PERIODICA POLYTECHNICA SER. TRANSP. ENG. VOL. 22, NO. 1, PP. 25-34 (1994)

IDENTIFICATION OF NONLINEAR VEHICLE DYNAMICS WITH UNOBSERVABLE INPUT

Péter VÁRLAKI and György TERDIK

Technical University of Budapest
H-1521 Budapest, Hungary

Received: Nov. 10, 1992

Abstract

The realization problem and identification procedure of simple nonlinear vehicle dynamics are studied using the estimated spectrum and bispectrum of the output (vertical acceleration) process when the input excitation is (in real time) unobservable.

Keywords: vehicle dynamics, identification.

1. Introduction

The realization and identification of nonlinear models for road vehicle dynamics can be applied to study ride quality and stability analysis of a specific vehicle. It is well known that the structural design of vehicles requires a detailed model with large degrees of freedom, but models with relatively few degrees of freedom can be applied to study ride quality and stability analysis of the vehicle. Nonlinear analysis, using pre-designed road excitation (input) signals, can often be performed by the use of two independent nonlinear dynamic models with twice two degrees of freedom. However, in some important practical cases we cannot use the INPUT/OUTPUT identification models because we have no 'effective' possibility for the measurement of the real (time) 'input' random excitation 'signal', though the stochastic characteristics of the road profile are known or principally can be known (may be should be known). In these usually 'real-time cases' (e.g. for semi-active suspension control processes) is it necessary to identify of nonlinear vibrating structure of vehicle axle system by only measurements of the 'output processes', i.e. the vertical acceleration of the axles. Solving this identification or filtering task we may use the modern theory of stochastic bilinear systems as a particular case of the general nonlinear Wiener model, i.e. a stationary L functional on a Wiener space generated by Gaussian white noise in discrete time and by a Wiener process in continuous time case. The paper considers the spectrum and bispectrum of the output (vertical accelerations) which is given by multiple Wiener-Ito

Integral Representation. The so called approximation process seems to be bilinearly realizable. Afterwards, we are considering the Maximum Likelihood (ML) estimators for the abstract parameters of nonlinear vibrating equation by the estimated spectrum and bispectrum using a Tuky window method, taking into consideration that the spectrum and the bispectrum estimator can give independent identical distributed Gaussian variables as a limit according to the different frequencies.

2. Physical Model of Vehicle System Dynamics

To perform this momentary analysis there is a very important simple dynamic model approach, that can be applied when the following condition for rigid body assumption is satisfied (see *Fig. 1* for illustration): $l_1 l_2 = I/m_v$, where m_v is the total mass and I is the moment of inertia of the whole body. This condition is satisfied or well approximated in many practical situations. In addition, if the measurements are carried out in laboratories (using e.g. Hydropulse apparatus), the experiment can be designed so that the above condition is satisfied. This approach allows the two-variable nonlinear identification for both the front and rear axle subsystems providing a reliable preliminary analysis of the nonlinear dynamic characteristics of the vehicle. In order to specify the nonlinearity characteristics $F_1(.)$, $F_2(.)$ assume that the axles have progressive damping and stiffness characteristics [2, 7]. These types of progressive nonlinearities are the most common in practice, and because of the symmetry of the ideal damping and stiffness curves [7]. These nonlinearities about a point, can be well approximated by the first three elements of their Taylor series. Finally, the nonlinear damping effect of wheel pneumatics will be neglected, because practical experiments proved their small significance. Under these considerations the differential equation of the simplified model of the vehicle consists of the differential equation of the front axle, of the connected *'front upper axle'* element, and of the rear axle connected to the *'rear upper axle'* element, the front- and rear-axle subsystems, are illustrated on *Fig. 1,* and the differential equations for the front-axle subsystem are given by

$$m_1\ddot{y}_1 + k_1(\dot{y}_1 - \dot{y}_i) + c_1(y_1 - y_i) + \alpha k_1(\dot{y}_1 - \dot{y}_i) + \beta c_1(y_1 - y_i)^2 =$$

$$c_1^g(u_1 - y_1) + k_1^g(\dot{u}_1 - \dot{y}_1)\,, \tag{1a}$$

$$m_i\ddot{y}_i + k_1(\dot{y}_i - \dot{y}_1) + c_1(y_i - y_1) - \alpha k_1(\dot{y}_i - \dot{y}_1)^2 - \beta c_1(y_i - y_1)^2 = 0\,, \tag{1b}$$

where linear road profile excitation was assumed. The dynamics of rear-axle subsystem can be described analogously.

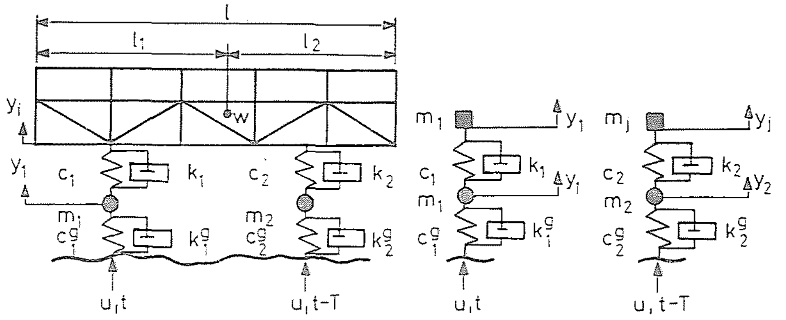

Fig. 1.

The solution of the above nonlinear differential equations can be obtained by a perturbation method, where the solution of the linear part ($\alpha = 0$, $\beta = 0$) is recurrently substituted into (1). This procedure can be described as follows. Since (1) represents a single input two-output system, the linear transfer functions for the two outputs y_{L1}, y_{Li} (displacements of axle and 'upper axle' elements) can be explicitly obtained by applying Laplace-transforms or Fourier-Transformation with zero initial conditions. With $\alpha = 0$, $\beta = 0$, one can write (now $i = f$),

$$Y_{L1}(s) = H_1(s)U(s)\,, \tag{2}$$

$$Y_{Lf}(s) = H_f(s)U(s)\,, \tag{3}$$

where $H_1(s)$, $H_f(s)$ are called linear transfer functions of the complex variable s,

$$H_1(s) = \frac{B_1(s)}{A(s)}\,, \tag{4}$$

$$H_f(s) = \frac{B_f(s)}{A(s)}\,. \tag{5}$$

The numerator and denominator polynomials can be expressed by the physical parameters as:

$$B_1(s) = b_3^1 s^3 + b_2^1 s^2 + b_1^1 s + b_0^1 ,$$

$$b_2^1 = \frac{k_1 k_1^g}{m_1 m_i} + \frac{c_1^g}{m_1}, \qquad b_1^1 = \frac{c_1 k_1^g + k_1 c_1^g}{m_1 m_i},$$

$$b_3^1 = \frac{k_1^g}{m_1}, \qquad b_0^1 = \frac{c_1 c_1^g}{m_1 m_i}, \tag{6a}$$

$$B_f(s) = b_2^1 s^2 + b_1^1 s + b_0^i ,$$

$$b_2^1 = \frac{k_1 k_1^g}{m_1 m_i}, \qquad b_1^i = b_1^1 ,$$

$$b_0^i = b_0^1, \tag{6b}$$

$$A(s) = s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 ,$$

$$a_3 = \frac{k_1 + k_1^g}{m_1} + \frac{k_1}{m_i}, \qquad a_2 = \frac{c_1 + c_1^g}{m_1} + \frac{k_1 k_1^g}{m_1 m_i} + \frac{c_1}{m_i},$$

$$a_1 = b_1^1 , \qquad a_0 = b_1^0. \tag{6c}$$

The solution $y_{Lf}(t)$, $y_{L1}(t)$ can be expressed by the convolutional integrals: $y_{L1}(t) = \int_0^\infty g_1(\tau)u(t-\tau)\mathrm{d}\tau$, $y_{Lf}(t) = \int_0^\infty g_f(\tau)u(t-\tau)\mathrm{d}\tau$, where the impulse response functions $g_1(t)$, $g_f(t)$ are the inverse-transforms of $H_1(s)$, $H_f(s)$, respectively. After recurrent substitutions of y_{L1}, y_L into (4) we obtain

$$\begin{aligned} y_1(t) \cong & \int_0^\infty g_1(\tau)u(t-\tau)\mathrm{d}\tau - \\ & - \alpha k_1 \int_0^\infty g_1^*(\tau) \left[\frac{\mathrm{d}}{\mathrm{d}t} \int_0^\infty [g_1(\tau) - g_f(\tau_1)]\, u(t-\tau-\tau_1)\mathrm{d}\tau_1 \right]^2 \mathrm{d}\tau - \\ & - \beta c_1 \int_0^\infty g_1^*(\tau) \left[\int_0^\infty [g_1(\tau_1) - g_f(\tau_1)]\, u(t-\tau-\tau_1)\mathrm{d}\tau_1 \right]^2 \mathrm{d}\tau + \dots . \end{aligned} \tag{7}$$

Similar result can be obtained for $y_f(t)$.

Explicit I/O relation between $y_1(t)$, $y_f(t)$ and $u(t)$ can be given in the frequency domain by applying one- and two-dimensional transformations [8]

$$Y_1(s_1, s_2) = H_1(s)U(s) + H_{21}(s_1, s_2)U(s_1)U(s_2) , \tag{8a}$$

$$Y_f(s_1, s_2) = H_f(s)U(s) + H_{2f}(s_1, s_2)U(s_1)U(s_2) , \tag{8b}$$

where with equivalent rearrangements, the two-variable transfer functions are obtained as

$$H_{21}(s_1, s_2) = \frac{B_{22}(s_1, s_2)}{A(s_1+s_2)} \frac{B_{21}(s_1)}{A(s_1)} \frac{B_{21}(s_2)}{A(s_2)}, \tag{9a}$$

where

$$B_{22}(s_1, s_2) = \frac{(s_1+s_2)^2(\beta c_1 + \alpha k_1 s_1 s_2)}{m_1},$$

$$B_{21}(s_k) = \frac{s_k^2(k_1^g s_k + c_1)}{m_1}, \qquad k = 1, 2,$$

and analogous result can be obtained for $H_{2f}(s_1, s_2)$ as well. It can be seen that the above transfer functions consist of the sum of a single variable and of a two-variable transfer function, thus the I/O relationships can be described in the time domain by second order Volterra functional series model, see e.g. the model for $y_1(t)$, as

$$y_1(t) = \int_0^\infty g_1(\tau)u(t-\tau)d\tau + \iint_0^\infty g_{21}(\tau_1, \tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_1, \tag{9b}$$

where the two-dimensional impulse response function $g_{21}(\tau_1, \tau_2)$ was obtained as the two-dimensional inverse transform of $H_{21}(s_1, s_2)$. The block structure of the dynamic nonlinear model associated with (9) is illustrated in *Fig. 2.*

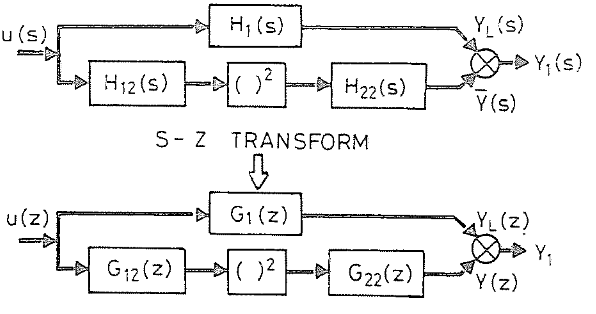

Fig. 2.

The equivalent discrete transfer functions $G_1(z)$, $G_{21}(z_1, z_2)$ can be obtained from $H_1(s)$, $H_{21}(s_1, s_2)$ by applying continuous-discrete, or $S - Z$ transform with e.g. zero-order hold assumptions on the input. The SS representations of the axle models can be derived, e.g. by the method suggested by GILBERT [4]. For minimal order realization of continuous-time two-power input-output maps, we may apply a direct realization procedure that is based on the elementary subsystem (ESS) decomposition of the transfer functions $G_1(z)$, $G_{21}(z)$, $G_{22}(z)$, the discrete equivalents of the transfer functions $H_1(s)$, $H_{21}(s) = B_{21}(s)/A(s)$, $H_{22}(s) = B_{22}(s)/A(s)$, respectively, [3, 9]. The equivalent discrete state-space identification model and its realization problems together with the ML structure and parameter estimation procedure in the time-domain were discussed in [9].

3. System Identification with Unobservable (Unmeasurable) Input Process

If the input is unobservable, the above nonlinear model with Volterra functional series can be reconsidered as a bilinear time series model. In this case we use the Wiener/Ito integral representation, whose w_t is a discrete white noise model with variance σ^2 and $W(\mathrm{d}w)$ is the stochastic spectral measure connected to the w_t by the spectral representation $w_t = \int_0^1 \exp(i2\pi\omega)W(\mathrm{d}\omega)$, or to the Wiener process w_t in continuous time [6].

The machinery we base our analysis on is the bispectrum of the process

$$B_Y(z_1, z_2) = \sum_{k,j=-\infty}^{\infty} c_{YY}(k, j) z_1^{-k} z_2^{-j}, \tag{10a}$$

where $z_1 = e^{i2\pi\lambda_1}$, $z_2 = e^{i2\pi\lambda_2}$, $\lambda_1, \lambda_2 \in [0, 1]$, and $c_{YY}(k, j) = \mathrm{E}Y_0 Y_k Y_j$.

B_Y exists for all $\lambda_1, \lambda_2 \in [0, 1]$ if

$$\sum_{k,l=-\infty}^{\infty} |c_{YY}(k, l)| < \infty .$$

The following symmetry properties fulfil for the third order moments c_{YY}

$$\begin{aligned} c_{YY}(k, l) &= c_{YY}(l, k) \\ &= c_{YY}(-k, l-k) = c_{YY}(l-k, -k) \\ &= c_{YY}(-l, k-l) = c_{YY}(k-l, -l) . \end{aligned} \tag{10b}$$

From the definition of B_Y and from (3) one can prove the following properties

$$B_Y(z_1, z_2) = \overline{B_Y(z_1^{-1}, z_2^{-1})},$$

$$\begin{aligned}B_Y(z_1, z_2) &= B_Y(z_2, z_1)\\ &= B_Y(z_1, z_1^{-1}z_2^{-1}) = B_Y(z_1^{-1}z_2^{-1}, z_1)\\ &= B_Y(z_2, z_1^{-1}z_2^{-1}) = B_Y(z_1^{-1}z_2^{-1}, z_2)\,.\end{aligned}$$

The method we use is the substitution of the solution for the linear equations (with $s = i\omega$)

$$y_{L1}(t) = \int_{-\infty}^{\infty} e^{i\omega t} G_{L1}(i\omega) W(\mathrm{d}\omega)\,, \qquad y_{Lf}(t) = \int_{-\infty}^{\infty} e^{i\omega t} G_{Lf}(i\omega) W(\mathrm{d}\omega) \tag{11a}$$

into the quadratic part of (10) and look for the solution with Hermite degree 2, where

$$G_{L1}(i\omega) = H_{L1}(i\omega)\frac{\alpha(i\omega)}{\beta(i\omega)}\,, \qquad G_{Lf}(i\omega) = H_{Lf}(i\omega)\frac{\alpha(i\omega)}{\beta(i\omega)}\,, \tag{11b}$$

furthermore $F_u(i\omega) = \frac{\alpha(i\omega)}{\beta(i\omega)}$ is the transfer function of $u(t)$ as a 'known' ARMA Gaussian stationary input. (Here $F(i\omega) = F^{+}(i\omega)F^{-}(i\omega)$ is the autospectrum of process $u(t)$ and $F^{+}(i\omega) = F_u(i\omega)$). Then, for the bilinear model we get

$$y_1(t) = y_{1L}(t) + \iint_{-\infty}^{\infty} e^{i(\omega_1+\omega_2)t} G_{1Q}(i\omega_1, i\omega_2) W(\mathrm{d}\omega_1, \mathrm{d}\omega_2)\,, \tag{12a}$$

$$y_f(t) = y_{fL}(t) + \iint_{-\infty}^{\infty} e^{i(\omega_1+\omega_2)t} G_{fQ}(i\omega_1, i\omega_2) W(\mathrm{d}\omega_1, \mathrm{d}\omega_2)\,. \tag{12b}$$

Evaluating the quadratic terms in (4) using the linear solutions we obtain:

$$(y_{1L} - y_{fL})^2 = \sigma_d^2 + \iint_{-\infty}^{\infty} e^{i(\omega_1+\omega_2)t} \left[G_{1L}(i\omega_1) - G_{fL}(i\omega_1)\right]\cdot$$

$$\left[G_{1L}(i\omega_2) - G_{fL}(i\omega_2)\right] W(\mathrm{d}\omega_1, \mathrm{d}\omega_2)\,. \tag{13a}$$

$$(\dot y_{1L} - \dot y_{fL})^2 = \sigma_d^2 + \iint_{-\infty}^{\infty} e^{i(\omega_1+\omega_2)} (i\omega_1)(i\omega_2) \left[G_{1L}(i\omega_1) - G_{fL}(i\omega_1)\right]\cdot$$

$$\left[G_{1L}(i\omega_2) - G_{fL}(i\omega_2)\right] W(\mathrm{d}\omega_1, \mathrm{d}\omega_2)\,. \tag{13b}$$

In this case

$$G_{1Q}(i\omega_1,\, i\omega_2) = H_{21}(i\omega_1,\, i\omega_2)F_u(i\omega_1)F_u(i\omega_2) =$$

$$= \frac{B_{22}(i\omega_1,\, i\omega_2)B_{21}(i\omega_1)B_{21}(i\omega_2)}{A(i\omega_1 + i\omega_2)A(i\omega_1)A(i\omega_2)} \frac{\alpha(i\omega_1)\alpha(i\omega_2)}{\beta(i\omega_1)\beta(i\omega_2)} . \tag{14}$$

If $y(t) = y_1(t) + \xi(t)$ then $\xi(t)$ is an additive ARMA stationary noise of measurement independent of the 'real' output process $y_1(t)$ with a transfer function $\frac{\beta_0(i\omega)}{\alpha_0(i\omega)}$. In discretized form $\xi_t = \frac{\beta_0(z)}{\alpha_0(z)}\varepsilon_t = G_0(z)\varepsilon_t$ where ε_t is a discrete white noise with the variance σ_0^2.

The equivalent discrete one and two dimensional time series model in frequency domain (according to *Fig. 2*) is

$$G_1(z) = \frac{B_1(z)}{A(z)} \frac{\beta(z)}{\alpha(z)} , \tag{15}$$

$$G_{1Q}(z_1,\, z_2) = \frac{B_{22}(z_1,\, z_2)}{A(z_1,\, z_2)} \frac{B_{12}(z_1)B_{12}(z_2)}{A(z_1)A(z_2)} \frac{\beta(z_1)\beta(z_2)}{\alpha(z_1)\alpha(z_2)} , \tag{16}$$

i.e. the discrete output time series model is

$$y_t = G_1(z)w_t + G_{1Q}(z_1,\, z_2)H_2\left(w_{t_1},\, w_{t_2}\right) + G_0(z)\varepsilon_t , \tag{17}$$

where $H_2\left(w_{t_1},\, w_{t_2}\right) = w_{t_1}w_{t_2} - \delta_{t_1=t_2}\sigma^2$.

On the basis of the above model we can compute the spectrum and bispectrum of the output process from the measured output time series using the following relationship [6].

$$\varphi_{y_1}(z) = \sigma_0^2 \left|\frac{\beta_0(z)}{\alpha_0(z)}\right|^2 + \sigma^2 \left|\frac{B_1(z)}{A(z)}\right|^2 \left|\frac{\beta(z)}{\alpha(z)}\right|^2 +$$

$$\frac{2\sigma^4}{|A(z)|^2} \int_0^1 \left|\frac{B_{1Q}(z_1,\, zz_1^{-1})B_{1L}(zz_1^{-1})}{A(z_1)A(z_1^{-1}z)}\right|^2 \mathrm{d}\omega_1 \tag{18}$$

for the autospectrum of discretised output process y_{1t}, and

$$\Phi_{y_1}(z_1,\, z_2,\, z_3) = 6\sigma^4 \mathrm{sym}\left[G_{1L}(z_1)G_{1L}(z_2)G_{1Q}(z_1^{-1},\, z_2^{-1})\right] +$$

$$8\sigma^6 \int_0^1 \mathrm{sym}\left[G_{1Q}(z_1^{-1};\, z_2^{-1}z^{-1})G_{1Q}(z,\, z^{-1}z)G_{1Q}(z^{-1},\, z_2 z)\right] \mathrm{d}\omega$$

for the bispectrum of y_{1t}, where sym denotes the symmetrization by the variables z_1, z_2 and z_3.

The above result offers the following frequency domain identification procedure if the input excitation process was not measurable.

1. Estimation and smoothing the spectrum and bispectrum of the measured output process.
2. Knowing the structure of the one- and two-variable transfer functions, estimate their free parameters (coefficients). This step includes a frequency domain function fitting and can be solved e.g. by a nonlinear least squares method.

Remark: This two-step procedure was applied if the input excitation was white noise or a known ARMA process. It can, however, be applied also if the transfer function of the ARMA input process has also to be estimated.

The identification of the above model in the frequency domain can be performed by estimating the spectrum and bispectrum from the measured y_t, $t = 1, \ldots, N$ data and formalizing the nonlinear least-squares problem as

$$\sum_k \left[\widehat{I}_k - f(\omega_k; \Theta)\right]^2 + [\mathrm{card}(\Delta)]^{-1} \sum_i \sum_j \left[\widehat{J}_{ij} - \Phi(\omega_i, \omega_j; \Theta)\right]^2 \rightarrow \min_{\Theta},$$

where $\widehat{I}_k$, $\widehat{J}_{ij}$ are the estimation of spectrum and bispectrum, respectively, Θ denotes the free parameters in the theoretical spectrum $f(z; \Theta)$ and bispectrum $\Phi(z_1, z_2; \Theta)$, the summation goes for all frequencies ω_i, $\omega_j \in \Delta$ and card (Δ) denotes the quantity of summands.

4. Conclusion

This paper presented two modelling approaches for identification of nonlinear dynamics of vehicle suspension systems. If the road profile excitation was measurable, then a second order Volterra model can be derived and identified in time domain. If the identification has to be carried out only from output (e.g. acceleration) measurements, the stochastic bilinear model can be applied and identified in the frequency domain from estimated spectrum and bispectrum of the output process.

Acknowledgement

This work was supported by the Hungarian Foundation of Sciences under Grant No. OTKA-816/1991.

References

1. MICHELBERGER, P. – BOKOR, J. – KERESZTES, A. – VÁRLAKI, P. (1984): Dynamic Modelling of Commercial Road Vehicle Structures from Test Data. *Proc. of XX. FISITA Congress,* Vienna, 4, 4.96–6.103.
2. MICHELBERGER, P. – BOKOR, J. – KERESZTES, A, – VÁRLAKI, P. (1986): Nonlinear Parametric Identification of Bus Dynamics for Ride Quality and Stability Analysis. *Proc. 21st FISITA Congress,* Belgrade, 2, 2.123–2.130.
3. MICHELBERGER, P. – BOKOR, J. – KERESZTES, A. – VÁRLAKI, P. (1987): Determination of Mass, Damping and Stiffness Matrices Using Structural and Parametric Identification of Linear Vehicle Frame Models. *Proc. 10th IAVSD-Symp.* Prague, CSSR, pp. 252–264.
4. GILBERT, E. G. (1983): Minimal-Order Realizations for Continuous-time Two-power Input-output Maps. *IEEE Trans, on Aut. Control,* AC–28, No. 4.
5. DASGUPTA, S. – ANDERSON, B. D. O. – KAYE, R. J. (1988): Identification of Physical Parameters in Structured Systems. *Automatica,* Vol. 24. No. 2, pp. 217–226.
6. TERDIK, GY. – MEAUX, L. (1991): The Exact Bispectra for Bilinear Realizable Processes with Hermite Degree 2. *Adv. Appl. Prob.,* Vol. 23. pp. 798–808.
7. APEATAUR, M. (1984): Properties of Non-linear Two-force Elements Used in vehicle Dynamic System under Stationary Stochastic Excitation. Part 1-Ideal Elements. *Vehicle Systems Dynamics,* Vol. 13, pp. 215–247.
8. GEORGE, D. A. (1959): Continuous Nonlinear Systems. *Technical Report,* MIT, USA.
9. MICHELBERGER, P. – BOKOR, J. – KERESZTES, A. – VÁRLAKI, P. (1991): Statistical Identification of Nonlinear Vehicle Vibrating Structures. *9th. IFAC/IFORS Symposium on Identification and System Parameter Estimation,* Budapest, Hungary, 1. pp. 358–362.
10. TERDIK, GY. (1991): Bilinear State Space Realization for Polynomial Systems. *Computer Math. Appl,* Vol. 22, pp. 69–83.