PERIODICA POLYTECHNICA SER. TRANSP. ENG. VOL. 23, NO. 1-2, PP. 37-44 (1995)

ON DYNAMICAL PROCESSES IN RAILWAY TRACTION UNITS CAUSED BY WHEELSET GRAVITY POINT ECCENTRICITIES

I. ZOBORY and V. ZOLLER

Technical University of Budapest
Department of Railway Vehicles
H-1521 Budapest, Hungary

Received: Nov. 9, 1994

Abstract

Authors analyse the effect of the wheelset gravity point eccentricity on the longitudinal dynamical processes of the vehicle by using a simple model describing the longitudinal motion of the wheelset and the bogie-frame interconnected through a linear spring/damper system. The torque driving the wheelset is given by a specified time function. The wheel/rail contact is treated by the approximate pure rolling condition. By using perturbation techniques, appoximate solutions of the equations of motion are determined.

Keywords: vehicle dynamics, simulation of operational loads, perturbation techniques.

1. Introduction

In case of railway vehicles certain gravity point eccentricity (GPE) always appears with the wheels due to inaccuracies in the machining or irregular wear on the running surfaces.

It is an important question in the design and operation of railway vehicles, how great the allowed GPE can be. It is obvious that the presence of non-zero GPE values implies the existence of rotating centrifugal forces in the course of the rotatory motion of the wheels [2]. As the wheelsets are elastically connected in horizontal direction with the bogie-frame of the railway vehicle, it is clear that, due to the horizontal component of the rotating centrifugal forces acting on the wheelset, under certain conditions excited vibrations can be generated in the wheelset/bogie-frame two mass system. The latter excited vibrations can cause unwanted excess loads in the drive system, and the running behaviour of the vehicle can become untolerable.

The actual problem which motivated the investigations described in this paper was connected with the undesirable longitudinal vibrations experienced in the running gear of a metro trainset during the test period [3]. As a possible source of the unwanted dynamic processes, the presence of certain GPE values was suspected. In order to check the possible motions caused by the excitation effect of the rotating centrifugal forces, a simple model was established, for which the strongly nonlinear problem could be treated by some approximation methods developed in the theory of nonlinear ordinary differential equations (see e.g. [1]).

2. The Model

In *Fig. 1* the simple in-plane longitudinal dynamical model of the wheelset/bogie-frame system is shown. Mass m_1 represents the bogie undergoing longitudinal translatory motion, while m_2 represents the mass of the wheelset having a GPE value designated by e. The moment of inertia of the wheelset is denoted by Θ, while s and k stand for the stiffness and damping coefficients of the connection, respectively. The longitudinal displacements x_1 and x_2, as well as the velocities $\dot{x}_1$ and $\dot{x}_2$ are also identified in *Fig. 1.* The rolling radius of the wheel is R, while c stands for the air drag coefficient. The time dependent torque M, acting on the wheelset, is in dynamical connection with the peripheral force F arising in the wheel–rail contact. The rolling motion of the wheelset is considered as pure rolling, i.e. creep dependent phenomena are omitted.

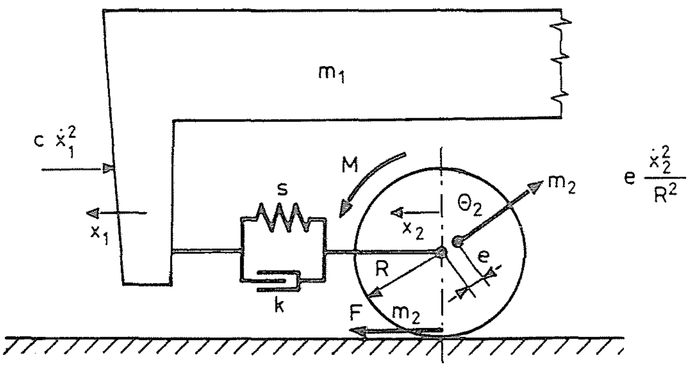

Fig. 1. The system model

3. Equation of Motion

The governing equations of the system have the form

$$m_1\ddot{x}_1 = s(x_2 - x_1) + k(\dot{x}_2 - \dot{x}_1) - c\dot{x}_1^2 , \tag{1}$$

$$m_2\ddot{x}_2 = -s(x_2 - x_1) - k(\dot{x}_2 - \dot{x}_1) + \frac{m_2 e}{R^2}\dot{x}_2^2 \cos\frac{x_2}{R} + F , \tag{2}$$

where, in case of pure rolling in the wheel–rail contact, force F can be determined by the equation

$$\Theta \frac{\ddot{x}_2}{R} = M - FR ,$$

and M stands for the reduced turning moment.

Acceleration a of the longitudinal fundamental motion of the mass gravity point can be obtained by the governing equation

$$(m_1 + m_2)a = \frac{M - \Theta a/R}{R} - cv^2$$

as

$$a = \frac{MR - cR^2v^2}{(m_1 + m_2)R^2 + \Theta} ,$$

where v is the speed of the fundamental motion.

Let us introduce the following notations:

$$\omega_1^2 := \frac{s}{m_1} , \quad \omega_2^2 := \frac{s}{m_2 + \Theta/R^2} , \quad \omega^2 := \omega_1^2 + \omega_2^2 ,$$

$$\kappa_1 := \frac{k}{m_1} , \quad \kappa_2 := \frac{k}{m_2} , \quad \kappa := \kappa_1 + \kappa_2 ,$$

$$\varepsilon := \frac{em_2R}{m_2R^2 + \Theta} \quad \text{and} \quad \delta := \frac{cR}{m_1} .$$

Using the above defined new parameters, the governing equations take the form

$$\ddot{x}_1 - \kappa_1\dot{x} - \omega_1^2 x = -\frac{\delta}{R}\dot{x}_1^2 , \tag{3}$$

$$\ddot{x}_2 + \kappa_2\dot{x} + \omega_2^2 x = \left(\frac{\omega}{\omega_1}\right)^2 a + \frac{\delta}{R}\left(\frac{\omega_2 v}{\omega_1}\right) + \frac{\varepsilon}{R}\dot{x}_2^2 \cos\frac{x_2}{R} , \tag{4}$$

where $x := x_2 - x_1$ stands for the relative displacement.

4. Perturbation Equations

We are looking for the solution of the set of differential equations (3–4) in the series form

$$x_i = u_{i0} + \varepsilon u_{i1} + \delta u_{i2} + \dots , \quad i = 1, 2,$$

where functions u_{ij}, $j = 0, 1, 2, \ldots$ are already independent of the dimensionless quantities ε and δ. Relative displacement x can be written into the form

$$x = u_0 + \varepsilon u_1 + \delta u_2 + \ldots$$

Omitting the terms involving degree greater than 1, we obtain the following six equations:

$$\ddot{u}_{10} - \kappa_1 \dot{u}_0 - \omega_1^2 u_0 = 0\,, \tag{5}$$

$$\ddot{u}_{20} + \kappa_2 \dot{u}_0 + \omega_2^2 u_0 = \left(\frac{\omega}{\omega_1}\right)^2 a\,, \tag{6}$$

$$\ddot{u}_{11} - \kappa_1 \dot{u}_1 - \omega_1^2 u_1 = 0\,, \tag{7}$$

$$\ddot{u}_{21} + \kappa_2 \dot{u}_1 + \omega_2^2 u_1 = \frac{1}{R}\dot{u}_{20}^2 \cos\frac{u_{20}}{R}\,, \tag{8}$$

$$\ddot{u}_{12} - \kappa_1 \dot{u}_2 - \omega_1^2 u_2 = -\frac{1}{R}\dot{u}_{10}^2\,, \tag{9}$$

$$\ddot{u}_{22} + \kappa_2 \dot{u}_2 + \omega_2^2 u_2 = \frac{1}{R}\left(\frac{\omega_2 v}{\omega_1}\right)^2 \tag{10}$$

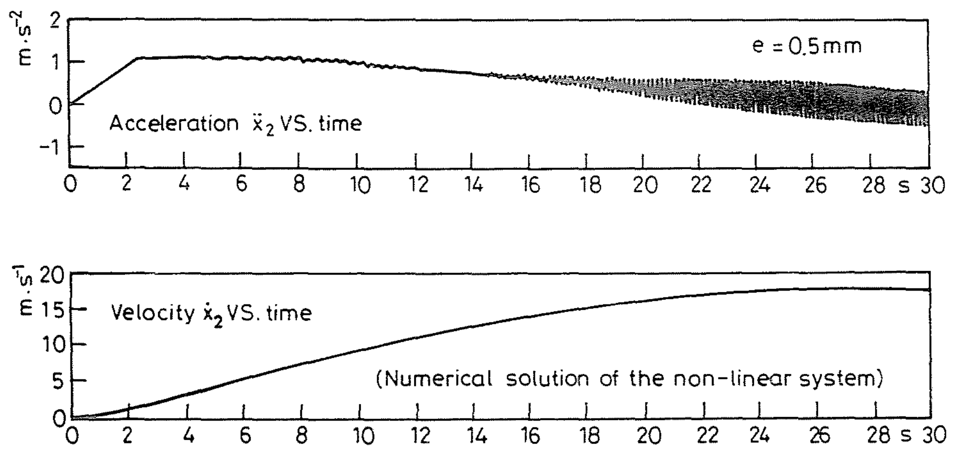

Fig. 2. Vibration form obtained by direct numerical solution

5. Exact Solution of the Perturbation Equations

If we subtract (5) from (6), (7) from (8), and (9) from (10), then we obtain system

$$\ddot{u}_0 + \kappa \dot{u}_0 + \omega^2 u_0 = \left(\frac{\omega}{\omega_1}\right)^2 a\,, \tag{11}$$

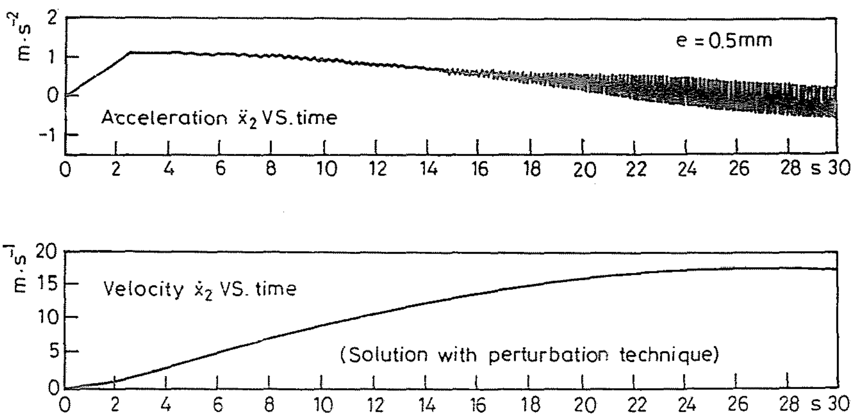

Fig. 3. Vibration form obtained by perturbational techniques

$$\ddot{u}_1 + \kappa \dot{u}_1 + \omega^2 u_1 = \dot{u}_{20}^2 \cos \frac{u_{20}}{R} , \tag{12}$$

$$\ddot{u}_2 + \kappa \dot{u}_2 + \omega^2 u_2 = \frac{1}{R} \left(\dot{u}_{10}^2 + \left(\frac{\omega_2 v}{\omega_1} \right)^2 \right) . \tag{13}$$

System (11–13) would be a system in three variables, if we could determine $\dot{u}_{10}$ and $\dot{u}_{20}$ with the help of functions u_j, $j = 0, 1, 2$. But $\dot{u}_{10}$ can be given by (5), and a rearrangement of (6) yields

$$\ddot{u}_{20} = \left(\frac{\omega}{\omega_1} \right)^2 a - \kappa_2 \dot{u}_0 - \omega_2^2 u_0 .$$

Hence u_{10} and u_{20} can be determined with the knowledge of u_0.

The solution of the perturbation equations can be given in the following way.

Step 1. Let us solve equation

$$\ddot{u}_0 + \kappa \dot{u}_0 + \omega^2 u_0 = \left(\frac{\omega}{\omega_1} \right)^2 a$$

by formula

$$u_0(t) = e^{-\kappa t/2} \left(C_1 \cos \frac{\lambda t}{2} + C_2 \sin \frac{\lambda t}{2} \right) +$$

$$+ \left(\frac{\omega}{\omega_1} \right)^2 \frac{2}{\lambda} \int_0^t a(\tau) e^{\kappa(t-\tau)/2} \sin \frac{\lambda}{2}(t - \tau) d\tau ,$$

where $\lambda^2 := 4\omega^2 - \kappa^2 > 0$ in practical cases, and constants C_1 and C_2 can be determined by the prescribed conditions as $C_1 = x(0)$ and $C_2 = \frac{1}{\lambda}(\kappa x(0) + 2\dot{x}(0))$.

Step 2. Let us evaluate

$$\dot{u}_{10}(t) = \kappa_1(u_0(t) - x(0)) + \omega_1^2 \int_0^t u_0(\tau)d\tau + \dot{x}_1(0)$$

by *Eq.* (5).

Step 3. Let us evaluate

$$\dot{u}_{20}(t) = \left(\frac{\omega}{\omega_1}\right)^2 (v(t) - v_0) - \int_0^t (\kappa_2 \dot{u}_0(\tau) + \omega_2^2 u_0(\tau)d\tau + \dot{x}_2(0)$$

and

$$u_{20}(t) = \left(\frac{\omega}{\omega_1}\right)^2 (s(t) - v_0 t) -$$

$$- \int_0^t \int_0^\theta (\kappa_2 \dot{u}_0(\tau) + \omega_2^2 u_0(\tau) + \omega_2^2 u_0(\tau))d\tau d\theta + \dot{x}_2(0)t + x_2(0)$$

by *Eq.* (6), where $s(t)$ is the longitudinal displacement of the fundamental motion.

Step 4. Let us solve equation

$$\ddot{x} + \kappa\dot{x} + \omega^2 x = \left(\frac{\omega}{\omega_1}\right)^2 a + \frac{\delta}{R}\left(\frac{\omega_2 v}{\omega_1}\right)^2 + \frac{\varepsilon}{R}\dot{u}_{20}^2 \cos\frac{u_{20}}{R} + \frac{\delta}{R}\dot{u}_{10}^2$$

for the relative displacement as

$$x(t) = \epsilon^{-\kappa t/2}\left(C_1 \cos\frac{\lambda t}{2} + C_2 \sin\frac{\lambda t}{2}\right) +$$

$$+\frac{2}{\lambda}\int_0^t \left(\left(\frac{\omega}{\omega_1}\right)^2 a(\tau) + \frac{\delta}{R}\left(\frac{\omega_2}{\omega_1}\right)^2 v(\tau)^2 + \right.$$

$$\left. + \frac{\varepsilon}{R}\dot{u}_{20}(\tau)^2 \cos\frac{u_{20}(\tau)}{R} + \frac{\delta}{R}\dot{u}_{10}(\tau)^2\right)\epsilon^{\kappa(\tau - t)/2}\sin\frac{\lambda}{2}(t - \tau)d\tau \; .$$

6. Steady-state Solutions

Suppose that the acceleration a of the fundamental motion is constant. If we are looking for a steady-state solution, then we can assume

$$u_0(t) = \frac{a}{\omega_1^2}, \quad \dot{u}_{10}(t) = v(t) \quad \text{and} \quad u_{20}(t) = s(t) \; .$$

This way we obtain equation

$$\ddot{x} + \kappa\dot{x} + \omega^2 x = \left(\frac{\omega}{\omega_1}\right)^2 a + \frac{\delta}{R}\left(\frac{\omega v}{\omega_1}\right)^2 + \frac{\varepsilon v^2}{R}\cos\frac{s}{R} \tag{14}$$

with solution

$$x(t) = \frac{a}{\omega_1^2} e^{-\kappa t/2}\left(\cos\frac{\lambda t}{2} + \frac{\kappa}{\lambda}\sin\frac{\lambda t}{2}\right) +$$
$$+\frac{2}{\lambda}\int_0^t \left(\left(\frac{\omega}{\omega_1}\right)^2 a + \frac{\delta}{R}\left(\frac{\omega}{\omega_1}\right)^2 v(\tau)^2 +\right.$$
$$\left. +\frac{\varepsilon}{R} v(\tau)^2 \cos\frac{s(\tau)}{R}\right) e^{\kappa(\tau - t)/2} \sin\frac{\lambda}{2}(t-\tau)d\tau ,$$

where $v(\tau) = v_0 + a\tau$ and $s(\tau) = v_0\tau + \frac{a}{2}\tau^2$.

Let us reparametrize *Eq.* (14) by introducing displacement s as a new variable:

$$v^2 x'' + (\kappa v + a)x' + \omega^2 x = \left(\frac{\omega}{\omega_1}\right)^2 a + \frac{\delta}{R}\left(\frac{\omega v}{\omega_1}\right)^2 + \frac{\varepsilon v^2}{R}\cos\frac{s}{R}, \tag{15}$$

where $'$ stands for derivation by s, and $v(s) = \sqrt{v_0^2 + 2as}$ holds.

Let us suppose also that condition $\omega_1 v_0 >> a$ is satisfied. Then we are able to introduce another 'small' dimensionless parameter $\alpha := \frac{a}{\omega_1 v_0}$. In this case, omitting the terms of degree greater than 1 in α, ε and δ, the first perturbational approximation provides differential equation

$$v_0 x'' + \kappa v_0 x' + \omega^2 x = \frac{\alpha\omega^2 v_0}{\omega_1} + \frac{\delta}{R}\left(\frac{\omega v_0}{\omega_1}\right)^2 + \frac{\varepsilon v_0^2}{R}\cos\frac{s}{R}. \tag{16}$$

The corresponding steady-state solution at any speed v has the form

$$x(s) = \frac{a}{\omega_1^2} + \frac{\delta v^2}{R\omega_1^2} + \frac{\varepsilon R v^2}{\sqrt{(R^2\omega^2 - v^2)^2 + (R\kappa v)^2}}\cos\left(\frac{s}{R} - \phi\right), \tag{17}$$

where $\tan\phi = \frac{R\kappa v}{R^2\omega^2 - v^2}$ is satisfied.

7. Conclusions

Our solution provides a very good approximation of the nonlinear resonance phenomena appearing due to the existence of a nonzero GPE value. Comparing *Figs. 2* and *3* one can see that the vibration forms obtained by our perturbational approach have the same characteristics as the solution

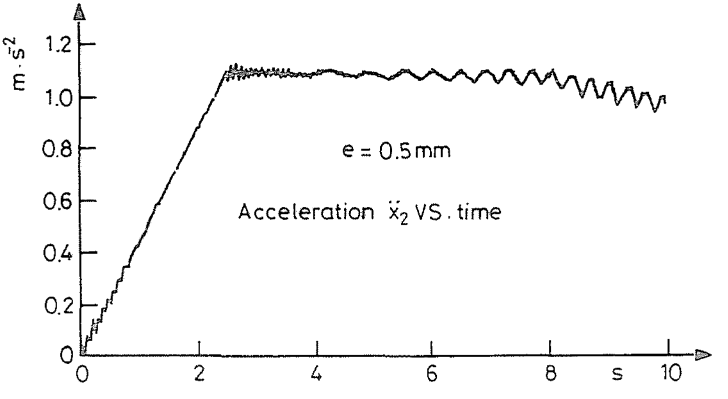

Fig. 4. Blown up section of the perturbational solution

achieved by the direct numerical solution to the equations of motion in case of a metro-vehicle example. Approximate formula (17) possesses almost all the important features of the vibration time functions shown in *Fig. 4*. In the course of further investigations the system model will be generalized to take into consideration the creep-dependence of the tangential force transmitted on the wheel in the wheel–rail rolling contact [4].

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