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EXPERIMENTAL AND NUMERICAL INVESTIGATION INTO NON-ROLLING TYRE

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Abstract

Tyre vibration is an important reason of travell reduction safety when increasing the speed of a vehicle. The tyre circumference can be seen as a loop shaped chain of elastic parts which have their portion of the mass of the tyre. This model is suitable for computer aided investigation on tyre vibration phenomena such as steady waves when rolling. Such an investigation is presented in this article.

Keywords: tyre vibration, tyre model.

1. Introduction

Nowadays the maximal power of car engines becomes very high, thus, many of these vehicles are able to travel at a speed of more than 200 km/h. The manufacturers contend with each other for the buyers, that's why they increase the maximal speed of their product. But it is a very dangerous process. The only contact between the vehicle and the road is the friction force acting at the base point of the rolling tyre. This force is our appliance to change the direction or speed of our vehicle's motion. Increasing the rolling speed, the maximal possible force is decreasing. This effect is partially due to the tyre vibration intensively investigated all over the World, for instance at the Technical University Delft in the Netherlands under the direction of Dr. Eng. H. B. Pacejka. In the summer of 1992 two friends of mine (György Zsiros and Zoltán Tuka) and myself had the possibility to spend a short term there, and carry out some measurement connected with this task. Our work consisted in the vibration test of a non-rolling tyre, a pneumatic tyre (Pirelli 175/70 R13) built up on a very stiff base, by the help some acceleration-sensors connected with charge amplifiers and the Hewlett Pacard modal analyzer. The modal analyzer is a computer controlled multi-channel analog-digital converter using the LMS program package for complicated mathematical calculations.

2. Model Creation

First of all, we have set up a simplified theoretical tyre model and a numerical simulation program which can demonstrate its motions. The purpose of the model creation was to get ideas on the further measurements. So we needed a model suitable for numerical investigation into the most important vibration phenomena of a pneumatic tyre.

In *Fig. 1* you can see the simplified model, which contains finite number (72) of mass elements with massless springs and dampers connecting the masses with the fixed rim, and with each other. The masses can move only in radial direction. They represent a little part of the circumference of the tyre. The springs represent the following effects:

- the stiffness of the tyre side,
- the influence of the internal air pressure and
- the stiffness-like effect between the neighbouring parts of the tyre because of the tangential force in the tyre band.

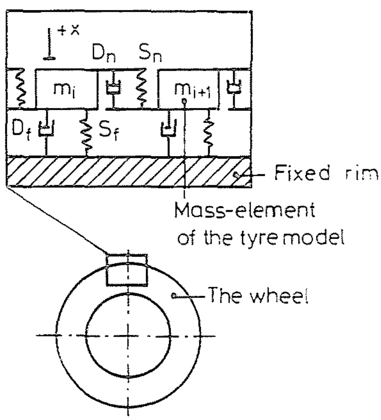

Fig. 1. The mechanical model of the tyre

The dampers modell the inner friction of the tyre material.

The dynamic behaviour of the model can be described by the following differential equations:

$$\ddot{x}_i = \frac{F_i}{m_i},$$

$$F_i = -S_f\, x_i - D_f\, \dot{x}_i - S_n\,(x_i - x_{i+1}) - S_n\,(x_i - x_{i-1})$$
$$-D_n\,(\dot{x}_i - \dot{x}_{i+1}) - D_n\,(\dot{x}_i - \dot{x}_{i-1}),$$

where:

m_{i-1}, m_i, m_{i+1} : the mass of three adjacent elements of the tyre model,

S_f, D_f : the stiffness and the damping of the mass element – wheel rim connection,

S_n, D_n : the stiffness and the damping between the neighbouring mass elements.

They can be solved also numerically by computer. If we take moving pictures based on the solution of the equations, then we can see what happens to the tyre during the real measurement. Therefore, we have written a computer program for the numerical solution of the previously mentioned equations using the simple Euler method to be fast enough for real time displaying. By the help of this program we could get ideas on the tyre vibration process and we could plan the further measurements. For example, simulating a hammer hitting on the tyre, we have seen that two wave fronts appear spreading away from the hitting point and moving in the opposite direction toward another point, as shown, in *Fig. 2.*

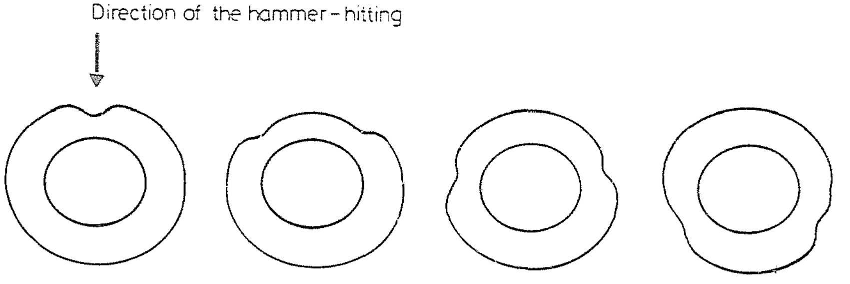

Fig. 2. Spreading wave fronts after a hammer hitting

We have observed that the speed of the wave front seemed to be independent of the hitting force but was mainly determined by the parameters of the tyre model. The program was able to measure the time delay while the wave fronts were moving from the hitting point to the other point. This option was very useful for identification of the wave speed in the case of our model.

If we know the time delay, then the wave speed can be calculated in the following way: $V_w = \frac{D\pi}{2T}[\mathrm{m/s}]$

where: V_w: the wave speed around the tyre

D : the diameter of the tyre

T : the time delay while the wavefront is moving from the hitting point to the other point.

Comparing this speed with the speed measured on the real tyre, we could check our model and correct its parameters.

3. The Eigenfrequencies of the Model

A further possibility of checking the accuracy of the model is to compare its eigenfrequencies with those of the real tyre.

In the program we could set the beginning conditions of different vibration eigenmodes of the tyre model to show how the tyre moves in the given eigenmode. The program can identify the different eigenfrequencies by measuring the time necessary for a given number of wave periods to pass. Having run the program with different parameters, we came to the following conclusions:

- decreasing the stiffness between the mass elements, the different ordered eigenfrequencies are getting closer to the first ordered one. Extremely: if the stiffness is zero, then all the eigenfrequencies are the same because the masses become independent (with no damping).
- decreasing the stiffness between the mass and the rim, the eigenfrequencies are spreading. Extremely: if this stiffness is zero, the eigenfrequencies are proportional to the order number of the given eigenmode. It means that the ratio between the 1st, 2nd, ..., n-th eigenfrequencies are 1, 2,..., n. In this case, the model is similar to a closed swinging string.

The real tyre is somewhere between the two previous cases, so if we measure the eigenfrequencies of it, then we can come to conclusions about the stiffness relations of the actual tyre.

4. Measurements on the Real Tyre

If we want to identify the most important eigenfrequencies and eigenmodes of the tyre then we must measure acceleration-time functions at more different points of the tyre, circumference during the same time interval. We used four piezo-electronic acceleration sensors in different arrangements to find the nodes of each eigenmode. Every measurement started with a hammer hit and was no longer than half a second. We applied three of the many data processing possibilities offered by the LMS software package:

- the SPECTRUM ANALYSIS,
- getting TIME RECORDS and
- the CROSS-CORRELATION FUNCTION.

5. Analysis of Spectrum Curves

The eigenfrequencies appear on the spectrum curves (*Fig. 3*) as amplitude peaks, but the ratio of the amplitudes are different depending on the place

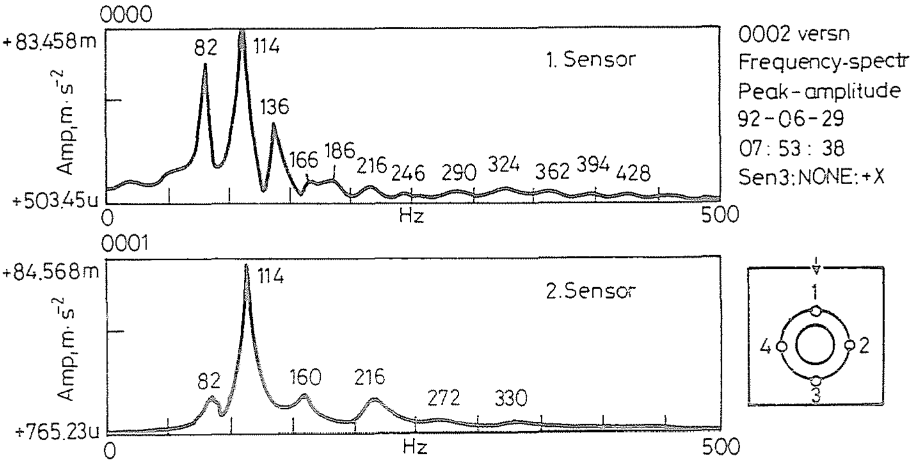

Fig. 3. Spectrum curves measured at different points of the tyre circumference

of the acceleration sensor. If the sensor is close to the node of some eigenmode, then the peak of the given eigenfrequency almost disappears, as you can see in *Fig. 3.* Changing the tyre pressure, the eigenfrequencies were shifting, so we could be sure that the peaks indicated the features of the tyre and were not due to some miscalculation or any physical disturbance. The spectrum curves show that the different eigenfrequencies are the following (in the case of 0.2 Mpa tyre pressure): 82, 114, 136, 160, 186, 216 Hz.

6. Identification of the Wave Speed around the Tyre

The other purpose of our measurements was to identify the wave speed around the tyre circumference. This can be a very important feature when rolling at a high speed. This wave speed is a limit like the sound speed in the aerodynamics, because reaching this speed the dynamic behaviour of the tyre basicall changes.

Rolling slowly, the continuous disturbance caused by the road generates a deformation spreading around the tyre. But rolling faster than this wave speed, the influence of the deformation is not able to move against the rolling direction, so steady waves appear on the tyre. These steady waves can cause a strong deformation dangerous for the tyre because of the intensive warm-building and fatiguing. The wave speed can be calculated by measuring the time delay while the influence of the hammer hit passes the given part of the tyre circumference. This time delay can be determined by using cross-correlation function between two time records given by two sensors or by comparing the simple time records.

7. Analysis of Cross-correlation Functions

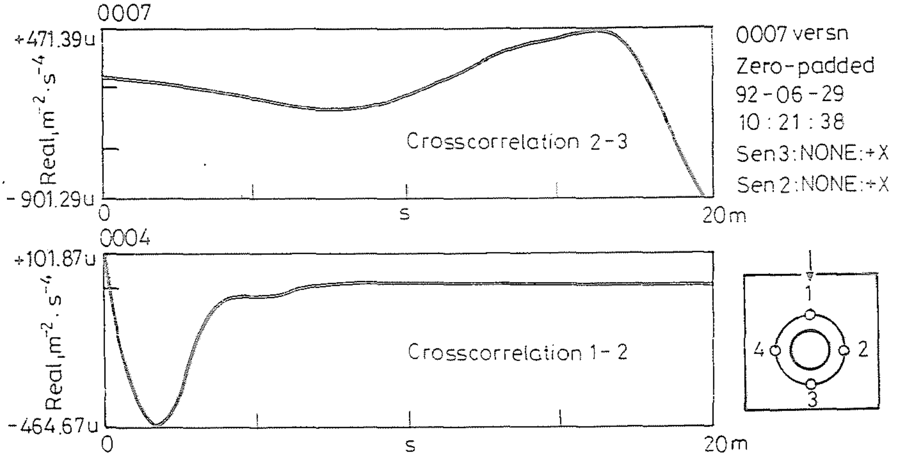

Fig. 4. Cross-correlation between time records measured at different points around the tyre circumference

The cross-correlation function (*Fig. 4*) has an extreme value at the mentioned time delay. The extreme value is a maximum or minimum depending on if the wave is inverting while moving or not.

8. Analysis of the Time Records

Observing the time functions, it is easy to find the time delay, as you can see in *Fig. 5*.

The first influence of the hammer hit measured by the first sensor appears before the other sensors measure the first peaks. The measured time delay was about 14-17 ms (in the case of 0.2 Mpa tyre pressure) but increasing the tyre pressure, the time delay was getting shorter. 14-17 ms determine a wave speed of about 200 km/h.

It means that if we increase the rolling speed of this tyre, then reaching the limit of 200 km/h, we could observe dangerous steady waves on the tyre circumference.

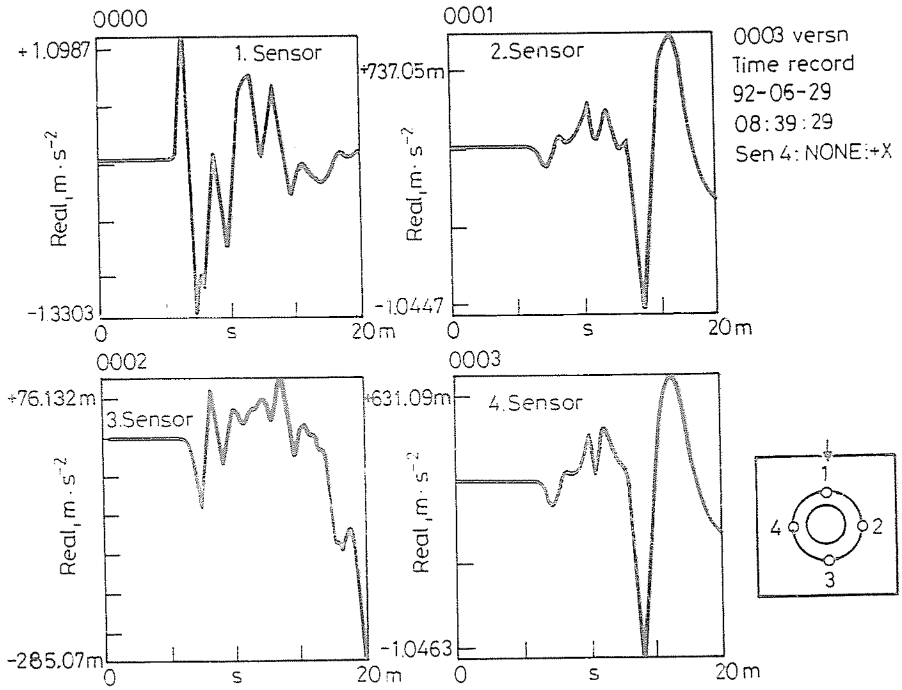

Fig. 5. Time records measured at different places

Now we know how to measure and how to calculate some features of a tyre by simulation so we have nothing more to do than to vary the parameters of the simulated model until the calculated time delay and frequencies converge on the really measured values. In this case, we can use the program for further investigations and we can hope that the results of the simulations approximate the behaviour of a real tyre without executing expensive researches.

The above mentioned value of the wave speed was identified on non-rolling tyre but the behaviour of the tyre changes when it is rolling. Its basic reasons are the following:

- the centripetal acceleration,
- the Coriolis acceleration and
- the effect that the place of the continuous disturbance caused by the road is moving around the tyre.

The latter is the most important effect, because this is the reason why steady waves appear. In the computer program we could simulate this phenomenon. We forced a deformation to a part of the tyre model, and moved this place at a rolling speed around the circumference. While the rolling speed was below the calculated wave speed limit, the deformation was similar to that of a non-rolling tyre but above this speed limit we could observe steady waves (*Fig. 6*). The steady wave form was similar to that eigenmode the wave speed of which was near to the current rolling speed. The wave speed of an eigenmode can be calculated in the following way: $V_w = \frac{D*\pi}{n} * f_n$ [m/s]

where: V_w: the wave speed around the tyre

D : the diameter of the tyre

f_n : the n-ordered eigenfrequency.

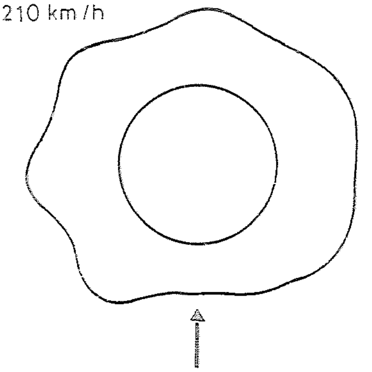

Fig. 6.

Table 1

Order number	Eigenfrequency Hz	Calculated wave speed km/h
1	82.2	558.0
2	95.4	323.7
3	114.6	259.4
4	135.8	230.4
5	160.2	217.5
6	183.8	207.9
7	208.3	201.9
8	235.8	200.0
9	260.4	196.3

For example, rolling at a speed of 230 km/h, the wave form is similar to the 4th eigenmode. Our program calculated the different wave speeds according to the eigenmodes and eigenfrequencies. The calculated eigenfrequencies and wave speeds are given in *Table 1*.

The wave speed is decreasing in sequence of the order numbers but a bottom limit seems to exsist for them that is near to the wave speed calculated from the time delay of influence of a hammer hit.

So, raising the rolling speed from a low value, first a very dense steady wave shape appears, and increasing the speed, the steady wave peaks spread apart.

9. Conclusions

Although our investigations were just a chain link in a long-term project of TU DELFT, some interesting conclusions can be mentioned. The eigenfrequencies of a pneumatic tyre have a little importance in everyday life but knowing them, we can approximate the rolling speed where dangerous steady waves appear on the tyre. It is important to know that if the tyre pressure decreases, then this speed limit also decreases making high-speed travel of the vehicle more dangerous.

10. Direction of Further Investigations

The measurements presented have been carried out on non-rolling tyre therefore we have to handle the conclusions carefully about the vibration phenomena connected with rolling tyre. So our investigation was just a first approximation which could give ideas for planning further research. If we want to have more exact results about the vibration process of rolling tyre, then we must extend the simulation program taking the Coriolis and centripetal acceleration into consideration and make measurements on rotating real tyres.