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INVESTIGATION INTO THE DYNAMICAL AND WEAR PROCESS OF A RAILWAY WHEELSET, HAVING TWO WHEELS WITH GRAVITY POINT ECCENTRICITIES

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Abstract

The goal of the article was to provide a limit allowed to the gravity point eccentricities (GPE) of wheels, which still does not have a major impact on the dynamics and running safety of the railway wheelset. The other goal of this investigation was to examine and to create a model which is capable to tackle the development of circumferential wear process (CWP) as the consequences of GPEs. During the investigation also the dynamical influence of irregularly worn wheels was analyzed.

The development of irregular circumferential wear process of wheels is of great importance, because of its great impact on the operation safety of high speed vehicles.

Keywords: nonlinear dynamics, railway dynamics, wear calculation.

Introduction

Recently many effort made in the field of wheelset motion on an ideal track [1], [4], [8], [9].

The aim of this investigation was to take into consideration an imperfect railway wheelset, namely to investigate the influence of GPEs into the dynamical and wear processes. Many transportation companies are facing with the problem of parasitic motion of railway vehicles in the range of operational speed. Such problems have occurred at the Budapest Mass Transport Company during the operation of the multipleunit underground trainsets (METRO) [1], [2] and similar problems were registered at the Danish State Railways. In the paper [6] the vertical dynamics of a suburban train were examined due to the effect of eccentric wheelsets. In literature [2] and [7] the authors have concluded an additional longitudinal and vertical acceleration, caused by GPEs of wheelsets. Although the dynamic influence of the eccentricity was clear, no research has been carried out in order to determine the development of circumferential wear due to the eccentricity. The present article deals only with the phenomenon of CWP, the profile wear was not taken into consideration.

Dynamical Model and Equations of Motion

The problem under consideration is that of a longitudinally and laterally suspended wheelset with GPE moving at a constant forward speed along a straight and ideal track, see *Fig. 1.* The magnitudes of the GPEs are different at the left and right wheels and there is a phase angle β in between, see *Fig. 2.* The wheel profiles are conical with conicity λ, and assumed to be circular if CWP is not considered. Both the wheels and the rails are made of steel of shearing modulus G. In case of the validity of Hertz's theory concerning the contact area between wheel and rail, the contact

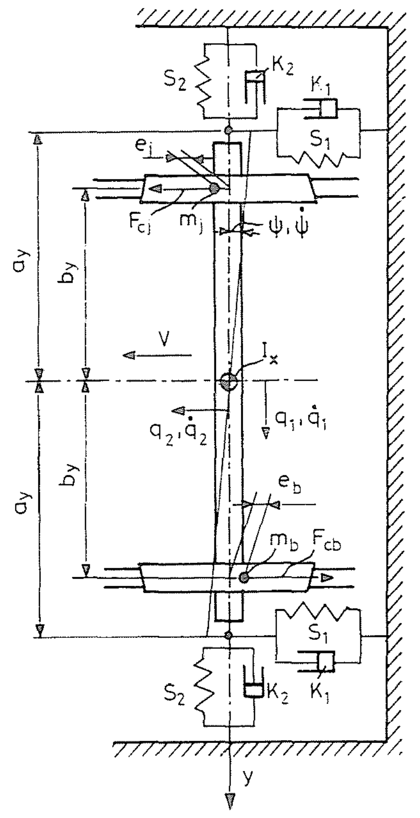

Fig. 1.

area supposed to be an ellipse with the semi-axes a and b. The creep term is based on Vermulen and Johnson theory [4], [8]. In the calculations the normalized creep term was used with respect to the forward speed, which is called creepage. Due to the fact that the wear distribution is different at the two wheels, the creepage terms are different, too, both at the left and right wheels. In the following only the L and R notations are used, see also *Fig 1.*

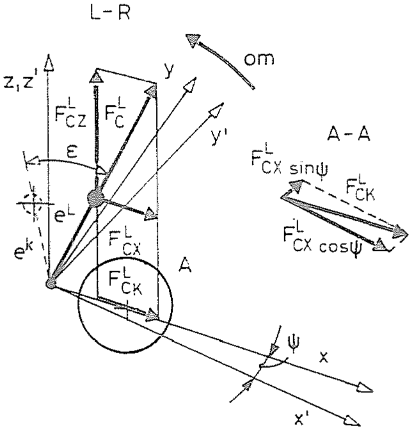

Fig. 2.

The lateral and longitudinal creepages are

$$\xi_x^{L,R} = \dot{y}/v - \varphi \ , \tag{1}$$
$$\xi_y^{L,R} = b_y\dot{\varphi}/v + \lambda y/r^{L,R} \ , \tag{2}$$

where the dot denotes the differentiation with respect to time. In the calculations the spin creepage was omitted.

For the creepages and creep forces the Vermulen and Johnson approximation [8] is used, where the resulting creepages $\xi_R^{L,R}$ are given by

$$\xi_R^{L,R} = \sqrt{(\xi_x^{L,R}/h_1)^2 + (\xi_y^{L,R}/h_2)^2} \ , \tag{3}$$

where h_1 and h_2 are the Hertzian longitudinal and lateral coefficients, respectively. The longitudinal and lateral creep-force components of the left and right wheels are determined by

$$F_x^{L,R} = (\xi_x^{L,R}/\varphi)F_R^{L,R}/\xi_R^{L,R} \ , \tag{4}$$
$$F_y^{L,R} = (\xi_y^{L,R}/\Phi)F_R^{L,R}/\xi_R^{L,R} \ . \tag{5}$$

Thus the resulting creep-forces are given by

$$F_R^{L,R}(u) = \mu N^{L,R} \begin{cases} u - \frac{1}{3}u^2 + \frac{1}{27}u^3 \ , & u < 3, \\ 1, & u > 3, \end{cases} \tag{6}$$

where u is different at the two wheels, according to the different $N^{L,R}$ normal loads and creepages, due to the influences of GPEs

$$u^{L,R} = (G\Pi ab/\mu N^{L,R})\xi_R^{L,R} \ . \tag{7}$$

Flange contact was also considered, when the wheelset exceeds the clearance δ in lateral direction, resulting the flange force F_T described by a linear spring and a dead band

$$F_T(y) = \begin{cases} k_0(y-\delta) \ , & \delta < y \ , \\ 0 \ , & -\delta < y < \delta \ , \\ k_0(y+\delta) \ , & y < -\delta \ . \end{cases} \tag{8}$$

The equations of motion which are based on Newton's 2nd theorem consist of the suspension, the constraints, the flange forces and the centrifugal forces due to GPEs. The magnitudes of the existing centrifugal forces $F_c^{L,R}$ are determined by the angular velocity, the equivalent masses of wheels $m^{L,R}$ and with the GPE of wheels $e^{L,R}$

$$F_c^{L,R} = m^{L,R} e^{L,R} \omega^{L,R} \ , \quad \text{where} \quad \omega = v/r_0 \ . \tag{9}$$

After the replacement of variables, $x_1 = \dot{q}_2$, $x_2 = \dot{\varphi}$, $x_3 = \dot{q}_1$, $x_4 = q_2$, $x_5 = \varphi$, $x_6 = q_1$ and $x_7 = 1$ the following 1st order differential equations occurred for the equations of motion of six degree of freedom (longitudinal, yaw and lateral) railway wheelset:

$$\dot{x}_1 = -2(k_x/m)x_1 - 2(s_x/m)x_4 - F_1/m \ , \tag{10}$$

$$\dot{x}_2 = -2(k_x a_y^2/I)x_2 - 2(s_x a_y^2/I)x_5 - F_2/I \ , \tag{11}$$

$$\dot{x}_3 = -2(k_y/m)x_3 - 2(s_y/m)x_6 - F_3/m \ , \tag{12}$$

$$\dot{x}_4 = x_1 \ , \tag{13}$$

$$\dot{x}_5 = x_2 \ , \tag{14}$$

$$\dot{x}_6 = x_3 \ , \tag{15}$$

$$\dot{x}_7 = 1 \ , \tag{16}$$

where the dot denotes the differentiation with respect to time and

$$F_1 = F^L_{cx} + F^R_{cx} + F^L_x + F^R_x, \tag{17}$$
$$F_2 = (F^L_{cx} + F^R_{cx})a_y + (F^L_x + F^R_x)b_y, \tag{18}$$
$$F_3 = (F^L_c - F^R_c)\sin\varphi + F^L_y + F^R_y + F_T . \tag{19}$$

For the notations and values see *Table 1.*

Table 1
Parameters of wheelset

Constant	Value	Description
$m^L = m^R$	511 kg	mass of one wheel and half axle
I	678 kg m^2	moment of inertia
a_y	0.716 m	half of wheel base
b_y	0.850 m	distance from wheel centre to S_x
a	6.578 mm	major semiaxis of contact path
b	3.934 mm	minor semiaxis of contact path
G	8.08 10e^8 N/m^2	shear modulus
S_x	1.823 MN/m	longitudinal stiffness
S_y	3.646 MN/m	lateral stiffness
K_x	2 KNs/m	longitudinal damping coefficient
K_y	2.92 KNs/m	lateral damping coefficient
S_0	14.6 MN/m	stiffness of flange
δ	9.1 mm	clearance
μ	0.15	coefficient of friction
N	66.670 KN	normal load of wheel
h_1	0.54219	longitudinal Hertzian coeff.
h_2	0.60252	lateral Hertzian coeff.
v		velocity of wheelset
r_0	0.4572m	nominal radius of wheel

The Wear Model

The Frictional Work

In order to create a model which is capable to investigate the circumferential wear process (CWP) of wheels, a good method was to be found which describes if [1], [2], [3], [5].

Generally, for the calculation of the specific wear, we shall assume a proportionality dependence between the specific wear w and the work of contact forces F_{fr} in the following form

$$w = k_w F_{fr} \ .$$

The factor of proportionality k_w (the wear rate) is determined from experiments [1].

In this research the wear model from KNOTHE and ZOBORY was used, where the specific frictional power is calculated from the creepage and creep-force multiplication. For the numerical analysis the circumference of wheels has been devided into equivalent unit surfaces (segments), see *Fig. 3.*

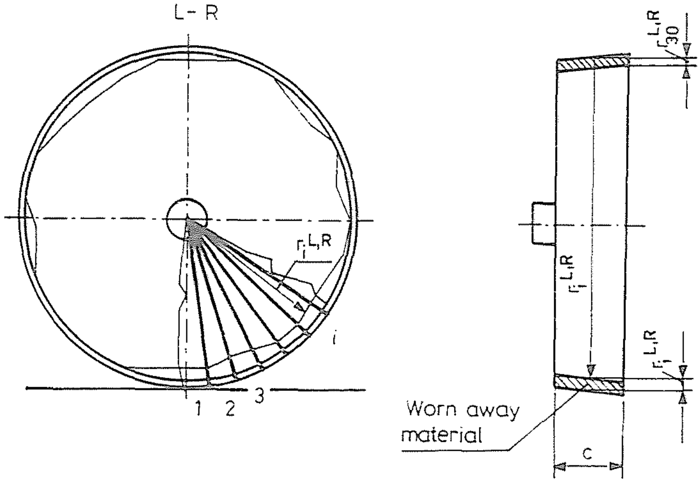

Fig. 3.

Therefore the specific frictional power is determined on unit surfaces, according to the following relation:

$$\frac{dW^{L,R}}{dl}(T) = w^{L,R} = k_w(\xi_x^{L,R} F_x^{L,R} + \xi_y^{L,R} F_y^{L,R}) \ , \tag{20}$$

where

T	: unit surface,
l	: distance travelled by the wheel over the rail,
k_w	: coefficient of wear,
$\xi_{x,y}, F_{x,y}$	: creepages and creep-forces.

For practical reasons the phenomenon of frictional work per unit surface per rotation of wheelset has been introduced,

$$w_v^{L,R} = \int_0^{tr} w^{L,R} v \, dt, \tag{21}$$

where

t_r : time needed to one rotation of wheelset,

v : rolling velocity,

$w_v^{L,R}$: frictional work per unit surface per rotation of wheelset.

On the basis of *Eq 21* the phenomenon of average frictional work per rotation can be introduced

$$w_{av,i}^{L,R} = \sum w_{v,i}^{L,R} / n \ , \tag{22}$$

n : number of revolution of wheelset.

The unit surface of wheel is determined by taking into consideration some geometrical aspects and supposing that the value of the wheel conicity is very small $\lambda = 0.05$, therefore the wheel can be replaced by a cylindrical one for the calculation of unit surface T

$$T = r_0 \alpha c \quad \text{and} \quad k_r = \rho T \ ,$$

where

c : thickness of the wheelblock,

α : angle (rad) belonging to one segment of the wheel,

r_0 : nominal radius of wheel,

k_r : coefficient of wear for the changing of radius,

ρ : density of steel.

From this the actual radius of wheel per unit surface can be determined

$$r_i^{L,R} = r_i^{L^0,R^0} - \Delta r_i^{L,R}$$

and

$$\Delta r_i^{L,R} = w_{av,i}^{L,R} / k_r \tag{23}$$

where

i : The no. of unit surfaces (segments), (see *Fig. 3*)

$\Delta r_i^{L,R}$: Decrement of radius at the unit surface.

The Rate of Wear

According to *Eqs.* (22) and (23) the wear can be defined in two ways:

- One way is the determination of mass of material worn away dimension: kg or more precisely μg: (1 μg =1 e^{-9}kg).

- The other way (*Eq.* 23) is the determination of the depth of part that was worn away (dimension: mm). In the calculations that was the radius decrement per unit surface $\Delta r_i^{L,R}$.

For calculations the second form is more suitable, it gives us the new shape of the wheel.

Method of Investigation

For the investigation of the dynamical model numerical methods were used such as the EULER and 4th order Runge-Kutta methods. A FORTRAN program was developed for dynamical and wear calculations.

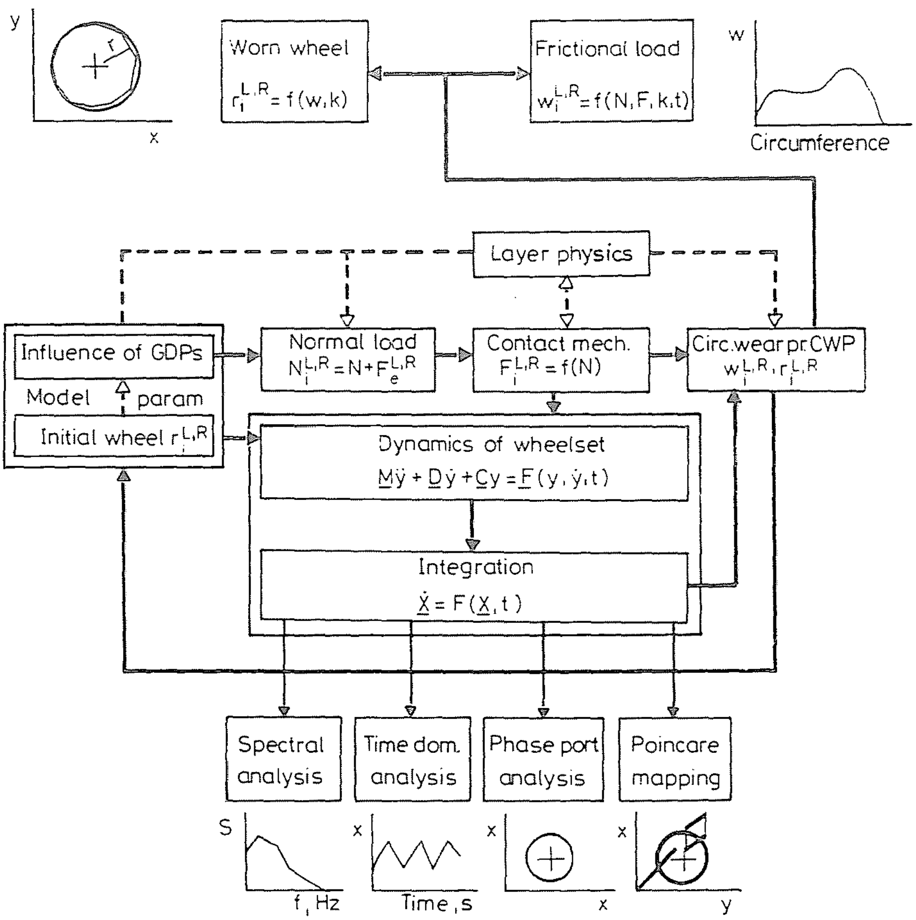

Fig. 4.

The schematic view of the program can be seen on *Fig 4*. Although the primary proposal of investigation was not to make bifurcation analysis, but it is possible by using numerically determined time series, phase space projection, Poincaré sections and spectral density analysis.

The Results

From practical point of view the strictly dynamical investigation and the wear-dynamical interaction problem was distinguished, because of the different integration steps is use. As it has already been mentioned the aim of investigation was to give limitations to the allowable magnitude of GPEs. Due to the dynamical behaviour of the wheelset with different GPEs a stability map can be drawn up according to *Fig 5*.

From this we can see that the critical value of GPE (e_{cr}), can highly influence the dynamic stability of the wheelset. Similar consequences were achieved from the linear stability analysis, where v_{cr} was between 44.2–52.7 m/s according to the different GPEs of magnitude 0–3 mm. For practical purposes and due to experiences at DSB (Denmark) and BKV (Hungary), the maximal value of GPE is limited to 0.3 mm (see also [2] and *Fig. 5*).

According to *Fig 6* the critical speed v_{cr} and the limit speed v_{lim} are shifted backwards on the horizontal axis due to GPE. This means that the unstable limit cycle will arise before the $v_{cr} > v > v_{lim}$ region, as well as instability can occur below v_{cr}. To illustrate further two spectral density functions are shown belonging to $v = 25$ m/s with and without GPE, see *Fig 7*. From the figure we can clearly see the impact of GPE, which enbodied in higher lateral acceleration of the railway wheelset.

In the following the wear calculations are presented. The calculations are attached to the nominal radius. At *Fig 8* the frictional power can be seen at 25 m/s, GPEs of 0.2 mm and phase angle of 45°. The figure shows a periodic wear load around the circumference of wheel, which was divided into 60 equidistant segments. At *Fig 9.a* and *b* the wear load and the radius of worn wheel can be seen at the given parameters of $v = 25$ m/s and GPEs of 0.4 mm and phase angle of 180°. From *Fig 9.a* we can see the influence of GPEs higher than e_{cr}, (considering the GPE limit due to dynamic instability), since the wear load is increasing by the number of rotation of wheelset, resulting an 'egg' form of wheel circumference, see *Fig 9.b*.

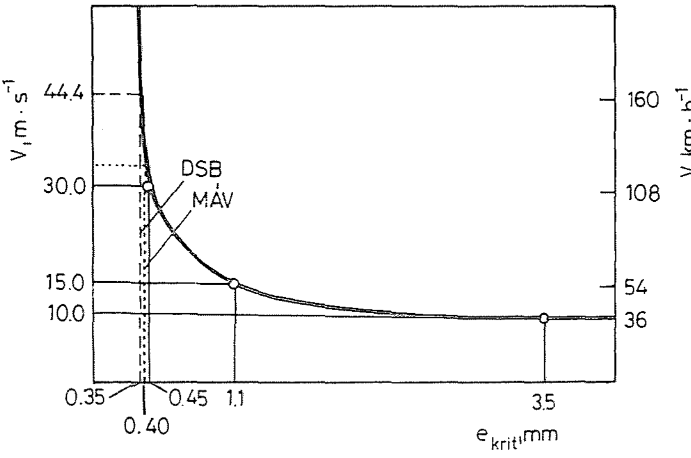
V, m · s⁻¹
44.4
30.0
15.0
10.0
DSB
MÁV
160
108
54
36
V, km · h⁻¹
0.35
0.45
1.1
3.5
0.40
e krit, mm

Fig. 5.

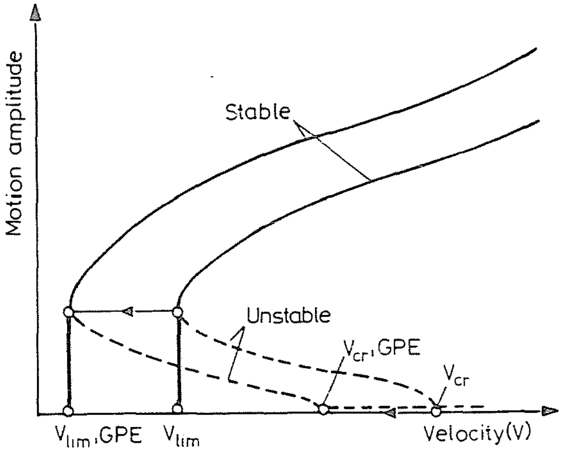
Motion amplitude
Stable
Unstable
Vcr, GPE
Vcr
Vlim, GPE
Vlim
Velocity(V)

Fig. 6.

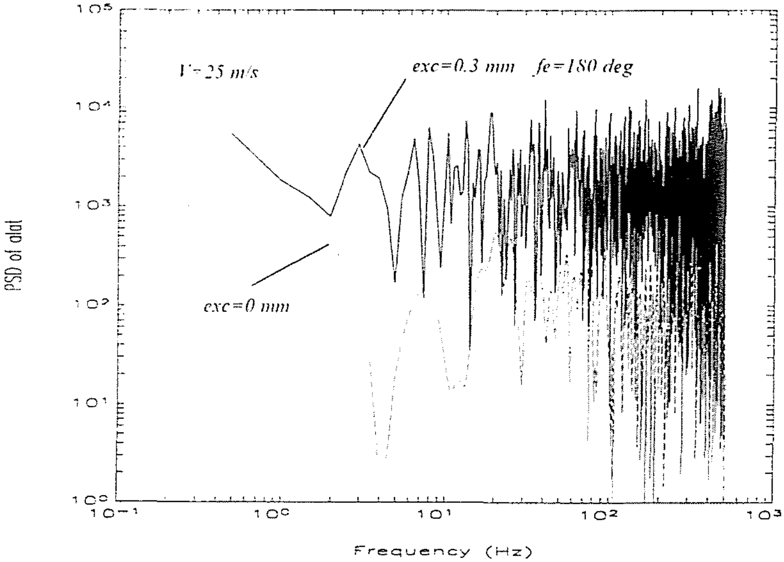
V=25 m/s
exc=0.3 mm fe=180 deg
exc=0 mm
PSD of dl/dt
Frequency (Hz)

Fig. 7.

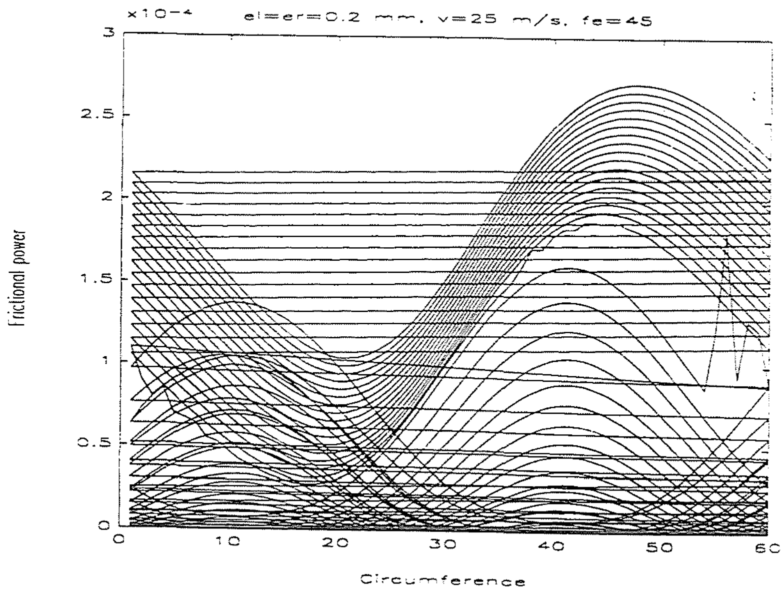
x10⁻⁴
el=er=0.2 mm, v=25 m/s, fe=45
Frictional power
Circumference

Fig. 8.

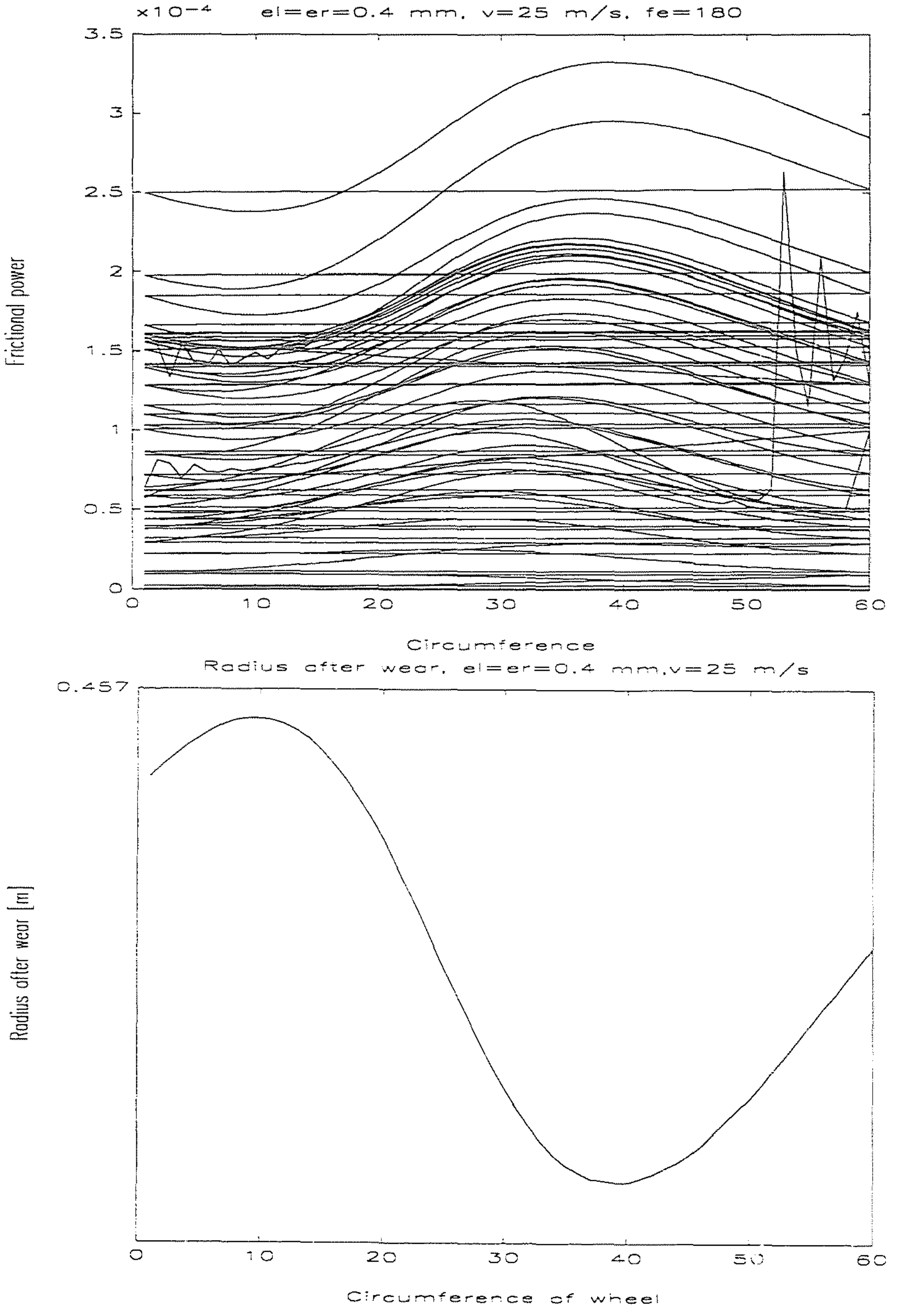
x10⁻⁴ el=er=0.4 mm, v=25 m/s, fe=180
3.5
3
2.5
2
1.5
1
0.5
0
0
10
20
30
40
50
60
Frictional power
Circumference
Radius after wear, el=er=0.4 mm,v=25 m/s
0.457
0
10
20
30
40
50
60
Radius after wear [m]
Circumference of wheel

Fig. 9.

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