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FLEXIBLE BUS STRUCTURES OBTAINED BY THE METHOD OF STATIC CONDENSATION

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Abstract

The determination of dynamic responses (accelerations, stresses) of linear systems with large number of degrees of freedom costs much work and time. Practically the same results can be obtained by using an appropriate method by which the given dynamic system can be reduced achieving less cost and time required for computation.

Retaining the structure of physical model the static reduction is the most frequently applied process. Elaboration of lumped mass matrix of bus and commercial vehicle models is heuristic, therefore only the stiffness matrix of the given system is problematic.

Considering the computational possibilities there are more ways to determine the stiffness matrix of a simplified model. A reduced stiffness matrix, elaborated from the results of dynamic analysis of finite element models, is competitive from the point of view of accuracy and computational costs.

Keywords: FEM analysis, structural dynamics, static condensation, vehicle dynamics.

1. Introduction

Concerning the calculated outputs of dynamic systems the increase of degrees of freedom (DOF) causes several problems, such as: the computational time increases exponentially, where the power term is about 3 and 4, and on the other hand the punctuality of computation is reduced. The reason of these failures is the more segmented, detailed model. Therefore the time or frequency domain functions can be calculated with smaller time or frequency sampling intervals and required time to calculate the independent variable, as the answer function is proportional with the 2nd-3rd power of the unknown functional.

By increasing the DOF of model, the information obtained by the computation also increases. Although the information must be considered very carefully, hence the possibility to measure data on a real system is very much limited in comparison with the calculated points of available model. The results obtained from computation are adjusted according to real measurements.

From the afore-mentioned facts it is clear that in the selection of the number of unknown parameters the designers should be careful, hence the computational facilities allow us much freedom. Therefore the models designed for static modelling must be simplified for dynamic analysis. From the reduction technics the method of static condensation is the most known. A practical implementation is shown in this paper, used to model vehicle undercarriage systems.

2. Dynamic Structure of Vehicle Undercarriage Systems

The description of the deformation of vehicles, vehicle undercarriages is based very often on the concept of discontinuity modelling. The required parameters, such as the mass, stiffness and damping parameters are sought from finite element (FEM) modelling. For example the mechanical model of a bus structure can be represented by 300–500 nodes, where each node has six DOF.

In the calculation of dynamic signals the number of DOF can be reduced significantly, on the one hand to half, if the nodes only modell mass points, and on the other hand to a further one third, if only the vertical dynamics of vehicle are considered. Here we suppose that from the vertical excitation no lateral force exaggeration exists, hence no lateral vibration occurs. (A small scale lateral displacement can only occur due to three-dimensional geometrical and stiffness constraints of the model). The DOF of dynamic model therefore is equal with the number of nodes.

The knowledge of mass distribution of system may lead to further simplifications. The size of useful load can be compared with the own mass of vehicle, where both loads are acting on the main frame of the vehicle. During the modelling phase the designers ought to concentrate on the exact description of the before mentioned phenomena concerning the dynamic impacts. Then the dynamic models of buses could be described by 140–200 DOF.

Although, we can state without going into details that the global equations of motion of a bus can be described with appropriate 40–60 DOF, moreover if we only consider the bending modes then this number can be reduced to further 8–12, which results in two times smaller DOF.

3. Derivation of Stiffness Matrix of the Simplified Dynamic Model

After the selection of the unknown parameters of dynamic model the system matrices of the static finite element model have to be transformed into the dynamic freedoms. In the following, only the reduced stiffness matrix is considered. In theory the solution is simple: after the participation of stiffness matrix the unimportant unknowns of the dynamic calculation are eliminated (static condensation, Guyan algorithm):

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix} , \quad (1)$$

$$x_2 = -S_{22}^{-1} S_{21} x_1 , \quad (2)$$

$$S_{red} x_1 = F , \quad (3)$$

$$S_{red} = S_{11} - S_{12} S_{22}^{-1} S_{21} . \quad (4)$$

The implementation of reduced stiffness matrix is not problemless, hence the FEM programs existing in the market have no such option, moreover the stiffness matrix of the system has no access. The stiffness matrix of dynamic systems thus can only be sought from the optional facilities of FEM programs. It means that the elements of the condensed matrix are built up from 5 digit, rounded results of the internal calculations. Although this data file consists the errors of numerical calculation, too.

The stiffness matrix of the dynamic model can be determined based on the concepts of

a, kinematic load,
b, flexibility matrix and inverse matrix theorems or instead of the concept of static load the matrix can be obtained as a result of dynamic analysis, as
c, the combination of eigenvectors and eigenvalues.

Concerning the above mentioned methods the *c* variant has given the best results obtained from the tests made on different underframe structures. This method needs less work and computational time and the obtained results are satisfactory concerning numerical punctuality, too (the comparison of different methods is summarized and presented at the GAMM'94 Conference in Braunschweig).

The meaning of stiffness matrix reduction based on the knowledge of modal parameters (eigenvalue and eigenvector) is as follows:

- the eigenvalue analysis of a mechanical system can only be done, if the mass matrix is non-singular.

If this condition is satisfied, then the system has to be transformed into the place of degrees of freedoms of the non-zero elements of the mass matrix, which means static condensation.

$$\begin{bmatrix} M_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \quad (5)$$

$$x_2 = -S_{22}^{-1} S_{21} x_1 , \quad (6)$$

$$\mathbf{M}_1\ddot{\mathbf{x}}_1 + \mathbf{S}_{red}\mathbf{x}_1 = \mathbf{0} , \tag{7}$$

$$\mathbf{S}_{red} = \mathbf{S}_{11} - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\mathbf{S}_{21} . \tag{8}$$

The solution of eigenvalue problem simply means the determination of the modal parameters of the simplified model. From the results, the missing elements of eigenvalues and that of the reduced mass matrix can be calculated by the inverse transformation method. If the mass matrix of the simplified model is identic, then the equation of motion described in the subcoordinate system is as follows:

$$\mathbf{T}^T\mathbf{E}\mathbf{T}\ddot{\mathbf{q}} + \mathbf{T}^T\mathbf{S}_{red}\mathbf{T}\mathbf{q} = \mathbf{0} , \tag{9}$$

$$\mathbf{E}\ddot{\mathbf{q}} + \Lambda\mathbf{q} = \mathbf{0} , \tag{10}$$

from which the searched condensed stiffness matrix:

$$\mathbf{S}_{red} = \mathbf{T}\Lambda\mathbf{T}^T . \tag{11}$$

where

- Λ – diagonal matrix, consisting the eigenvalues (square values of own frequencies),
- $\mathbf{T}$ – normated eigenvector matrix sought from the non zero elements of mass matrix.

The way to determine the reduced stiffness matrix is as follows: in the places of degrees of freedoms to be reduced, identity mass distribution is considered and then the eigenvalue problem is solved as an option of FEM analysis. At the end, when the modal parameters are selected, the matrix is built up and the required operation is carried out.

4. Numerical Investigation and its Results

In the afore-mentioned paragraph the obtained condensed matrix can have different errors, such as the errors of the numerical procedure, which can be introduced through the process of condensation of mass matrix, the iteration procedure of eigenvalue analysis and from the truncated and rounded presentation of modal parameters (eigenvalue, eigenvector) in the data sheet of FEM analysis. While the error mentioned at last can be approximated, the errors of numerical calculation are unknown (besides the eigenvalues).

In the evaluation procedure of reduced mass matrix punctuality, the static equilibrium equations can be considered as the basis. If the flexible constraints of the system are removed then a free system is obtained, whose stiffness matrix only has internal contact forces. The force system of the kinematical loads existing in the rows of the stiffness matrix must fulfill the conditions of weight point and moment. This means that the sum of the elements in the rows of the matrix and the moments calculated to the weight points must be zero. Although the satisfaction of the equilibrium condition is only a necessary condition. Further information can be obtained to the reliability of the condensed stiffness matrix, if the modal parameters are calculated to the free systems, too. Without going into details it is evident that the eigenvectors used as kinematical loads must fulfill the equilibrium conditions. The shape of eigenvectors, the number of nodes, i.e. lines, and its situation given for a construction (for example bus system), gives additional information for the experts in order to check the computed results. Hence, in this way the contents of errors can be detected.

The numerical analysis is carried out for a grid system of a bus and in the other hand four FEM bus models were under investigation by the application of SUPERSAP software package. The reduction has been made in the vertical plane of the underframe of the vehicles, according to the imagined vertical deformations of beam elements.

This flexible beam represents the average reduced stiffness of logitudinal underframe structures, incorporating the effect of lateral beams, too. Then the global bending stiffness parameters are given.

The eigenvalue problem is only solved to the free system consisting a grid structure. The applied FEM program is only capable to handle the modal parameters of constrained system, therefore the model is fixed with small value of stiffness.

The goal of investigation is to prove the applicability of the mentioned theorem. The most important information of the given figures in the appendix can be summarized as:

- the DOF of the system and its reduced counterpart,
- the size of the elements of reduced matrix,
- the geometrical location of weight point and
- the sum of errors in one row of the matrix, which is defined as the sum of elements and their moments.

5. Conclusions

1. The reduced stiffness matrix practically satisfies the equilibrium conditions. The values of the sums derived in every row have not reached the size of rounded errors.
   For example concerning the grid model of bus system, the maximal rounded error 0.5, due to the 5 digit displaying (while in case of 5th order approximation, the roundoff error is 5), while the error consequence concerning the moment is not else than the maximal distance multiplied by the value of error $0.5^{*}480 = 290$.
2. Concerning the last model the values of errors are greater and the sum of moments shows a large scale deviation in a range, which can

be located geometrically. The probability of this phenomenon can be found in the fault of FEM program made for statistical analysis. This comes from the investigation so that the errors deduced for the moment are very close to the already determined values by the application of kinematical load (a, variant). Although by this the usefulness of the concept, i.e. that the matrix is derived by the modal approach can be counteracted with the results obtained by the static condensation method, moreover it can be deduced that the relative error is within the range of round off errors.

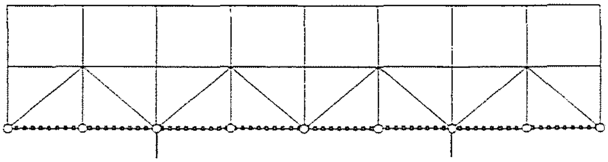
3. The above statements are concerned with the simplification of an order of two models, therefore one can conlude that such a large size of reduction can be used in numerical way.

References

[1] SZŐKE, D.: Zusammensetzung der reduzierten Steifigkeitsmatrix, *GAMM Tagung* 1994, Braunschweig.

6. Appendix A

Model of the side wall of the bus

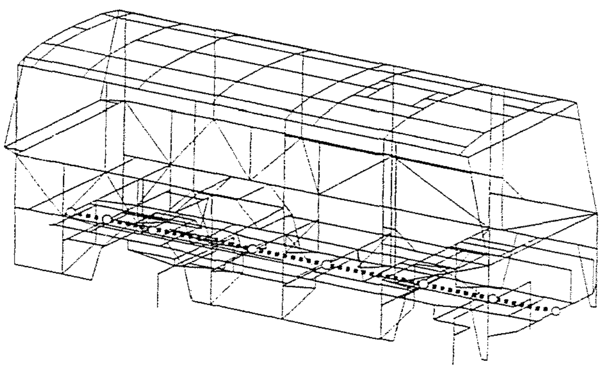


static model: 171 DOF
dynamic model: 9 DOF

S_{max}= 5.5180E4 S_{min}= 121.6 [·10 N/cm] x_{max}= 480 [cm]
max. round off error: Δ F= 0.5 Δ M= 240

DOF	1	2	3	4	5	6	7	8	9
Δ F	0.37	-0.77	0.96	-0.55	0.23	-0.55	0.96	-0.77	0.37
Δ M	41	-135	102	-28	0.0	28	-102	135	-41

FE model of the Midi-bus

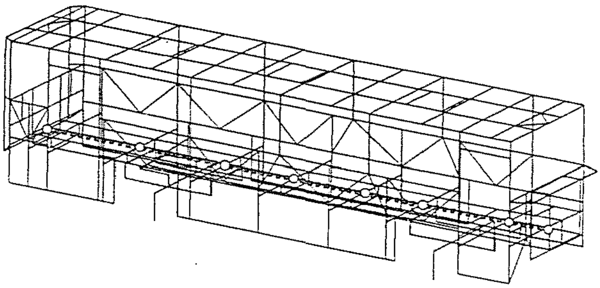


static model: 1969 DOF
dynamic model: 8 DOF

S_{max}= 6.4682E4 S_{min}= 1.0784 [·10 N/cm] x_{max}=330 [cm]
max. round off error: Δ F= 0.5 Δ M= 165

DOF	1	2	3	4	5	6	7	8
Δ F	-0.05	0.03	-0.02	0.36	-0.56	0.36	-0.02	-0.08
Δ M	-4	16	-95	140	-85	81	-67	13

FE model of the citybus (vers. A.)

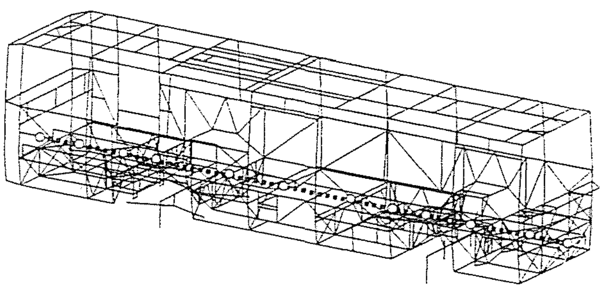


static model: 1737 DOF
dynamic model: 8 DOF

S_{max}= 6.9082E4 S_{min}= 9.7424 [·10 N/cm] x_{max}= 527 [cm]
max. round off error: Δ F= 0.5 Δ M= 263

DOF	1	2	3	4	5	6	7	8
Δ F	0.06	-0.11	0.19	0.36	-1.08	0.98	-0.28	0.03
Δ M	-29	82	-225	296	-277	191	-102	43

FE model of the citybus (vers. B.)



static model: 2668 DOF
dynamic model: 12 DOF

S_{max}= 1.7078E5 S_{min}= 5.3274 [·10 N/cm] x_{max}= 517 [cm]
max. round off error: Δ F= 5 Δ M= 2.58E3

DOF	1	2	3	4	5	6	7	8	9	10	11	12	
Δ F	0.09	-0.08	0.64	-0.19	-0.76	0.74	0.42	-1.33	3.31	-2.21	-0.74	0.84	
Δ M	-0.58	-1.51	2.91	-0.94	0.14	-0.18	0.14	-0.24	0.50	-0.24	0.07	-0.07	·E4
Δ M_A	-0.57	-1.53	2.91	-0.92	0.12	-0.16	0.13	-0.22	0.39	-0.15	0.08	-0.08	·E4