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A NEW APPROACH TO THE SIMULATION OF RANDOM ROADS

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Abstract

The stochastic simulation of random road surfaces as well as of parallel tracks is considered. Starting from the spectral density

$$S(\alpha) = \frac{\sigma^2}{\pi} \frac{\gamma}{\gamma^2 + \alpha^2}\,; \quad \gamma > 0\,,$$

a fast simulation method is derived and demonstrated for the surfaces as well as their derivatives. Thereby the theory of weakly correlated functions supplies the theoretical background.

Keywords: stochastic simulation, random road surfaces, weakly correlated functions.

1. Introduction

Considering the influence of roads on the behaviour of vehicles the mathematical modelling results in differential equation systems containing the road surfaces and their derivatives as random excitations. In this paper we are concerned with a new simulation procedure of road surfaces. In the literature several principles and methods can be found for simulation and application in vehicle dynamics (cf. for instance [7], [8] [9]). We are especially interested in a more general method supplying a fast (on-line-) simulation procedure as well as the basis for a theoretical stochastic analysis.

Using the concept of weakly correlated functions VOM SCHEIDT and WÖHRL derived some approximate models of random road profiles (see [10]–[14]). In section 2 the main results of these approximations are summarized and analyzed for our purposes. Thereby we will see that these models are also suitable to get two correlated parallel tracks.

Subsequently in section 3 our simulation procedure is derived on the basis of the simulation of sufficiently smooth weakly correlated processes. Finally in section 4 some numerical results are given to show the usefulness and efficiency of the simulated realizations. Applications of this new simulation to vehicle dynamics can be found in [3], [5] and [11]. A closed presentation of the simulation methods described in this paper, a comparison with theoretical results and also more concrete applications are included in [11].

2. Mathematical Models and Statistical Adaptation

Starting from the often used spectral density of road profiles f

$$S(\alpha) = \frac{\sigma^2}{\pi} \frac{\gamma}{\gamma^2 + \alpha^2}, \quad \gamma > 0, \tag{1}$$

with the corresponding correlation function

$$R(t) = \sigma^2 e^{-\gamma|t|} \tag{2}$$

an approximation of f is derived in WÖHRL [14] and VOM SCHEIDT; WÖHRL [12] in form of a linear functional

$$f(t,\omega) = \int_{-\infty}^{t} e^{-\gamma(t-s)} f_\varepsilon(s,\omega)ds, \tag{3}$$

where $f_\varepsilon(s,w)$ is a wide-sense stationary and weakly correlated process. Weakly correlated processes are random functions without 'distant effect' or functions of 'noise-natured character'. The exact definition and the resulting limit theorems or expansions of stochastic characteristics are given in [10]. Especially, their expectation function is zero. the correlation function of such processes has the form

$$\langle f_\varepsilon(s_1) f_\varepsilon(s_2)\rangle = \begin{cases} R_\varepsilon(s_1,s_2) & \text{for } |s_1 - s_2| \le \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

and the so-called intensity is in case of wide sense stationary processes defined by

$$a = \lim_{\varepsilon \downarrow 0} \frac{1}{\varepsilon} \int_{-\varepsilon}^{\varepsilon} R_\varepsilon(z)dz. \tag{4}$$

Then the following limit theorem

$$\lim_{\varepsilon \downarrow 0} \frac{1}{\varepsilon} \langle f(t_1) f(t_2)\rangle = \frac{a}{2\gamma} e^{-\gamma|t_2-t_1|}$$

results in the approximation for small values of the correlation length $\varepsilon > 0$

$$\langle f(t_1) f(t_2)\rangle \approx \frac{a\varepsilon}{2\gamma} e^{-\gamma|t_2-t_1|}, \tag{5}$$

which corresponds to the desired correlation function (2).

Whereas the original correlation function (2) is not differentiable, the approximation (3) is twice differentiable if f_ε is continuously differentiable for $\varepsilon > 0$. It follows

$$
\begin{aligned}
\dot{f}(t,\omega) &= f_\varepsilon(t,\omega) - \gamma \int_{-\infty}^{t} e^{-\gamma(t-s)} f_\varepsilon(s,\omega)ds \, , \\
\ddot{f}(t,\omega) &= \dot{f}_\varepsilon(t,\omega) - \gamma f_\varepsilon(t,\omega) + \gamma^2 \int_{-\infty}^{t} e^{-\gamma(t-s)} f_\varepsilon(s,\omega)ds \, .
\end{aligned}
\tag{6}
$$

Because of the appearance of these explicit derivatives $\dot{f}_\varepsilon$ and $\ddot{f}_\varepsilon$ VOM SCHEIDT [10] introduced a smoothing function (polynomial) $Q_0(t,\delta)$ depending on the parameter $\delta > 0$ and having the properties $Q_0(0,\delta) = Q_0'(0,\delta) = 0$. Putting

$$
f(t,\omega) = \int_{-\infty}^{t} Q(t-s,\delta) f_\varepsilon(s,\omega)ds \, ,
$$

where $Q(t-s,\delta) = Q_0(t-s,\delta)e^{-\gamma(t-s)}$, the approximation corresponding to the correlation function (3)

$$
\langle f(t_1)f(t_2)\rangle \approx \frac{a\varepsilon}{2\gamma} e^{-\gamma|t_2-t_1|}
$$

is true if $\delta \downarrow 0$. Here we have only linear functionals of f_ε

$$
f^{(k)}(t,\omega) = \int_{-\infty}^{t} Q^{(k)}(t-s) f_\varepsilon(s,\omega)ds \, , \quad k = 0,1,2
$$

as derivatives. This second model is especially advantageous for the theoretical stochastic analysis of random vibration systems (cf. for instance [10], [11] and [13]). In our former papers (cf. [1], [2]) we also used this model. But, it needs some special efforts with respect to the numerical calculations because of the structure of the smoothing function Q_0. Therefore, we use now the first model (3) to derive a quicker simulation procedure. To this end we need in (6) also simulations of differentiable weakly correlated processes. In section 3 the resulting procedure is given.

After modelling the random road profile as linear functional (3) and subsequently its derivatives by (6) the next problem is to specify the model

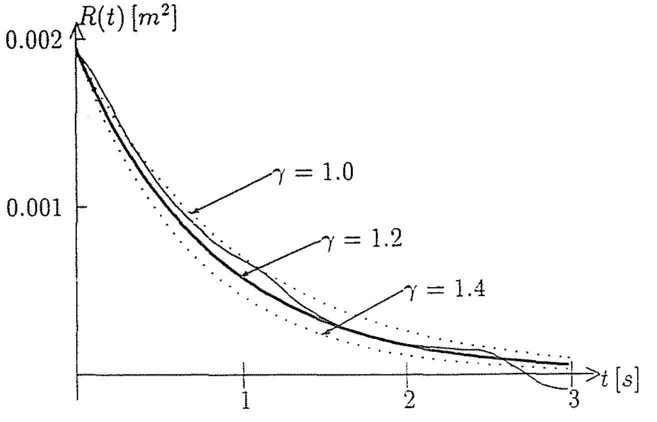

Fig. 1. Adaptation of the scale parameter

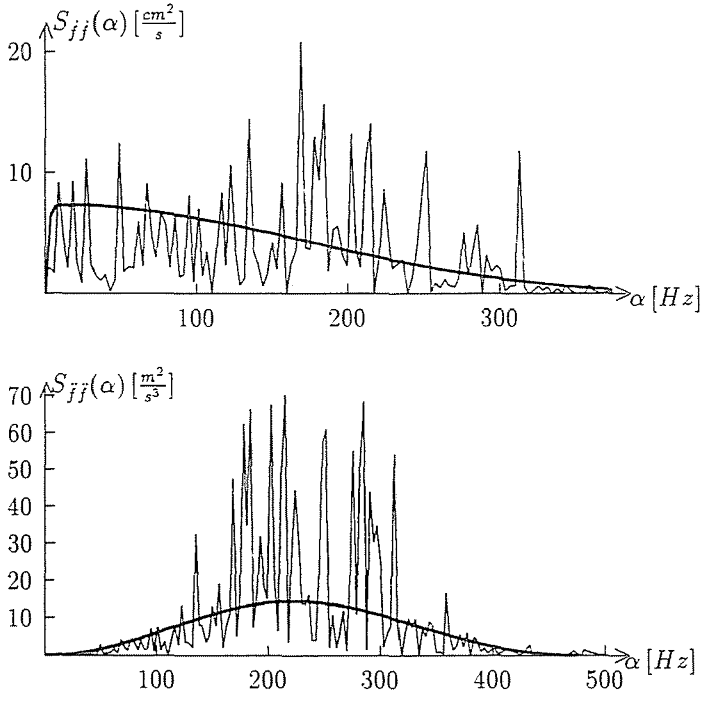

Fig. 2. Estimated and adapted spectral densities

parameters γ, ε and a by comparisons with statistical estimates from measurements of real roads. This can be carried out by means of an interactive procedure. Firstly, the scale parameter γ is adapted by consideration of the correlation function $R(t)$ (see *Fig. 1*).

Secondly, the correlation length ε is determined by calculating the theoretical spectral densities of the 1^{st} and 2^{nd} derivative

$$\begin{aligned} S_{\dot{f}\dot{f}}(\alpha) &= S_{f_\varepsilon f_\varepsilon}(\alpha) - \frac{\varepsilon a}{2\pi}\frac{\gamma^2}{\gamma^2+\alpha^2} , \\ S_{\ddot{f}\ddot{f}}(\alpha) &= (\alpha^2-\gamma^2)S_{f_\varepsilon f_\varepsilon}(\alpha) + \frac{\varepsilon a}{2\pi}\frac{\gamma^4}{\gamma^2+\alpha^2} \end{aligned}$$

and comparing them with the estimates of the measured road. The final result is plotted in *Fig. 2*.

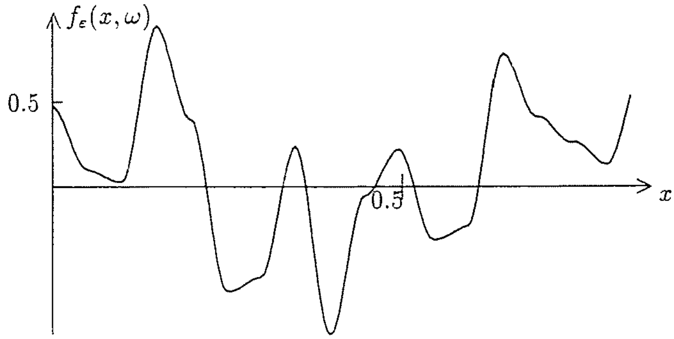

Fig. 3. Realization of a differentiable weakly correlated process

After all, the intensity is calculated from relations (2) and (5) by $a = 2\hat{\sigma}^2\gamma/\varepsilon$, where $\hat{\sigma}^2$ denotes the estimated dispersion of the measured road.

The mathematical models can be extended to considerations of two parallel tracks using methods described e.g. in PARKHILOVSKII [6] and SCHIEHLEN [8]. Taking into account the concepts mentioned above two correlated excitations (tracks) $f_L(t,\omega)$ and $f_R(t,\omega)$ with distance b and an orthotrop behaviour, i.e.

$$R_{f_L f_R}(t_1,t_2) = \langle f_L(t_1) f_R(t_2)\rangle = \sigma^2 e^{-\gamma(|b|+|t_2-t_1|)} ,$$

can be derived. They have again the form of linear functionals

$$f_L(t,\omega) = \int_{-\infty}^{t} e^{-\gamma(t-s)}[f_{1\varepsilon}(s,\omega) + f_{2\varepsilon}(s,\omega)]ds ,$$

$$f_R(t,\omega) = \int_{-\infty}^{t} e^{-\gamma(t-s)}[f_{1\varepsilon}(s,\omega) - f_{2\varepsilon}(s,\omega)]ds$$

with independent weakly correlated processes $f_{1\varepsilon}(s,\omega)$ and $f_{2\varepsilon}(s,\omega)$. The derivation and some further considerations with respect to the coherence function can be found in [2] and [11].

3. Simulation Procedure

Now we turn to the simulation of the mathematical models (3) and (6). First of all, the simulation of a differentiable weakly correlated process $f_\varepsilon(s,\omega)$ is given. Thereby, a bounded domain $s \in [\alpha,\beta]$ of interest is decomposed into n intervals $[a_i, a_{i+1}]$ with length $h = (\beta - \alpha)/n$ and $a_i = \alpha + ih$, $i = 0,1,\ldots,n$. Further $\{\xi_i(w)\}_i$ and $\{\overline{\xi}_i(w)\}_i$, $i = 0,1,\ldots,n$, denote two sets of independent, identically distributed random variables with $< \xi_i >= < \overline{\xi}_i >= 0$ and $< \xi_i^2 >= \sigma_\xi^2$ for all i.

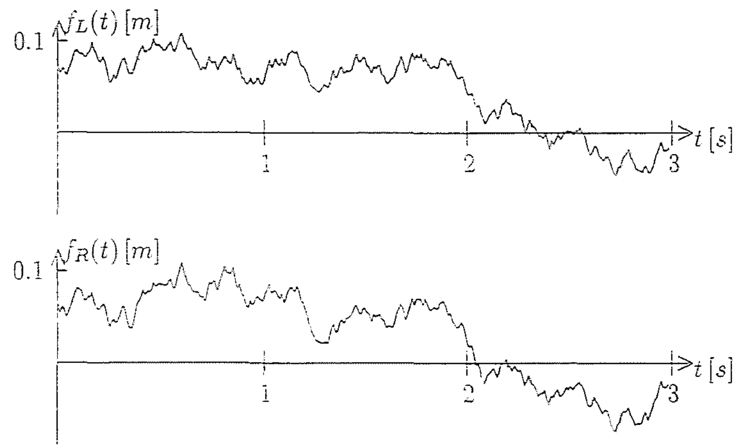

Fig. 4. Simulation of two tracks

Setting

$$\begin{aligned} f_\varepsilon(s,\omega) &= g_i(s,\omega) && (7)\\ &= p_i(s-a_i)^3 + q_i(s-a_i)^2 + u_i(s-a_i) + v_i \end{aligned}$$

for $s \in [a_i, a_{i+1}]$ and demanding

$$\begin{aligned} g_i(a_i) &= \xi_i\,, & g_i(a_{i+1}) &= \xi_{i+1}\,,\\ \dot{g}_i(a_i) &= \overline{\xi}_i\,, & \dot{g}_i(a_{i+1}) &= \overline{\xi}_{i+1} \end{aligned}$$

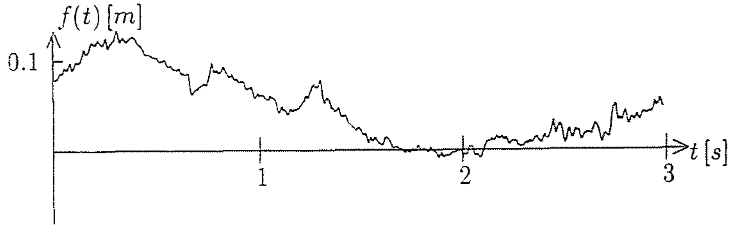

Fig. 5. Measured profile

the coefficients in (7) are determined by

$$
\begin{aligned}
p_i &= \frac{2(\xi_i - \xi_{i+1}) + (\overline{\xi}_i + \overline{\xi}_{i+1})h}{h^3}, \\
q_i &= \frac{3(\xi_{i+1} - \xi_i) + (2\overline{\xi}_i + \overline{\xi}_{i+1})h}{h^2}, \\
u_i &= \overline{\xi}_i \quad \text{and} \quad v_i = \xi_i .
\end{aligned}
$$

In consequence of the independence of the random variables ξ_i and $\overline{\xi}_i$ the so defined process f_ε is weakly correlated with correlation length $\varepsilon = 2h$. Its intensity can be determined by $a = \sigma_\xi^2/2$, i.e. the intensity depends only on the stochastic behaviour of ξ. Hence, a simulation of a differentiable weakly correlated process can be obtained by simulation of the random variables ξ_i and $\overline{\xi}_i$. In *Fig. 3* a realization of such a simulation is drawn with $\varepsilon = 0.1$.

Now we consider the simulation of the approximation of f, $\dot{f}$ and $\ddot{f}$ according to (3) and (6). To this end, we need the determination of the integral or linear functional, respectively. Firstly, we separate this integral

$$
f(t,\omega) = \int\limits_{-\infty}^{\alpha} e^{-\gamma(t-s)} f_\varepsilon(s,\omega)ds + \int\limits_{\alpha}^{t} e^{-\gamma(t-s)} f_\varepsilon(s,\omega)ds ,
$$

where α has to be chosen such that the first integral is neglectably small. There are possibilities to support this choice by some mathematical estimates. Secondly, the integral over $[\alpha, t]$ is determined by

$$
\int\limits_{\alpha}^{t} e^{-\gamma(t-s)} f_\varepsilon(s,\omega)ds = \sum_{i=0}^{n_t-1} \int\limits_{a_i}^{a_{i+1}} e^{-\gamma(t-s)} g_i(s)ds + \int\limits_{a_{n_t}}^{t} e^{-\gamma(t-s)} g_{n_t}(s)ds
$$

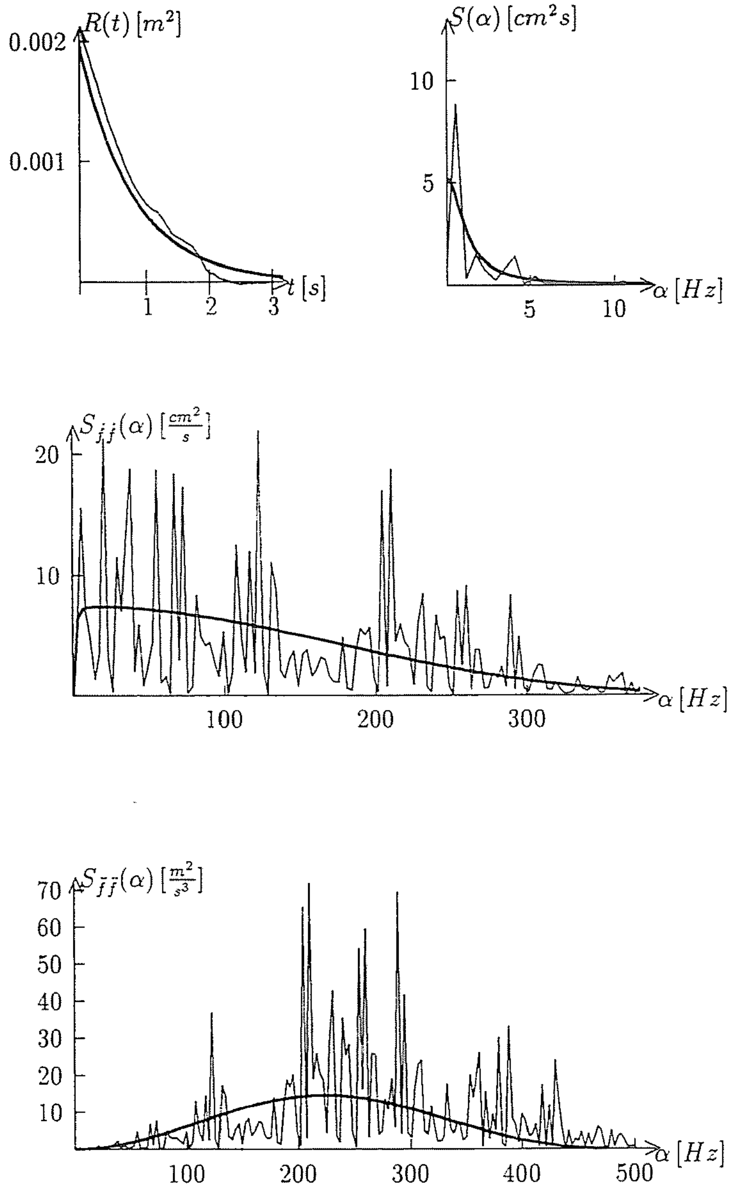

Fig. 6. Estimated and adapted correlation function and spectral densities

with $n_t = \text{entier}\,[(t-\alpha)/h]$. Some straightforward calculations lead to

$$f(t,\omega) \approx \int_{\alpha}^{t} e^{-\gamma(t-s)} f_{\varepsilon}(s,\omega)\,ds = \sum_{i=0}^{n_t-1} c_i e^{-\gamma(t-a_i)} + c_{n_t}(t) e^{-\gamma(t-a_{n_t})}\,, \qquad (8)$$

where

$$c_i = e^{\gamma h}\left[p_i\left(\frac{h^3}{\gamma}-\frac{3h^2}{\gamma^2}+\frac{6h}{\gamma^3}-\frac{6}{\gamma^4}\right)+q_i\left(\frac{h^2}{\gamma}-\frac{2h}{\gamma^2}+\frac{2}{\gamma^3}\right)\right.$$
$$\left.+u_i\left(\frac{h}{\gamma}-\frac{1}{\gamma^2}\right)+v_i\frac{1}{\gamma}\right]+\left[\frac{6p_i}{\gamma^4}-\frac{2q_i}{\gamma^3}+\frac{u_i}{\gamma^2}-\frac{v_i}{\gamma}\right]$$

and $c_{n_t}(t)$ corresponds to c_i if h is substituted by $t - a_{n_t}$.

4. Numerical Simulation

Finally we present some concrete simulation results. According to a realized adaptation the values $\gamma = 1.2$, $\varepsilon = 0.021$ and $a = 0.222$ are chosen. To demonstrate two parallel tracks the distance b is put $b = 0.0675$ s which corresponds to 1.5 m ($v = 80$ km/h). In the *Fig. 4* the simulated profiles $f_L(t)$ and $f_R(t)$ are drawn and in *Fig. 5* the measured profile $f(t)$ is drawn for a visual comparison.

A good coincidence can also be stated investigating the characteristics of the simulated profile. In *Fig 6* the estimated correlation function and spectral densities of the simulated profiles are drawn in comparison with the adapted characteristics.

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