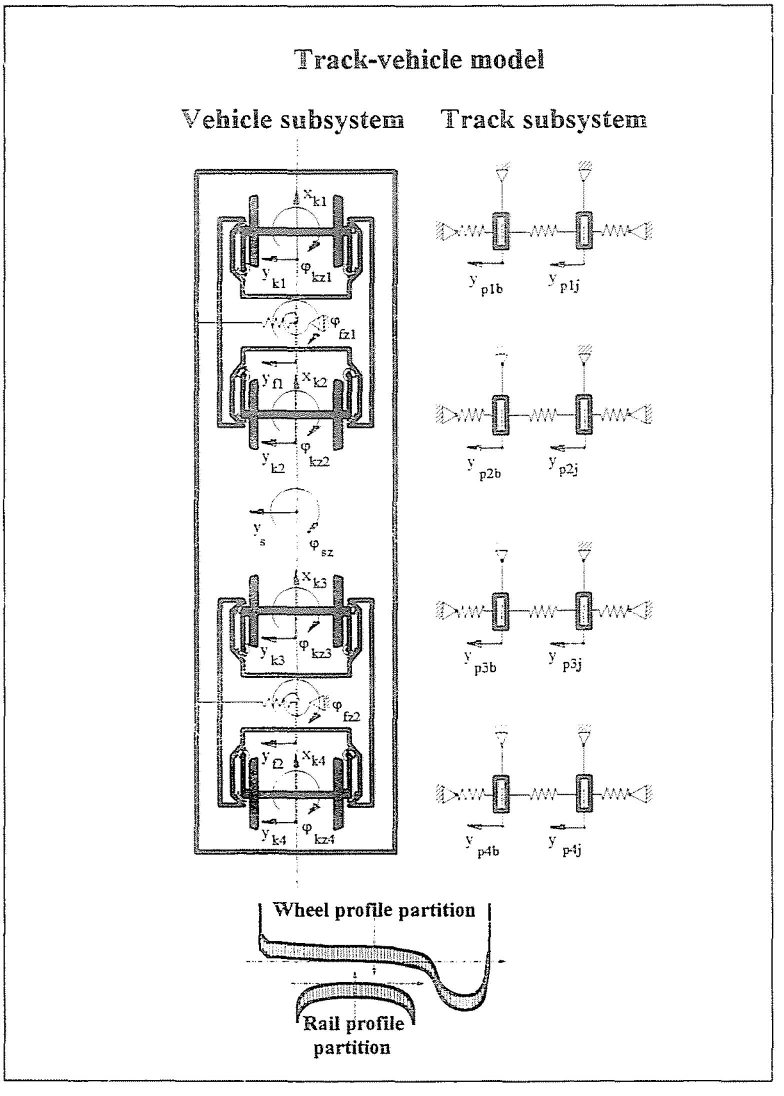


Fig. 1. Lumped-parameter track–vehicle model

2. Summary of the Wheel-Wear Simulation on a Specified Railway Line

2.1. Introductory Remarks

The possibility of predicting the wheel wear process of a railway vehicle is connected with the numerical feasibility of simulations based on:

- appropriate dynamical models,
- rolling contact theory, and
- wear hypothesis.

The early investigations into the wear phenomena of railway wheels focused on the wear propagation under simple and steady operation conditions, e.g. constant velocity operation on an ideal straight track [2], [3], [4], [5], [6].

The most frequently used wear hypothesis was formulated in terms of the proportionality between the specific energy dissipated over the contact surface and the specific mass removal for the unit of the distance covered [6].

In the following, the model elaborated for simulating the wear propagation on the wheels running on a specified railway line consisting of straight and curved sections will be summarized.

This model was the basis of the research into the more complicated wear process simulation concerning the stochastic operation on a specified railway network.

2.2. The Track-Vehicle Dynamical Model

The lumped-parameter track–vehicle model is shown in *Fig. 1.*

The vehicle body, the bogies, the wheel sets and the discrete masses representing the track inertia are modelled as rigid bodies. The spring structural connections between the rigid bodies mentioned are modelled by piecewise linear characteristics, while the dampers supposed to be parallel to the former ones are definitely linear. The used wheel and rail profiles can be practically arbitrary, the only requirement is to give their points on a laterally equidistant sequence of 1 mm spacing.

The track can be composed of specified straight and curved sections coming one after another in an arbitrary order, see *Fig. 2.* The transition curves can be approximated by circular arcs of non-equal radii.

- The track can be laterally 'imperfect', i.e. a stochastic lateral irregularity can be taken into consideration. On the basis of the measured spectral density function, lateral irregularity realization functions are generated for the two rails as based on simulation technique by random number generation [8]. The lateral irregularities as excitation sources are taken into consideration in both the straight and the curved track sections.

- The vertical wheel loads are constants on the straight sections, while in curves the quasi-static compensation of the centrifugal forces is carried out [7].
- The wheel–rail contact on the wheel tread is treated as a creep-dependent force transfer spot, the creep coefficients are treated by Kalker's linear theory [9].
- The longitudinal and lateral contact forces, as well as the spin moment are bounded by the values based on the constant sliding friction coefficient.
- The flanging is considered as a conditional, laterally elastic and damped linear connection between the wheel set and the mass representing the rail inertia.
- In the model, a specified constant torque is acting on each wheel set, which represents the resultant of the rolling resistance, the journal friction and the eventually acting tractive or braking torques.
- It is always assumed that no braking torque is exerted by frictional tread braking, i.e. the wear phenomenon on the wheel tread and the flange are caused exclusively by the wheel–rail contact.

2.3. Remarks on the Simplified Operation Conditions

The vehicle–track model in question takes into consideration track section-wise constant travelling velocities and torques on each wheel set. It is reasonable to use a constant average travelling velocity along the whole railway line examined, and also an average torque to act on the wheel sets on the

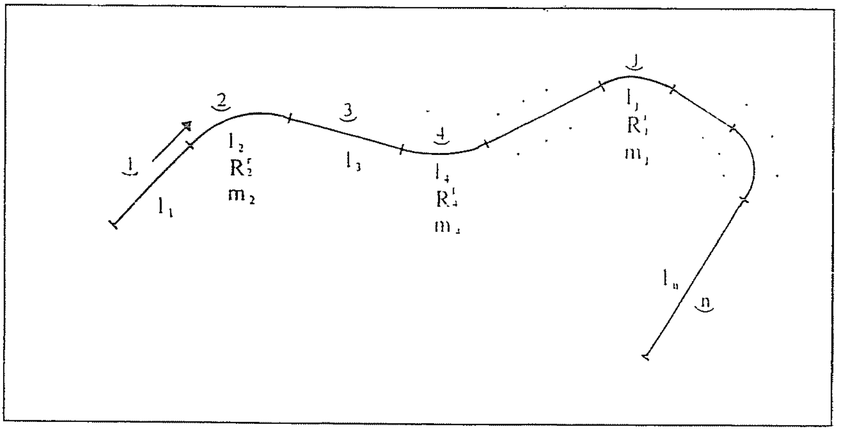


Fig. 2. Straight and curved sections of the track

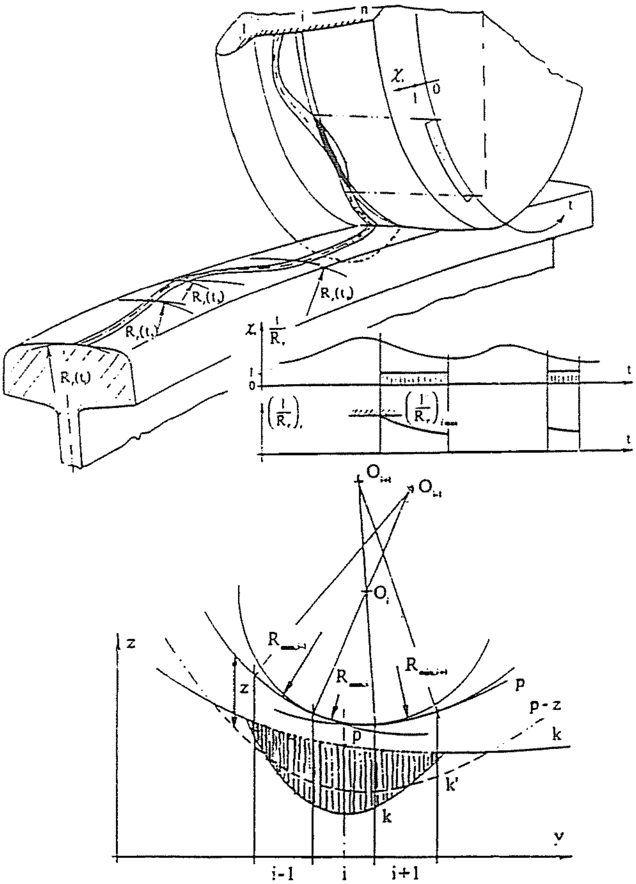


Fig. 3. The principle of physical smoothing

basis of a preliminary 'speed – distance covered' analysis. Tractive and braking torques vs. distance covered functions can also be determined for the whole length L of the line between terminals A and B.

Average velocity and torque can be calculated by formulae

$$\nu_{aAB} = \frac{1}{L}\int_A^B |\nu(s)|ds \quad \text{and} \quad M_{raAB} = \frac{1}{L}\int_A^B |M_r(s)|ds \ . \tag{1}$$

2.4. Wheel Profile Alteration Due to Wear

The wheel profile alteration due to wear caused by the rolling contact of the wheel and railhead is a rather slow process. Debris continuously leaves the wheel tread and recurrently the flange surface being in sliding contact with the railhead.

Principally, each angular displacement increment $d\varphi$ of the wheel implies a certain material removal Δdm from the contact spot on the wheel. This material loss is very slow, and a recognisable variation in profile geometry appears only over hundred km-s of distance covered by the wheel. This survey is the basis of the discretization technique used in the elaborated wear simulation procedure.

The fundamental idea is to consider the actual wheel profile and railhead profile as the basis of the contact geometrical and dynamical operations. The latter ones result in the wear-load distribution along the wheel meridional profile. This wear-load distribution is considered valid for a given distance covered by the wheel, and the profile alteration, i.e. the reduction in rolling radii is carried out by using appropriate smoothing procedures.. In this way, a discrete step of profile alteration due to wear is done [10].

The resulted new wheel profile takes over the role of the initial profile, and a subsequent discrete step can be done, etc. The outlined procedure means that the material is removed step-wise, whereas the contact conditions are considered to be constant in each step.

The errors caused by the discretization are balanced by the physical and mathematical smoothing procedures, which are built up on the basis of the rail and wheel profile compatibility and C_2 spline method.

Regarding the tread contact, the used wear hypothesis connects the specific mass removal $\partial m/\partial s$ from the contact band with the specific work done by the creep forces by proportionality

$$\frac{\partial m}{\partial s} = k_r \left(F_x \nu_x + F_y \nu_y \right) . \tag{2}$$

In the formula, k_r is the wear coefficient, F_x and F_y are the longitudinal and lateral creep forces, while ν_x and ν_y are the longitudinal and lateral creepages, respectively. In case of flange contact

$$\frac{\partial m}{\partial s} = k_s F_f \frac{\Delta \nu_s}{\nu} . \tag{3}$$

Here k_s is the sliding wear coefficient, F_f is the friction force on the flange, $\Delta \nu_s$ is the sliding velocity and ν is the travelling velocity.

Mass removal Δm from the contact surface A over a small distance of rolling Δs can be calculated based on relationship

$$\Delta m = \frac{\partial m}{\partial s} \Delta s . \tag{4}$$

Mass Δm should be distributed along that interval(s) of the wheel profile which intersects the contact area.

The Hertzian pressure ellipsoid is divided into 'slices' by parallel planes being in a lateral distance Δy from each other. The volume of the pressure ellipsoid is V, while that of the i-th slice is V_i. Then, weighting factors $\lambda_i = V_i/V$; $i = 1, 2, \ldots, n$ are defined, and the i-th 'slice' will have a mass fraction removed by wear

$$\Delta m_i = \lambda_i \Delta m = \lambda_i \frac{\partial m}{\partial s} \Delta s \ . \tag{5}$$

The wear load for the partition elements along the profile is

$$w_i = \frac{1}{S} \sum \Delta m_i \ : \quad i = 1, 2, \ldots, n \ , \tag{6}$$

where S is the total distance covered by the wheel. The unit of measure of w_i is mg/m. With the knowledge of the topology of the railway line the conditional wear-load distributions can be calculated for each straight and curved section:

$$w_{ij} \ ; \quad j = 0, 1, 2, \ldots, m \ . \tag{7}$$

The resultant wear distribution is yielded in the form

$$W_i = \sum_{j=0}^{m} w_{ij} l_j \ . \tag{8}$$

In formula (8) l_0 is the total length of the straight track sections and l_j; $j = 1, 2, \ldots, m$ are the total lengths of the curved ones of radii R_j; $j = 1, 2, \ldots, m$.

The decrement in radius of the i-th slice due to wear is

$$\Delta r_i = W_i / \rho \ 2\pi \ r_i \ \Delta y \ , \tag{9}$$

where ρ stands for the material density of the wheel and r_i is the initial radius of the wheel for partition element Δy. The sequence of worn profile radii are

$$r_i^w = r_i - \Delta r_i \ ; \quad i = 1, 2, \ldots, n \ . \tag{10}$$

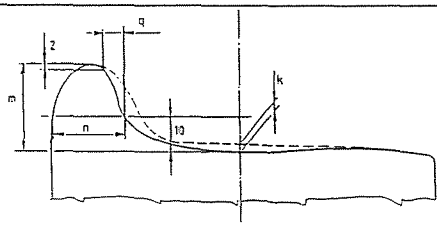
Discretized profile alterations are carried out by 1500 km distances. The errors caused by profile incompatibility should be balanced by smoothing.

Profile incompatibility can occur in certain altered wheel profile point(s) if the local curvature yielded is greater than the maximum rail profile curvature contacted.

The maximum rail profile curvatures contacted by the partition elements of the wheel profile are continuously computed.

For the wheel profile smoothing the following steps are required [7]:

- Construction of 'circle arc chain' (see *Fig. 3*).
- Pushing the circle arc chain into the profile to be smoothed.
- Mathematical smoothing by using C_2 spline.
- Checking of the curvature compatibility with the smoothed profile. If necessary, repetition of the above points up to achieving compatibility.



1	k	$0 \le k \le k_{meg} = 5$mm
2	n	$n \ge n_{min} = 27$mm
3	m	$m \le m_{meg} = 35$mm
4	q	$q \ge q_{min} = 6.5$mm

Fig. 4. Permitted profile dimensions

Distance covered M, which belongs to the exhaustion of one of the four conditions prescribed for permitted rail profiles (see *Fig. 4*) defines the mileage performance of the wheel under examination [10], [11].

2.5. Numerical Optimization of Mileage Performance

The mileage performance M belongs to the fixed axle-box guidance stiffness values taken into consideration. Define axle-box guidance stiffness vector $\mathbf{s} = [s_x, s_y]^T$, and seek for the conditional maximum of function $M(\mathbf{s})$ over the permitted domain E of rectangle form (see *Fig. 5*). If the initial stiffness vector is designated by $\mathbf{s}_0$, then – based on the gradient method-vector $\mathbf{s}_1$ can be computed by formula:

$$\mathbf{s}_i = \mathbf{s}_{i-1} + \Delta s \frac{\mathbf{grad}\ M(\mathbf{s}_{i-1})}{|\mathbf{grad}\ M(\mathbf{s}_{i-1})|} . \tag{11}$$

3. Specification of the Railway Network

The railway network can be interpreted as a set of nodes (line junctions) and lines interconnecting the nodes or radially leaving them as single branches. A simple network consisting of only two nodes and three lines is shown in *Fig. 6*.

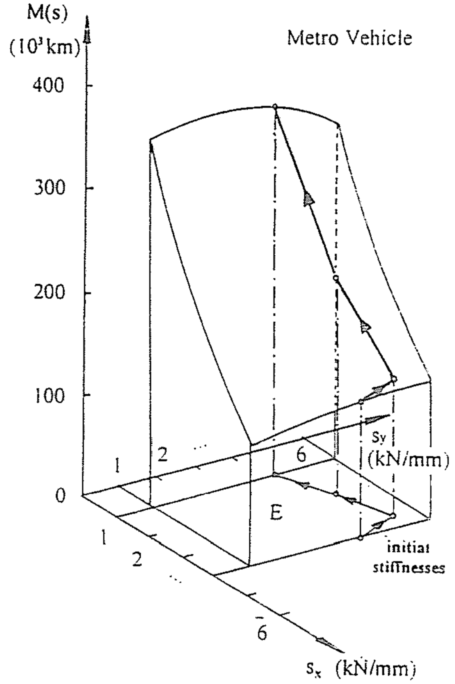


Fig. 5. Objective function $M(s)$ and the maximization procedure over domain E

4. Approximate Description of the Operation Conditions by a Semi-Markovian Stochastic Process

Let us denote the sequence of the states of the semi-Markovian process by

$$u_1, u_2, \dots, u_n, u_{n+1} \dots . \tag{12}$$

The sequence of random durations spent in the appropriate states is

$$\tau_1, \tau_2, \dots, \tau_n, \tau_{n+1} \dots . \tag{13}$$

The state transition probabilities are defined by the following conditional probabilities:

$$p_{ij} = P\left\{\{u_{n+1} = j\} | \{u_n = i\}\right\} \ ; \quad i, j = 1, 2, \dots, N \ , \tag{14}$$

where N stands for the number of line sections in the railway network considered. The system of transition probabilities can be represented by defining

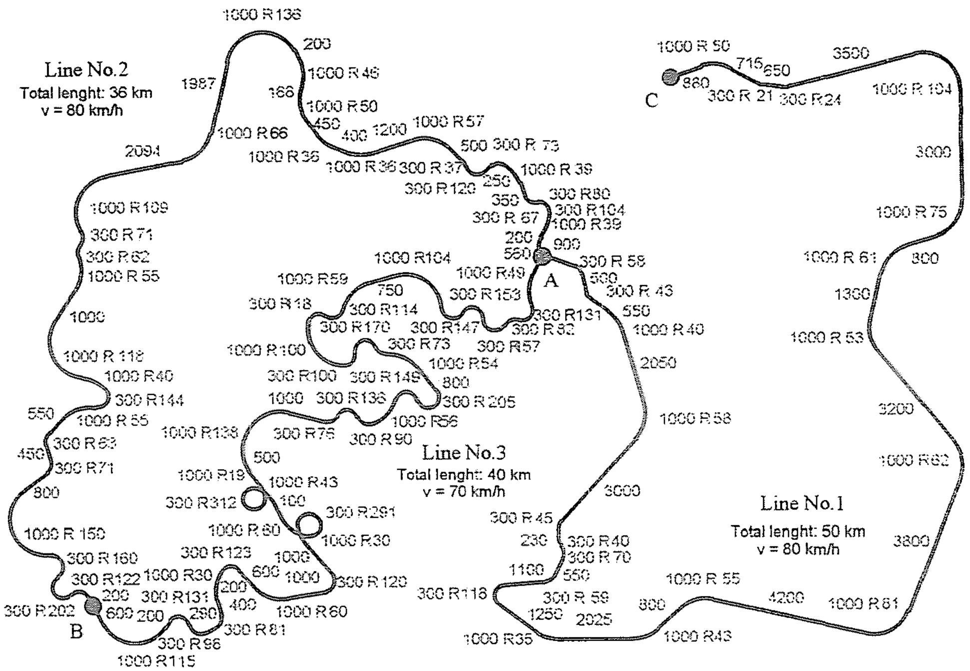


Fig. 6. Topology of the railway network (Example)

the $N \times N$ stochastic matrix Π as follows

$$\Pi = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1N} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2N} \\ p_{31} & p_{32} & p_{33} & \cdots & p_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{N1} & p_{N2} & p_{N3} & \cdots & p_{NN} \end{bmatrix} . \tag{15}$$

For the systematic process description, also the 'staying-in-state' probabilities should be dealt with. In case of semi-Markovian processes, a matrix-valued function built up from the conditional probability distribution functions $F_{ij}(t)$ plays a decisive role [12]. Functions $F_{ij}(t)$ are non-zero only for those index pairs which also non-zero transition probabilities belong to. Function $F_{ij}(t)$ means the conditional probability that staying in the j-th state is less than duration t, under the condition that the previous state was the i-th one.

According to the aforesaid, the durations spent in the appropriate states are characterized by the set of conditional probability distribution functions:

$$F_{ij}(t) = P\left\{\{\tau_{n+1} < t\}|\{u_n = i\}\bigcap\{u_{n+1} = j\}\right\} \; ; \quad i,j = 1,2,\dots,N \; . \quad (16)$$

Using the convenient matrix formulation, the $N \times N$ matrix-valued conditional distribution function $\mathbf{F}(t)$ is yielded:

$$\mathbf{F}(t) = \begin{bmatrix} F_{11}(t) & F_{12}(t) & F_{13}(t) & \dots & F_{1N}(t) \\ F_{21}(t) & F_{22}(t) & F_{23}(t) & \dots & F_{2N}(t) \\ F_{31}(t) & F_{32}(t) & F_{33}(t) & \dots & F_{3N}(t) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ F_{N1}(t) & F_{N2}(t) & F_{N3}(t) & \dots & F_{NN}(t) \end{bmatrix} . \quad (17)$$

The railway operation taking place on the network plotted in *Fig. 6* is characterized by semi-Markovian stochastic process $\xi(t)$ describing the transitions between the following five 'states':

1. Operation on line 1 from A to the end of line and back,
2. Operation on line 2 from A to B,
3. Operation on line 2 from B to A,
4. Operation on line 3 from A to B,
5. Operation on line 3 from B to A.

The state transitions will be controlled by the transition probability matrix $\Pi = \{p_{ij}\}$. The entries of matrix Π should be determined by using the operation time table or traffic statistics. In our example the following transition probability matrix was used:

$$\Pi = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.9 & 0 & 0.1 \\ 0.05 & 0.9 & 0 & 0.05 & 0 \\ 0 & 0 & 0.1 & 0 & 0.9 \\ 0.05 & 0.05 & 0 & 0.9 & 0 \end{bmatrix} . \quad (18)$$

In accordance with the definition, entries in matrix Π are conditional probabilities, e.g. p_{ij} stands for the conditional probability of occurring state j in the course of the state transition, under the condition that prior to the transition the state was i.

In the following tractation, variable $x \geq 0$ means the number of running cycles along a 'state-defining' line in the network considered. As states 2,3,4,5 are defined by performing a single running cycle along line 2 and line 3, respectively, the characteristic number of cycles will be 1 for the states mentioned, irrespective of the previous state i. So functions F_{ij} can be expressed by shifted unit jump functions U (Heaviside functions) as follows:

$$F_{ij}(x) = U(x-1) \; ; \quad j = 2,3,4,5 \; . \quad (19)$$

If state 1 is realized, then function $F_{ij}(x)$ should be defined as a piece-wise linear distribution function over the positive axis with break points at the positive integers. The actual variation of $F_{ij}(x)$ can depend on the previous state. In our example matrix-valued function $\mathbf{F}(x)$ takes the following form:

$$\mathbf{F}(x) = \begin{bmatrix} 0 & U(x-1) & 0 & U(x-1) & 0 \\ 0 & 0 & U(x-1) & 0 & U(x-1) \\ F_{31}(x) & U(x-1) & 0 & U(x-1) & 0 \\ 0 & 0 & U(x-1) & 0 & U(x-1) \\ F_{51}(x) & U(x-1) & 0 & U(x-1) & 0 \end{bmatrix} . \quad (20)$$

When simulating the number of cycles, uniformly distributed random number η is generated, and x_{ij} is obtained by relationship

$$x_{ij} = F_{ij}^{-1}(\eta) . \quad (21)$$

In *Fig. 7* the operation process of a vehicle moving on the specified railway network is visualized. On the horizontal axis the distance covered by the vehicle is represented, while on the vertical axis positive integers 1, 2, 3, 4 and 5 identify the states of the embedded Markovian chain. These states indicate the actual motion of the vehicle on the line sections, as they are defined in specification (18). The total length in km belonging to operation

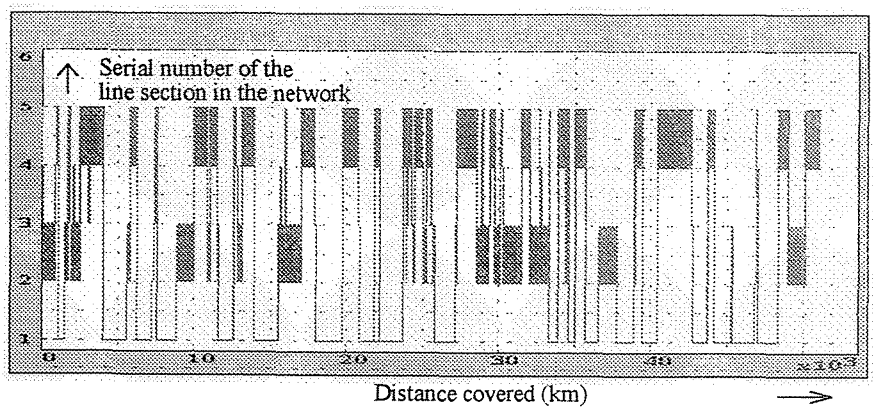


Fig. 7. Realization of the semi-Markovian stochastic process of the operation on the specified railway network

in state 1 can uniquely be read off from the Figure, by determining the total length of intervals fitting on level 1. The total length of operation in the other states can be determined similarly, by reading off the summarized length of the related intervals fitting on levels 2, 3, 4 and 5. In the Figure due to the horizontal scale applied unfortunately the intervals belonging to the latter states can hardly be recognised because of the considerable width of the vertical lines visualizing the actual state transitions between the adjacent levels.

5. Simulation of the Wheel Profile Wear Stochastic Process

With the knowledge of matrices Π and $\mathbf{F}(x)$ operation process realization sections of 1500 km length can be simulated one after another, sequentially, up to the exhaustion of at least one condition prescribed for the permitted wheel profiles. The sequence of the altered wheel profiles can be determined for the realization function in question, see *Fig. 8.* The alteration of the wheel profile meridional curves in the course of the stochastic operation conditions examined becomes a stochastic process, namely a two-parameter stochastic field $\zeta(S, y)$.

Profile realizations for distance covered $S = 24000$ km are visualized in *Fig. 9* for the case of stochastic operational process determined by matrices (19) and (21). Expected value and standard deviation of the profile realizations can be evaluated, and so can the one-dimensional marginal probability density functions of the profile.

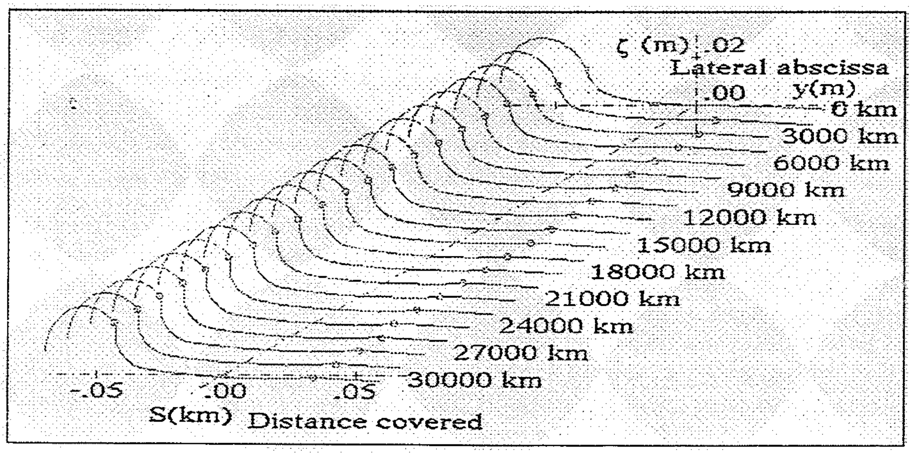


Fig. 8. Sequence of altered wheel profiles

6. Evaluation of the Stochastic Mileage Performance

In case of stochastic operational process also the mileage performance becomes a stochastic field defined on stiffness parameters s_x and s_y. Stochastic

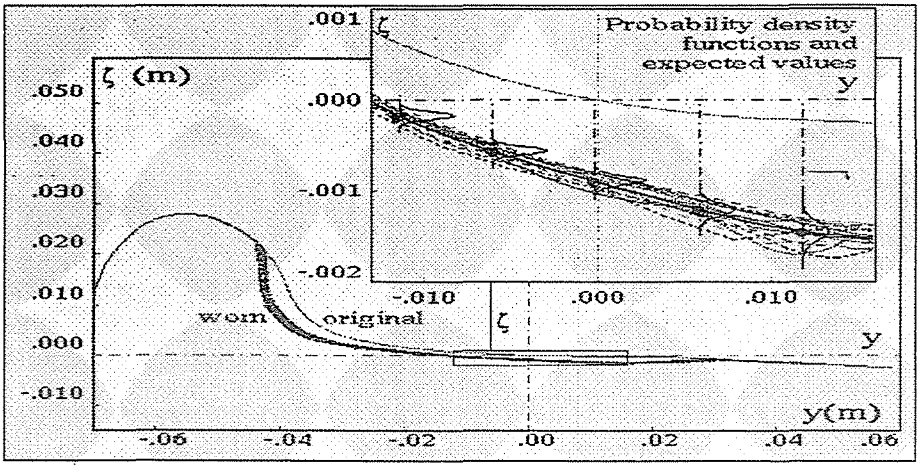


Fig. 9. Worn profile realizations after 24000 km distance covered in stochastic operation

mileage performance $\mu(s_x, s_y)$ can be defined as infimum of four random variables, namely of $\mu_1(s_x, s_y)$, $\mu_2(s_x, s_y)$, $\mu_3(s_x, s_y)$ and $\mu_4(s_x, s_y)$. The latter are the distances covered up to the individual exhaustion of the permitted profile dimensions in *Fig. 4*. According to the aforesaid

$$\mu(s_x, s_y) \stackrel{\text{def}}{=} \inf\ \{\mu_1(s_x, s_y), \mu_2(s_x, s_y), \mu_3(s_x, s_y), \mu_4(s_x, s_y)\}$$

is a bivariate stochastic field. In a spatial co-ordinate frame, over the rectangle lying in the parameter plane the one-dimensional marginal probability density functions are plotted on the basis of simulation results obtained for the simple network shown in *Fig. 6*.

It is obvious to determine also the expectation and standard deviation of $\mu(s_x, s_y)$ as a function of s_x and s_y over the parameter domain, as it is indicated in *Fig. 10*.

7. Possibility of Axle-Box Guidance Optimization

The objective function of the mileage performance optimization can be the expected value function of the field μ in the form:

$$M(\mathbf{s}) = \mathbf{E}\mu(s_x, s_y) = \max!\,. \tag{22}$$

The numerical procedure can be again the gradient method described by formula (11). As a supplementary objective, the requirement of low standard

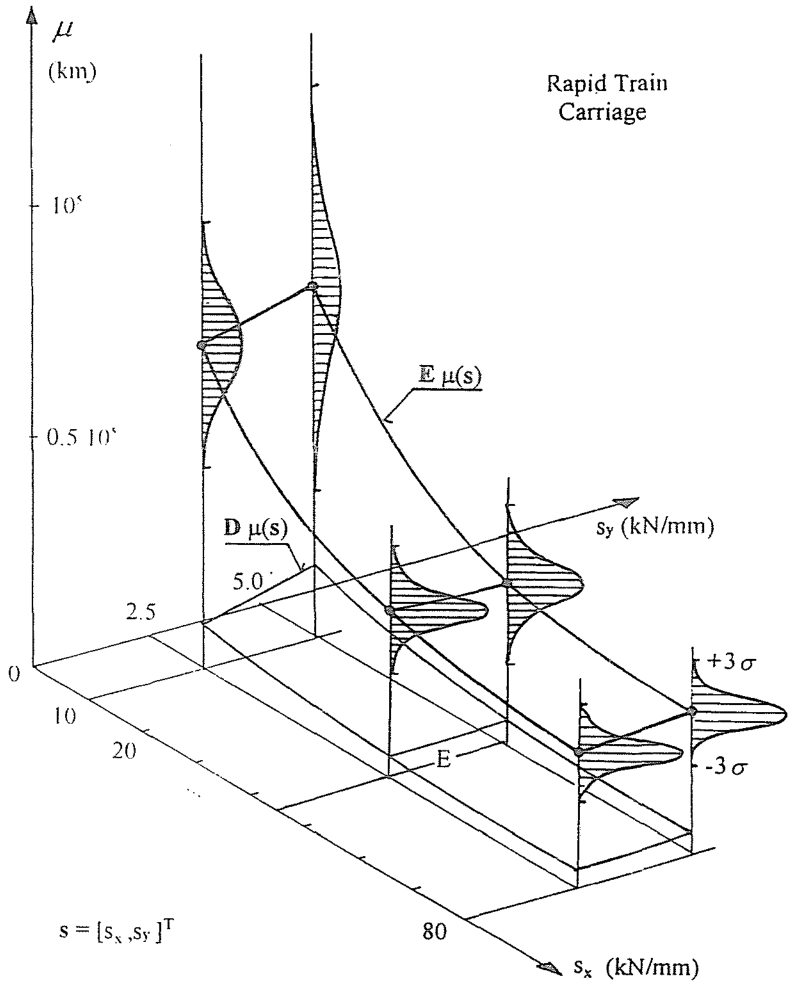


Fig. 10. Expectation, standard deviation and marginal density functions of stochastic field $\mu(s)$

deviation of mileage performance can be written as follows:

$$D(\mathbf{s}) = \mathbf{D}\mu(s_x, s_y) = \min! . \tag{23}$$

Objective functions (18) and (19) determine a region of the stiffness parameter plane in which in average high and relative homogeneous mileage performances can be predicted.

8. Concluding Remarks

On the basis of our theoretical investigations and numerical simulations, the following conclusions can be drawn:

- The operation of the vehicle can be approximately described by using the semi-Markovian stochastic model, based on the state transition probability matrix and the matrix-valued function of the conditional probability distribution functions of the durations spent on certain line sections after specified state transitions,
- The description of wheel wear propagation by a two-parameter stochastic field represents a new approach to the analysis,
- The stochastic field reflecting the mileage performance is introduced, its expectation and standard deviation functions form the objective functions of the traditional running-gear optimization, so the lateral and longitudinal axle-guidance stiffnesses can be optimized by a numerical procedure, also in case of stochastic operation on a specified railway network,
- Further investigations are required to take into consideration also the wheel wear caused by tread braking.

Authors are working on the prediction of rail wear propagation by means of similar tools presented in this paper.

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