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# HYSTERESIS OF NON-LINEAR VEHICLE VIBRATING SYSTEMS INDUCED BY DAMPING

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## Abstract

Dampers are usually built into the vehicles not in vertical position. This method of incorporating 'under angle' is primarily verified by the demand of increasing the transversal stability of shafts. Shall $\alpha_0$ be the angle of damper to the vertical.

Because of the use of $\alpha_0 = 8° - 40°$ angle in the suspension system, a damping force characteristic different from the actual member characteristic prevails. In the literature dealing with vehicle swinging, the different angles of the damper to the vertical are not considered in the recalculation of the damping force characteristic.

In the investigation to be introduced, the vertical damping force is, as per the above facts, a function of two variables depending on the relative displacement and the relative velocity. In this case even the damper of linear characteristic looses its linearity because of its hysteresis (see *Fig. 1*).

Taking into consideration real vehicle parameters, the suspension systems of rigid axle and those of swinging lever are investigated separately. The excitations are of two types, too, namely purely sinusoidal and real stochastic road profiles.

Taking into consideration the various speeds of vehicles and the various road conditions, the effect of hysteresis must not be neglected first of all at rigid axles in case of $\alpha_0 > 30°$ at inferior road conditions and at higher speed. keywords non-linear vehicle vibration, damping.

## 1. Introduction

The demand for more and more precise description of processes in the dynamic systems requires setting up of more precise mathematic models by the researchers. There are two possibilities to improve the mathematic models using them either together or separately:

1. Increasing the degree of freedom of the system.
2. Efforts to giving better than ever description of the facts when studying the subphenomena.

(Sometimes, even the two methods cannot be easily separated as the example of changing from the discrete models for the continuum models shows.)

Our study belongs to the circle of the 2. type investigations. When examining the swinging of the vehicle, the power transmitted by the dampers

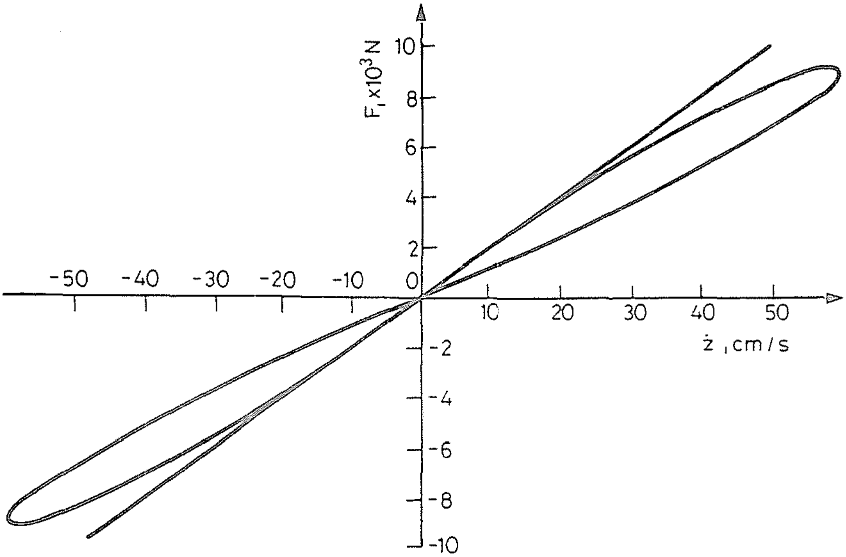


*Fig. 1.* Linear damping characteristic, swinging lever suspension $r = 20$ cm, $\alpha_0 = 30°$, amplitude of the periodic excitation = 10 cm, its angular frequency $\omega = 6$ rad/s.

is considered by the researchers as relative speed dependent. (Here the self-adjusting dampers sensing the load and having automatically varying characteristic are not considered.)

In the reality, the dampers built in aslant are continuously changing their spatial position during the vehicle swinging. As a consequence, in case of equal relative velocity — but in spatial positions of different $\alpha$ angle to the vertical — different damping forces arise during the vertical swinging.

Thus, the damping forces of aslant built in dampers are dependent on the relative displacements, too. In the literature dealing with vehicle swinging the researchers fail to take into consideration that the vertical damping force is a function of two variables dependent on the relative velocity and the relative displacement, or, with other words, a surface.

This phenomenon is now investigated in cases of suspension systems with rigid axle and with transverse link.

An interesting similarity can be proved between the leaf spring member and the aslant built in damper member. As it is well known, during the operation of the leaf spring beyond the spring force a Coulomb damping force also arises. The aslant built in damper, however, generates an additional spring force. The common thing in the two phenomenon is, that the arising force is a function of two variables (surface) in both cases. In the literature regarding swinging, in case of the leaf spring for the sake of simple handling the surface points are projected on the relative velocity — force plane, since here basically a spring is investigated. Thus, let us follow this principle in the case of aslant built in dampers, too. In this case the surface points are suitable to be projected on the relative velocity — force plane since here basically the damping force is investigated.

In both cases the common thing is, that the projection on plane resulted in the phenomenon of hysteresis.

## 2. Actual Characteristics of Aslant Built in Damper Regarding Vertical Swinging

During our investigations suspensions of rigid axle (*Fig. 2*) and of transverse link (*Fig. 4*) were dealt with.

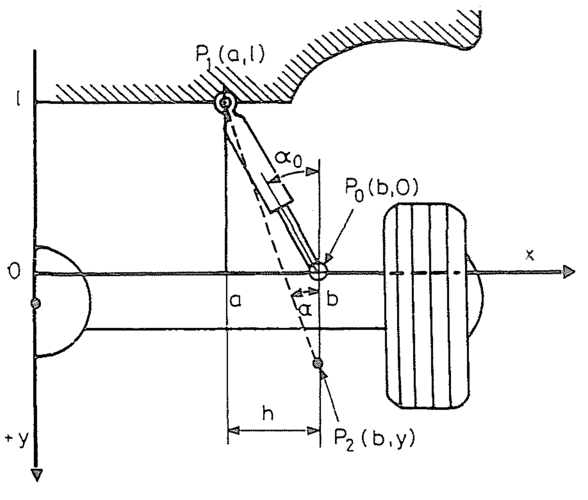


*Fig. 2.*

In this section the function of two variables of the damping force depending on the relative displacement between the chassis and the axle ($y$) and the relative velocity ($\dot{y}$) is derived.

### *2.1. The Case of Rigid Axle*

Assumptions: (1.) The damper is moving only in the plane of the drawing.
(2.) During the movement the little shunt of $P_1$ and $P_2$ in $x$ direction is neglected, thus, '$a$' and '$b$' coordinates are fixed constants.

Let us derive the following designations:

$$h = b - a$$

$$\overline{P_1 P_2} = z.$$

Since $y$ changes with time: $y = y(t)$, thus also $z$ — is a function of time: $z = z(t)$.

On the basis of the $\mathrm{tg}\alpha_0 = h/l$ relationship between the $\alpha_0$, $l$ and $h$ parameters any two parameters can be used during the derivation. Further on we use the '$l$' and '$h$' parameters.
Then:

$$\overline{P_1 P_2} = z = \sqrt{h^2 + (y - l)^2}. \tag{1}$$

Since $z$ depends on time, the following equation can be set up:

$$z(t) = \sqrt{h^2 + [y(t) - l]^2}. \tag{2}$$

The derivative by $t$ of distance $z(t)$ gives the value of relative velocity between points $P_1$ and $P_2$ at moment $t$:

$$\frac{dz(t)}{dt} = \dot{z}(t) = \frac{d\sqrt{h^2 + [y(t) - l]^2}}{dy} \frac{d(y)}{dt}. \tag{3}$$

$$\dot{z}(t) = \frac{y(t) - l}{\sqrt{h^2 + [y(t) - l]^2}} \, \dot{y}(t) \ . \tag{4}$$

The damping member characteristic shown by *Fig. 3* generates force $F$ by the effect of relative velocity $\dot{z}$, and this vector has an angle of $\alpha$ to the vertical.

$$F_Y = F \cos\alpha; \qquad (F_X = F \sin\alpha); \qquad \cos\alpha = \frac{y - l}{\sqrt{h^2 + (y - l)^2}}$$

Thus, the $F_Y$ damping force working in vertical direction, in case of displacement $y(t)$ depends not only on the relative vertical velocity $\dot{y}(t)$ but on the relative displacement $y(t)$, too:

$$F_Y\Big(y(t), \dot{y}(t)\Big) = \frac{y(t) - l}{\sqrt{h^2 + [y(t) - l]^2}} \, F\left(\frac{y(t) - l}{\sqrt{h^2 + [y(t) - l]^2}} \, \dot{y}(t)\right) \ . \tag{5}$$

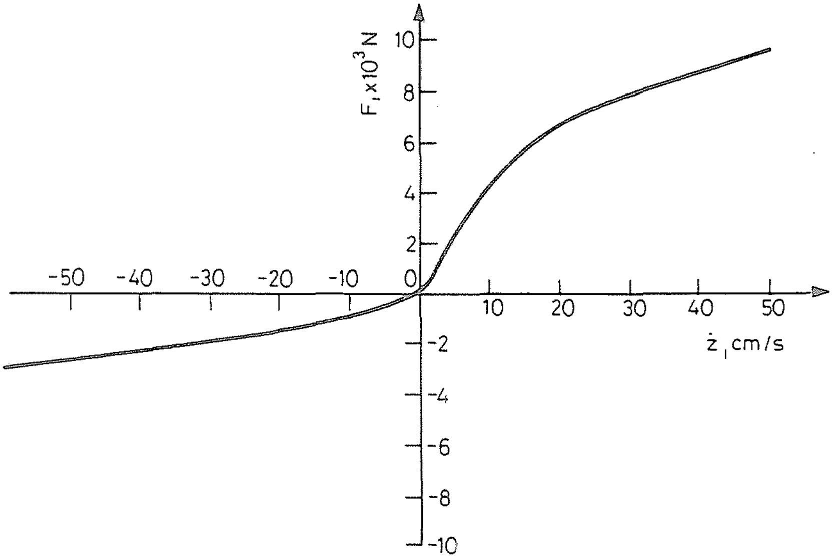


*Fig. 3.*

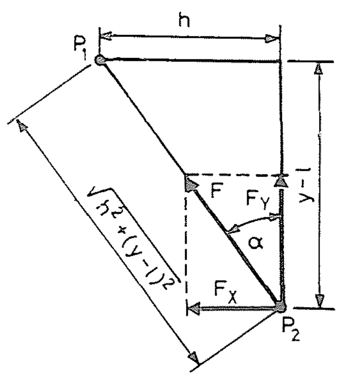


*Fig. 4.*

## *2.2. The Case of Suspension with Swinging Lever*

Based on the equation of the circle: $x_2^2 + y^2 = r^2 \Rightarrow x_2 = \sqrt{r^2 - y^2}$.

Assumptions: (1.) The damper moves in the plane of the drawing only.

(2.) The shunts of $P_1$ in $x$ — direction are neglected, only the shunts of $P_2$ in $x$ — direction are considered.

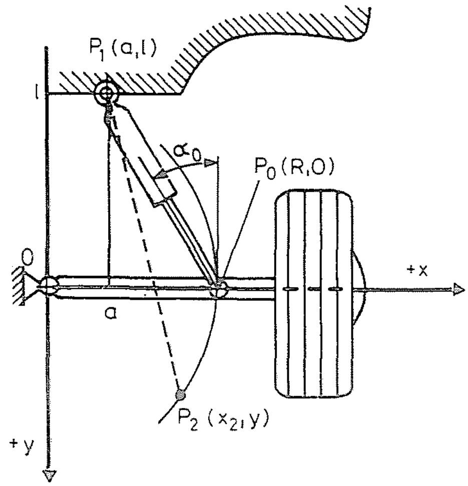


*Fig. 5.*

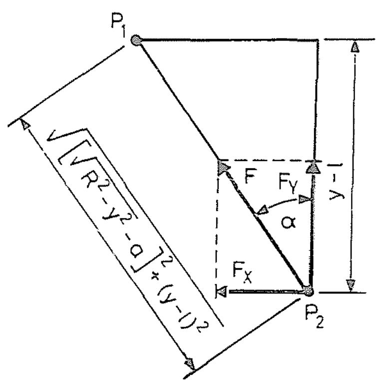


*Fig. 6.*

Following the previous way of thinking:

$$\overline{P_1P_2} = z(t)$$

$$z(t) = \sqrt{\left[\sqrt{r^2 - y^2(t)} - a\right]^2 + (y(t) - l)^2} \tag{6}$$

$$\dot{z}(t) = \frac{a\,y(t) - l\sqrt{r^2 - y^2(t)}}{\sqrt{r^2 - y^2(t)}\sqrt{\left[\sqrt{r^2 - y^2(t)} - a\right]^2 + [y(t) - l]^2}}\,\dot{y}(t). \tag{7}$$

$$\cos\alpha = \frac{y(t)-l}{\overline{P_1P_2}} = \frac{y(t)-l}{\sqrt{\left[\sqrt{r^2-y(t)^2}-a\right]^2+[y(t)-l]^2}}.$$

$$F_Y\Big(y(t),\dot{y}(t)\Big) = F\left(\frac{ay(t)-l\sqrt{r^2-y^2(t)}}{\sqrt{r^2-y^2(t)}}\times\right.$$

$$\left.\frac{1}{\sqrt{\left[\sqrt{r^2-y^2(t)}-a\right]^2+[y(t)-l]^2}}\,\dot{y}(t)\right) \tag{8}$$

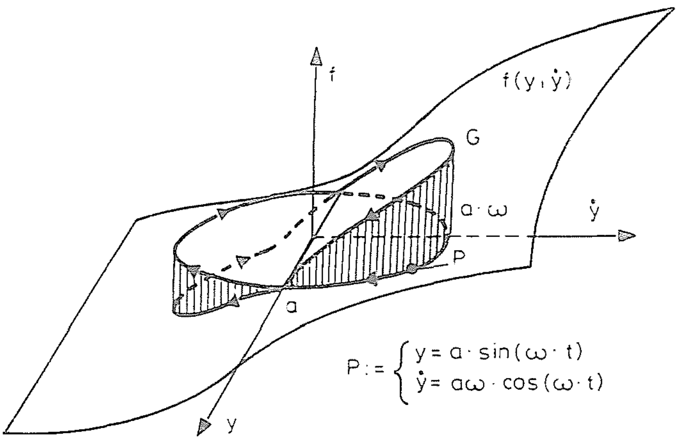


*Fig. 7.*

Our result is illustrated by *Fig. 7*, that shows that during the vertical swinging the damping force is moving above the plain established by the relative displacement $y$ and the relative velocity $\dot{y}$ on a single surface.

If this surface is neglected when modelling vehicle swinging, modelling error occurs. The error is proportional with the alteration of the surface in $y$ direction that depends on the type of suspension (rigid axle or swinging lever), the angle of building in of the damper and the amplitude of the excitation.

Now, let us see in case of a real damper that fragment of the surface that may be taken into account under real operating conditions. May this fragment of surface be projected perpendicularly on the force-relative

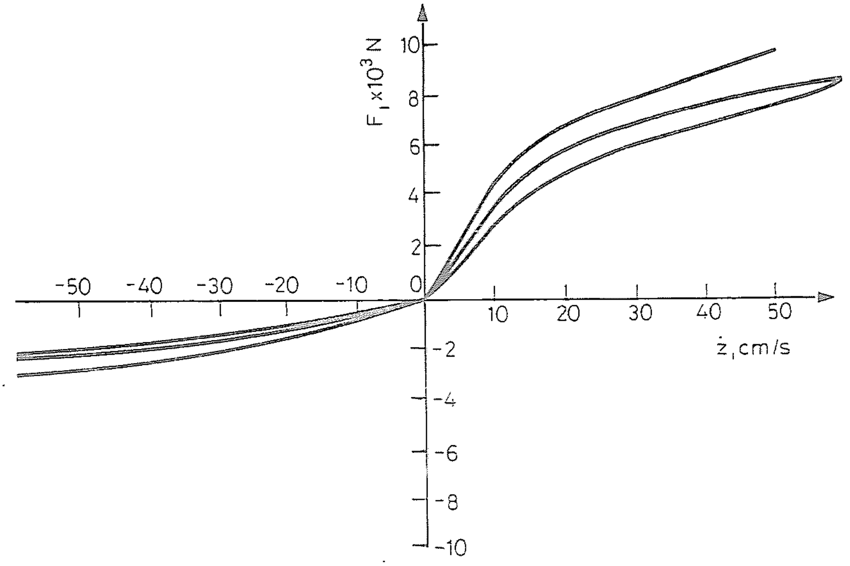


*Fig. 8.* Solution with rigid axle, $\alpha = 30°$, amplitude of the periodic excitation = 10 cm, angular frequency $\omega = 6$ rad/s.

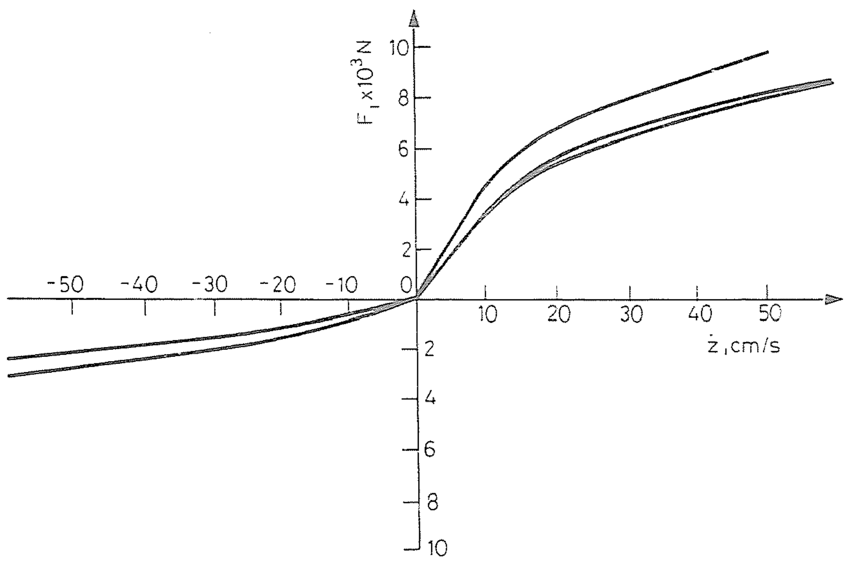


*Fig. 9.* Solution with swinging lever $\alpha = 30°$, amplitude of the periodic excitation = 10 cm, angular frequency $\omega = 6$ rad/s.

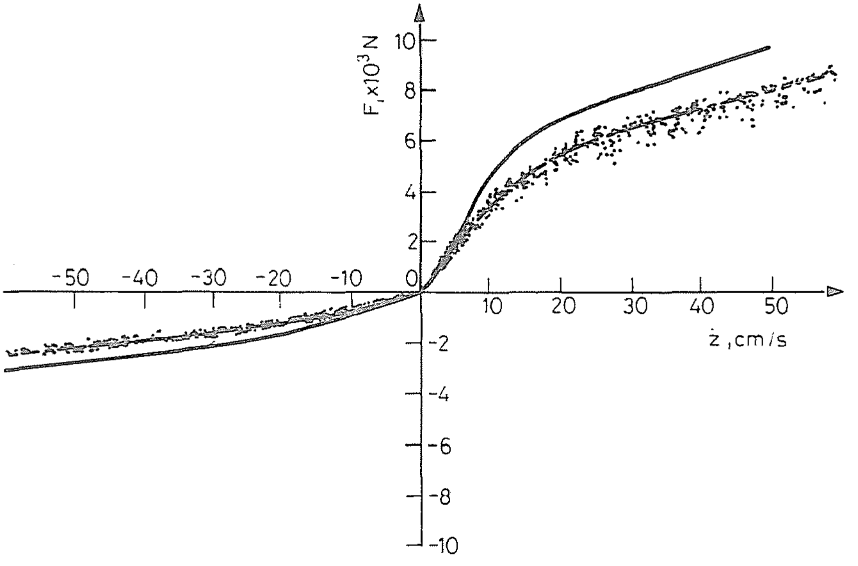


*Fig. 10.* Solution with rigid axle, $\alpha = 30°$, the vehicle runs on earth road, standard deviation of the road profile = 6 cm, velocity of the vehicle = 45 km/h.

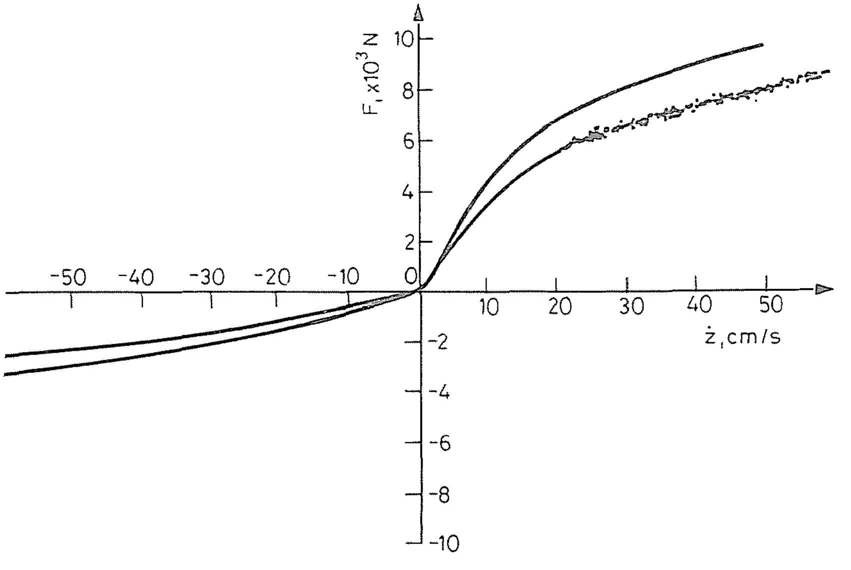


*Fig. 11.* Solution with swinging arm, $\alpha = 30°$, the vehicle runs on earth road, standard deviation of the road profile = 6 cm, velocity of the vehicle = 45 km/h.

velocity plane. In this case a range of damping forces is established insted of the previous damping force characteristic (*Fig. 8* and *9*). The figures illustrate the hysteresis ranges produced in case of periodic excitation.

This range shows well that under real circumstances deviances of which extent arise, if the phenomenon occurring because of the aslant building in is neglected.In *Fig. 10*, and *11* the original member characteristic is marked with continuous line. The range consisting of scattered dots illustrates the damping forces during real operation when the vehicle runs on the stochastic road profile. The calculations show the phenomenon that in case of rigid axle the range of dots is much wider than that in case of swinging lever.

In the examined cases the deviation of the central line of the dot-ranges compared to the real member characteristics took 16 to 20 [%]. The maximum deviation of the dots of the dot-range from their own centre line was 7 to 12 [%] in case of rigid axle and 2 to 5 [%] in case of swinging lever.

Table 1. shows the results of a digital simulation calculation at an asphalt road of 1 [cm] standard deviation and at a velocity range of $V = 40, \dots, 120$ [km/h].

$D(\ddot{z})$ shows the standard deviation of vertical acceleration of the center of gravity of the chassis.

$ST = D(z - g)/Z_{\text{stat}}$ [%] denotes the stability index of a wheel, where $D(z - g)$ is the standard deviation of the relative displacement between the wheel axle and the road profile, $Z_{\text{stat}}$ is the static deflection of the wheel.

The markings and the calculation methods are detailed in the literature [1, 2]. The $E$ column of the table shows the results of the calculations made with the original damping characteristic. The results of calculations taking into consideration of the aslant building in can be seen in columns I–II (I — the case of the rigid axle, II — the case of the solution with swinging lever).

The results show that — compared to the real values — in case of swinging acceleration a deviation of 9.42 to 9.51 [%] while in case of stability index even a deviation of 18.05 to 18.44 [%] can arise if the building in in aslant position is not taken into consideration. At last, how sensitively the swinging system reacts to the alteration of its one or more parameters are shown by the sensitivity functions of the system. According to the function for the maximum sensitivity [3]:

$$\varepsilon(\mathbf{p}) = \text{abs}(\mathbf{grad}W(\mathbf{p})) \quad \text{abs}(\mathbf{p})/W(\mathbf{p})$$

Where: $W$ is the scalar vector function qualifying the system and $\mathbf{p}$ is the parameter vector of the system.

Table 1

| $V$ [km/h] | $D(\ddot{z})$ [cm/s$^2$] | | | [%] deviation | |
|---|---|---|---|---|---|
| | $E.$ | $I.$ | $II.$ | $I.$ | $II.$ |
| 40 | 169.25 | 155.18 | 155.15 | 8.31 | 8.33 |
| 60 | 275.78 | 249.80 | 249.54 | 9.42 | 9.51 |
| 80 | 338.22 | 314.80 | 314.69 | 6.92 | 6.96 |
| 100 | 385.90 | 358.58 | 358.51 | 7.08 | 7.10 |
| 120 | 419.88 | 382.80 | 382.57 | 8.83 | 8.89 |
| % of mean deviation: | | | | 8.11 | 8.16 |

| $V$ [km/h] | $ST$ [%] | | | [%] deviation | |
|---|---|---|---|---|---|
| | $E.$ | $I.$ | $II.$ | $I.$ | $II.$ |
| 40 | 18.17 | 14.89 | 14.82 | 18.05 | 18.44 |
| 60 | 27.80 | 28.03 | 28.05 | 0.83 | 0.90 |
| 80 | 38.04 | 39.60 | 39.66 | 4.10 | 4.26 |
| 100 | 46.32 | 48.03 | 48.10 | 3.69 | 3.84 |
| 120 | 53.03 | 54.36 | 54.45 | 2.51 | 2.68 |
| % of mean deviation: | | | | 5.84 | 6.02 |

The investigations prove that the sensitivity of the system is too big if the chassis/axle mass rate is too small, the angular self-frequency of the axle is big, the angular self-frequency of the chassis is small and the relative damping of the system is small.

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