# SOFTWARE STOPSIM FOR STOCHASTIC SIMULATION OF MOTION AND LOADING PROCESSES OF VEHICLES 

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#### Abstract

The software to be introduced aims to simulate the motion and loading processes of vehicles by utilizing the information based on operation measurements, namely the transition probabilities of the tractive and braking effort control process, as well as the conditional probability distribution functions of the dwelling times on different control levels. The simulation procedure is based on statistical-dynamical principles. The evaluation of the time realizations generated can lead to the most important statistical characteristics concerning the future loading conditions of the vehicle in the period of design, so the dimensioning can be much more reliable.


Keywords: stochastic simulation, vehicle dynamics.

## 1. Introduction

The software to be introduced is called STOPSIM. It is written in FORTRAN, and the name is abbreviated from the full title 'STochastic OPeration SImulator'. Though the present paper assigned its goal as the simulation of the operational motion and loading process of vehicles, it is to be emphasized that the applied principles determining the structure of the software are of more general character, i.e. the software can be used for the simulation of operation process of a very wide class of machinery.

## 2. The Dynamical Model

The dynamical model itself is a relatively simple one. It is a rigid body of single degree of freedom performing a translatory motion process of varying velocity under the influence of resistance, driving and braking forces. The action lines of the latter forces are identical, i.e. all the force vectors in question are fitting onto a common carrier straight line. The resistance force is described as a velocity and outer force dependent quantity:

$$
F_{R}\left(v, \sum F\right)= \begin{cases}(\operatorname{sign} v)\left[a v^{2}+b \||v|+c\right], & \text { if }|v|>0  \tag{1}\\ \left(\operatorname{sign} \sum F\right) \min \left\{c,\left|\sum F\right|\right\}, & \text { if } v=0\end{cases}
$$

where $a, b$ and $c$ are non-negative constants, $v$ stands for the magnitude of the velocity of the mass, while $\sum F$ is the resultant of the non-resistance forces acting on the mass. The driving force acting on the mass is considered as a bivariate function, namely it depends on control $u$ and velocity $v$. So a function $F_{D}(u, v)$ is to be defined in a form appropriate for the numerical treatment. Without the considerable restriction of the practical applicability of the computation procedure it can be assumed that the variable $u$, which is called the control parameter of the system, takes its values only from a finite set $\left\{u_{0}, u_{3}, u_{2}, \ldots, u_{M}\right\}$ of non-negative integers. Value $u_{0}$ belongs to the case of exerting zero driving force (no tractive effort), while if $u=u_{M}$, the maximum driving force (tractive effort) is applied. Velocity $v$ can take its values from the non-negative bounded interval $\left[0, v_{\max }\right]$. As for the treatment of function $F_{D}(u, v)$ in case of the mentioned restrictions concerning its variables, the set of performance curve visualized in Fig. 1 can always be used. The numerical representation of the performance curves is made by linear interpolation method. The first step is the discretization of the velocity range, i.e. one should consider a sequence $\left\{v_{0}=0, v_{1}, v_{2}, \ldots, v_{n}=v_{\max }\right\}$. In the real applications this sequence is usually (but not necessarily) an equidistant one. Let's consider an arbitrary control level $u_{i}$, then the tractive effort at velocity $v_{j}$ is $f_{i j}=F_{D}\left(u_{i}, v_{j}\right)$. In accordance with the aforesaid, to control $u_{i}$ two numerical sequences are joined, namely $v_{0}=0, v_{1}, v_{2}, \ldots, v_{n}=v_{\text {max }}$ and $f_{i 0}, f_{i 1}, f_{i 2}, \ldots, f_{i n}$. Using this train of thoughts for each possible control level, a set of numerical data is yielded for characterizing the tractive effort vs. velocity performance curves. The number of numerical data is obviously $L=(M+1) 2(n+1)$.

The braking force acting on the mass is also a bivariate function, it depends on brake control $u^{*}$ and velocity $v$. So a function $F_{B}\left(u^{*}, v\right)$ is to be defined. Without the considerable restriction of the wide applicability, it can be assumed that variable $u^{*}$ takes its values only from a finite set $\left\{u_{0}^{*}, u_{1}^{*}, u_{2}^{*}, \ldots, u_{N}^{*}\right\}$ of non-negative integers. Value $u_{0}^{*}$ belongs to the case of exerting zero braking force, while if $u^{*}=u_{N}^{*}$, the maximum braking force is applied. Velocity $v$ can take its values from the non-negative bounded interval $\left[0, v_{\max }\right]$ as introduced above. The treatment of function $F_{B}\left(u^{*}, v\right)$ under the mentioned restrictive conditions concerning its variables is possible by defining the set of brake performance curves visualized in Fig. 2. The numerical representation of the brake performance curves is made by linear interpolation method. The discretization of the velocity range has been


Fig. 1. Drive performance curves


Fig. 2. Brake performance curves
carried out, i. e. considered again the sequence $\left\{v_{0}=0, v_{1}, v_{2}, \ldots, v_{n}=\right.$ $\left.n_{\max }\right\}$. Let's consider now an arbitrary brake control level $u_{i}^{*}$, then the braking force at velocity $v_{j}$ is $g_{i j}=F_{B}\left(u_{i}^{*}, v_{j}\right)$. In accordance with the aforesaid, two numerical sequences are joined, to control $u_{i}^{*}$ namely $v_{0}=$
$0, v_{1}, v_{2}, \ldots, v_{n}=v_{\max }$ and $g_{i 0}, g_{i 1}, g_{i 2}, \ldots, g_{i n}$. Using this train of thoughts for each possible brake control level, a set of numerical data is yielded for characterizing the braking effort vs. velocity performance curves. The number of the latter numerical data is obviously $K=(N+1) 2(n+1)$.

As it has been mentioned the mass performs a translatory motion, therefore the effect of the rotating components in the system having an angular velocity proportional to the velocity of the vehicle (i.e. wheels, components of the transmission system, etc.) should also be reckoned with. The rotating mass factor $\gamma$ is defined as the ratio of the mass $m_{r}$ reduced on the periphery of the wheels and the mass $m$ of the vehicle, in formula: $\gamma=m_{r} / m$.

The control process of the system is composed of the controls $u$ and $u^{*}$ by accepting that at a given time $t$ either $u$ or $u^{*}$ can be effective only, i.e. either tractive effort exertion or braking force exertion can be realized. Only one exception exists, namely $u_{0}=u_{0}^{*}$, this case identifies the state without exertion tractive and braking effect. Due to this remark, the resultant unified integer valued control process $U$ has the following possible states:

$$
\begin{gather*}
U \in\left\{U_{1}, U_{2}, \ldots, U_{N}, U_{N+1}, U_{N+2}, \ldots, U_{N+M+1}\right\} \\
\text { Number of states }=D=N+M+1, \tag{2}
\end{gather*}
$$

where

$$
U_{i}=\left\{\begin{array}{ll}
-u_{i}^{*}+N+1 & \text { if } i \in\{1,2, \ldots, N\} \\
N+1 & \text { if } i=N+1 \\
u_{i}+N+1 & \text { if } i \in\{N+2, \ldots, D\}
\end{array} .\right.
$$

Control process $U(t)$ having number $D$ integer states is considered as a semi-Markovian stochastic process, its realizations are integer valued stepfunctions, see Fig. 3. It is obvious that state $u_{0}^{*}=u_{0}=0$ identifying the state free from tractive effort and braking force exertion is corresponding to state $U_{N+1}$. Process $U(t)$ spends a certain time $\tau$ in each achieved state $U_{j}$, and $\tau$ is a non-negative random variable being in stochastic dependence on the achieved new state $U_{j}$ and on the previous state $U_{i}$ left in the least state transition at the beginning of duration $\tau$.

The state transitions of the control process $U(t)$ are described by the so-called embedded Markovian chain. The latter is a homogeneous Markovian chain which can be represented by the transition probability matrix $\mathbf{P}_{i}$, containing the conditional probabilities

$$
\begin{equation*}
p_{i j}=\mathbf{P}\left\{U\left(t_{a}\right)=U_{j} \mid U\left(t_{p}\right)=U_{i}\right\} \tag{3}
\end{equation*}
$$



Fig. 3. A realisation of the control process
where $t_{p}$ is the time just prior to the state transition and $t_{a}$ is that after the state transition, while $U_{i}$ is the previous state and $U_{j}$ is the following state of process $U(t)$. For the sake of characterizing the duration in which process $U(t)$ remains in a constant state, let us consider the random variable $\tau$ describing the duration mentioned. The conditional probability distribution function $F_{i j}(t)$ is defined as follows, by using the simpler designations (i.e. the indices $i$ and $j$ instead of $U_{i}$ and $U_{j}$ ):

$$
\begin{equation*}
F_{i j}(t)=\mathbb{P}\left\{\tau<t \mid U\left(t_{p}\right)=i \cap U\left(t_{a}\right)=j\right\} \tag{4}
\end{equation*}
$$

It is obvious that for the possible combinations of indices $i$ and $j$ a matrixvalued time function is obtained, on the basis of which the semi-Markovian process $U(t)$ is fully characterized.

The equation of motion of the dynamical system can be formulated by using Newton's second law:

$$
\begin{equation*}
F_{D}(u, v)-F_{B}\left(u^{*}, v\right)-F_{R}\left(v, \sum\right)=(1+\gamma) m \frac{d v}{d t} \tag{5}
\end{equation*}
$$

Taking into consideration the definition of $U(t)$, the equation of motion and the initial condition belonging to it can be formulated as the following initial-value problem:

$$
\begin{gather*}
\frac{d v}{d t}=f(v, U(t)) \\
v\left(t_{0}\right)=v_{0} \tag{6}
\end{gather*}
$$

This first-order non-linear differential equation will be solved numerically in the knowledge of the actual variation of the simulated control process realization $U(t)$. The initial velocity of the motion can be zero, while after the state transitions the new initial velocities are equal to those of the vehicle prior to the transitions. In this way the numerical solution of the motion equation can be sequentially carried out with ease over the time intervals in which control $U(t)$ takes its simulated constant values. The program STOPSIM uses the Eulerian integration method. Input stepfunction $U(t)$ and the speed-response function $v(t)$ of the dynamical model is shown in Fig. 4.


Fig. 4. Input and output functions of the dynamical model

## 3. Simulation Principles

The basic information concerning control process $U(t)$ is contained by the following $D \times D$ matrix and $D \times D$ matrix-valued time function

$$
\begin{equation*}
\mathbf{P}_{i}=\left[p_{i j}\right] \quad \text { and } \quad \mathbf{F}(t)=\left[F_{i j}(t)\right] \tag{7}
\end{equation*}
$$

The actual state transition is simulated on the basis of generation of a sequence of uniformly distributed pseudo random numbers. Let $\eta$ be a uniformly distributed random number in interval $[0,1]$. Let us assume that a transition should be simulated from the known state $i$ to the temporary unknown state $j$. After generating $\eta$ the unknown index $j$ assigning the new state after the state transition can be determined by checking relation

$$
\begin{equation*}
\sum_{k=0}^{j-1} p_{i k}<\eta \leq \sum_{k=0}^{j} p_{i k}, p_{i 0}=0 \tag{8}
\end{equation*}
$$

With the knowledge of the new state $j$, the duration $\tau$, in which control $U(t)$ takes value $u_{j}$ after state transition $i \rightarrow j$ can be simulated based on the conditional distribution function $F_{i j}(t)$ defined in relation (4). Function $F_{i j}(t)$ is assumed to be approximated over its range by a piece-wise linear continuous function of strictly monotonic property for each index pair $i, j$.

In this way, the inverse functions $F_{i j}^{-1}(t)$ always exist. Random duration $\tau$ in question is derived from a uniformly distributed random number $\eta$ as follows. It is obvious that the distribution function of random variable

$$
\begin{equation*}
r=F_{i j}^{-1}(\eta) \tag{9}
\end{equation*}
$$

is $F_{i j}(t)$, i.e. it is identical to the conditional distribution function which describes the duration in state $j$, after transition $i \rightarrow j$. Since $\eta$ can be generated by a standard subroutine, the appropriate $\tau$ realization can be computed.

The numerical treatment of the transition probability matrix $\Pi$ is obvious by using an appropriate two-dimensional array, while the non-zero elements of the matrix-valued time function $\boldsymbol{F}(t)$ are represented by sequences of co-ordinate pairs determining the break-points of the piece-wise linear conditional distribution functions $F_{i j}(t)$. It is to be noted that in case $p_{i j}=0$ also function $F_{i j}(t)$ is taken as an identically zero function. This property makes it possible to deal only with those index pairs for which $p_{i j}$ are non-zero. The derivation of duration $\tau$ from the uniformly distributed random number $\eta$ is visualized in Fig. 5. As it can be seen, functions $F_{i j}\left(\right.$. ) and $F_{i j}^{-1}($.$) can identically be treated by linear interpola-$ tion based on the number $L$ break-point co-ordinate pairs representing the conditional distribution function in question.


Fig. 5. Derivation of the duration $\tau$

## 4. Structure of the Data System

The data set consists of the following quantities:

1. Coefficients of the specific traction resistance of the vehicle/train:
$A F$, in unit of measure: $\frac{\mathrm{N} \mathrm{s}^{2}}{\mathrm{kN} \mathrm{m} \mathrm{m}^{2}}$,
$B F$, in unit of measure: $\frac{\mathrm{N} \mathrm{s}}{\mathrm{kN} \mathrm{m}}$,
$C F$, in unit of measure: $\frac{\mathrm{N}}{\mathrm{kN}}$.

Comment: In (1) we defined the resistance force. Coefficients $a, b$ and $c$ belong to a resistance force having a unit of measure N. For the practical computations it is more advantageous to use the specific resistance $f_{R}$ with respect to the unit weight $F_{G}$ of the vehicle. The unit of measure of $f_{R}$ is $\mathrm{N} / \mathrm{kN}$. The coefficients of $f_{R}\left(v,\left|\sum F\right|\right)$ will be $A F, B F$ and $C F$, instead of $a, b$ and $c$ in (1). If the mass of the vehicle/train is $m$ in unit kg , and the gravity acceleration is $g$ in unit $\mathrm{m} / \mathrm{s}^{2}$, then the following relationship holds:

$$
F_{R}\left(v,\left|\sum F\right|\right)=\frac{m g}{1000} f_{R}\left(v,\left|\sum F\right|\right)
$$

2. A small positive bound $E P S$, which is used as 'practical' zero, i.e. if $-E P S \leq x \leq E P S$ then $x$ is considered as zero. The order of magnitude of $E P S$ is less than $10^{-5}$.
3. The mass of the vehicle/train:
$T O M$, in unit of measure kg.
4. The rotating mass factor:
$G A M$, numerical ratio $\mathrm{kg} / \mathrm{kg}$.
5. Time step length of the numerical integration:
$D T$, in unit of measure s.
6. Braking force function $-F_{B}\left(\mathbf{u}^{*}, v\right)$ and tractive effort function $F_{D}(u, v)$ are unified into function $V F(U, v)$ after having introduced the unified control $U$ defined in (2).

Function $V F(U, v)$ is given by its substitutional values at different integer values of $U \in\{1,2, \ldots, D\}$ and in 11 fixed points of the speed axis. The first speed point is always the zero speed, while the last one is the top speed $v_{\max }$. The number of points in which function $V F(U, v)$ is determined is $11 \times D$. For the sake of clearer explanation consider the case $U=1$, when the most intensive braking effect is set in. The data required are as follows:

- control $U=1$
- sequence of non-negative speeds (11 points) in m/s:

$$
v_{1,1}=0, v_{1,2}, v_{1,3}, \ldots, v_{1,11}=v_{\max }
$$

- sequence of braking forces with negative sign in $k N$ :

$$
V F_{1,1}<0, V F_{1,2}<0, V F_{1,3}<0, \ldots, V F_{1,11}<0 .
$$

For control levels $U=2,3, \ldots, N$ the structure of the data is quite similar, the $V F$ values are of negative sign. In case of control level $U=N+1$ no braking and no tractive effort is applied. The special data required are the following:

- control $U=N+1$
- sequence of non-negative speeds (11 points) in $\mathrm{m} / \mathrm{s}$ :

$$
v_{N+1,1}=0, v_{N+1,2}, v_{N+1,3}, \ldots, v_{N+1,1 i}=v_{\max }
$$

- sequence of vanished braking and tractive efforts (zero sequence):

$$
V F_{N+1,1}=0, V F_{N+1,2}=0, V F_{N+1,3}=0, \ldots, V F_{N+1,11}=0
$$

For control levels $U=N+2, N+3, \ldots, N+M+1$ the exertion of nonnegative tractive effort is set. The structure of the data is similar again. The $V F$ values are of positive sign. For visualization in the case of $U=$ $N+M+1=D$, i.e. for the maximum tractive effort exertion the data required are the following:

- control $U=D$
- sequence of non-negative speeds (11 points) in $\mathrm{m} / \mathrm{s}$ :

$$
v_{D, 1}=0, v_{D, 2}, v_{D, 3}, \ldots, v_{D, 11}=v_{\max }
$$

- sequence of positive tractive efforts in kN :

$$
V F_{D, 1}>0, V F_{D, 2}>0, V F_{D, 3}>0, \ldots, V F_{D, 11}>0
$$

7. Transition probability matrix $\Pi$ is a stochastic matrix, i.e. the sum of the elements in an arbitrary row of $\Pi$ is equal to 1 . The elements $p_{i j}$ are read in due to the data format as follows:

| $p_{1,1}$ | $p_{1,2}$ | $\cdots$ | $p_{1, D}$ |
| :---: | :---: | :---: | :---: |
| $p_{2,1}$ | $p_{2,2}$ | $\cdots$ | $p_{2 . D}$ |
| $\vdots$ | $\vdots$ |  | $\vdots$ |
| $p_{D, 1}$ | $p_{D, 2}$ | $\cdots$ | $p_{D, D}$ |

8. Matrix valued time function of the dwelling time (duration-) conditional distribution functions. The functions $F_{i j}(t)$ are given by the ordinate sequence increasing strictly monotonously and belonging to a time-point sequence of $L=10$ points increasing strictly monotonicly as well. The data structure is like the following:
time points:

$$
t_{i j, 1}, t_{i j, 2}, \ldots, t_{i j, 10}
$$

$$
t_{i j, 1}=0 \text { and } t_{i j, .}>0 \text { for all } i j \text { pairs }
$$

$F_{i j}$ values:

$$
\begin{gathered}
F_{i j, 1}, F_{i j, 2}, \ldots, F_{i j, 10} \\
F_{i j, 1}=0 \text { and } F_{i j, 10}=1 \text { for all } i j \text { pairs. }
\end{gathered}
$$

Based on the two sequences linear interpolation is used, and in this way the conditional distribution functions are approximated by piece-wise linear functions. It is to be mentioned that for those index pairs, for which $p_{i j}=0$, the time-point and $F_{i j}$ sequences can be arbitrary, but they should obey the requirement of monotonic change and should satisfy the boundary conditions $\dot{t}_{i j, 1}=0, F_{i j, 1}=0$ and $F_{i j, 10}=1$.

## 3. Numerical Example

Program STOPSIM was applied to analyze the motion and loading conditions of a Diesel locomotive hauling a passenger train. The data used were the following:

1. Coefficients of the specific traction resistance vs. speed function of the train:

$$
A F=0.007 \mathrm{Ns}^{2} / \mathrm{kNm}^{2}, B F=0.0 \mathrm{Ns} / \mathrm{kNm}, C F=2.5 \mathrm{~N} / \mathrm{kN}
$$

2. A small positive bound: $E P S=10^{-5}$.
3. The mass of the train consisting of 1 locomotive and 5 passenger carriages: $T O M=260000.0 \mathrm{~kg}$.
4. The rotating mass factor: $G A M=0.08$.
5. Time step length of the numerical integration: $D T=0.5 \mathrm{~s}$.
6. Braking force and tractive effort functions vs. speed $v$ and control $U$ can be seen in Fig. 6. The number of states of the unified control variable $U: D=N+M+1=2+12+1=15$.
7. Transition probability matrix $\Pi$ is specified in Table 1. The numerical values of the matrix-elements were evaluated from an operational


Fig. 6. Tractive/braking force functions


Fig. 7. Simulated control process
measurement carried on between Budapest and Esztergom on a Diesel locomotive M41 which hauled a train consisting of 5 carriages.


Fig. 8. Simulated speed response process


Fig. 9. Probability distribution of the unified control process
8. The matrix valued time function of the dwelling time (duration) conditional distribution functions will be specified only for those index


Fig. 10. Probability density function of the speed response
pairs, for which the transition probabilities don't vanish. The strictly monotonic time-point and conditional probability sequences of length $L=$ 10 are specified in Tables $2 a$ and $2 b$. The numerical values of the sequences were evaluated from the operational measurement results mentioned above.

The simulated realization functions of the unified control process $U(t)$ and speed response process $v(t)$ of the dynamical system can be seen in Figs. 7 and 8. Both diagrams are defined over a time interval of 1000 s . A number of test computations shows that the stability of the relative frequencies reflecting the on-level and inter-level dwelling duration resp. the realization functions appears in a satisfactory measure, if the time domain of the simulation exceeds the limit of 70000 s, i.e. it reflects about 20 hours operation time. This requirement can be satisfied by software STOPSIM at a computer-duration about 20 minutes on an AT386 configuration having arithmetical co-processor.

The evaluated probability distribution of the control process can be seen in Fig. 9, whilst the probability density function of the speed response of the train-defined dynamical system is visualized in Fig. 10.

Table 1

| 0.0 | 0.2 | 0.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.8 | 0.0 | 0.2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.5 | 0.0 | 0.0 | 0.0 | 0.0233 | 0.0465 | 0.0697 | 0.1396 | 0.1628 | 0.0582 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.9 | 0.1000 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.7368 | 0.1053 | 0.0 .526 | 0.0 | 0.0 | 0.0526 | 0.0 .526 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.4286 | 0.0 | 0.3333 | 0.0476 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1905 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.2759 | 0.0 | 0.0690 | 0.4138 | 0.10 .34 | 0.0 | 0.0 | 0.1034 | 0.0345 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1429 | 0.4643 | 0.1429 | 0.0 | 0.0357 | 0.1786 | 0.0 | 0.0 | 0.0357 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0476 | 0.8095 | 0.1429 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0588 | 0.7059 | 0.0588 | 0.0 | 0.1176 | 0.0588 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.5000 | 0.0 | 0.0 | 0.2500 | 0.2500 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.5000 | 0.2500 | 0.0 | 0.2500 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.3333 | 0.0 | 0.0 | 0.6667 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 | 0.0 | 0.0 |

Table 2/a

| t 12 | 0.0 | 6.0 | 12.0 | 18.0 | 24.0 | 36.0 | 42.0 | 48.0 | 54.0 | 60.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F12 | 0.0 | 0.03 | 0.08 | 0.18 | 0.38 | 0.62 | 0.82 | 0.92 | 0.97 | 1.0 |
| t 13 | 0.0 | 6.0 | 12.0 | 18.0 | 24.0 | 36.0 | 42.0 | 48.0 | 54.0 | 60.0 |
| F13 | 0.0 | 0.03 | 0.08 | 0.18 | 0.38 | 0.62 | 0.82 | 0.92 | 0.97 | 1.0 |
| t 21 | 0.0 | 6.0 | 12.0 | 18.0 | 24.0 | 36.0 | 42.0 | 48.0 | 54.0 | 60.0 |
| F21 | 0.0 | 0.03 | 0.08 | 0.18 | 0.38 | 0.62 | 0.82 | 0.92 | 0.97 | 1.0 |
| t 23 | 0.0 | 6.0 | 12.0 | 18.0 | 24.0 | 36.0 | 42.0 | 48.0 | 54.0 | 60.0 |
| F23 | 0.0 | 0.03 | 0.08 | 0.18 | 0.38 | 0.62 | 0.82 | 0.92 | 0.97 | 1.0 |
| t 32 | 0.0 | 6.0 | 12.0 | 18.0 | 24.0 | 36.0 | 42.0 | 48.0 | 54.0 | 60.0 |
| F32 | 0.0 | 0.03 | 0.08 | 0.18 | 0.38 | 0.62 | 0.82 | 0.92 | 0.97 | 1.0 |
| t 36 | 0.0 | 0.8 | 1.2 | 1.6 | 2.0 | 20.0 | 38.0 | 56.0 | 74.0 | 92.0 |
| F36 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| ¢ 37 | 0.0 | 8.8 | 12.0 | 14.0 | 16.0 | 21.2 | 26.4 | 46.6 | 81.8 | 117.0 |
| F37 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 38 | 0.0 | 10.0 | 16.0 | 19.6 | 22.0 | 24.0 | 28.0 | 46.0 | 202.0 | 427.0 |
| F38 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 39 | 0.0 | 18.9 | 29.4 | 36.0 | 38.9 | 67.6 | 79.6 | 82.6 | 135.8 | 165.0 |
| F39 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| ¢ 310 | 0.0 | 36.2 | 42.2 | 43.0 | 57.0 | 62.4 | 68.4 | 70.0 | 100.0 | 161.0 |
| F310 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 311 | 0.0 | 2.0 | 3.5 | 5.0 | 8.0 | 11.0 | 38.0 | 65.0 | 68.0 | 71.0 |
| F311 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 43 | 0.0 | 1.5 | 2.25 | 52.0 | 126.2 | 200.5 | 256.3 | 275.2 | 294.1 | 313.0 |
| F43 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 53 | 0.0 | 8.9 | 47.5 | 69.4 | 82.0 | 123.0 | 174.0 | 186.2 | 224.0 | 416.0 |
| F53 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 54 | 0.0 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 |
| F54 | 0.0 | 0.2 | 0.3 | $0 . \div$ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 62 | 0.0 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 |
| F62 | 0.0 | 0.999 | 0.9991 | 0.9992 | 0.9993 | 0.9994 | 0.9995 | 0.9996 | 0.9998 | 1.0 |
| t 63 | 0.0 | 27.7 | 39.0 | 61.2 | 91.0 | 115.0 | 131.4 | 157.8 | 181.8 | 210.0 |
| F63 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 64 | 0.0 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| F64 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 67 | 0.0 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 |
| F6i | 0.0 | 0.999 | 0.9991 | 0.9992 | 0.9993 | $0.999 \pm$ | 0.9995 | 0.9996 | 0.9998 | 1.0 |
| t 68 | 0.0 | 4.2 | 6.3 | 8.4 | 10.5 | 12.6 | 14.7 | 16.8 | 18.9 | 21.0 |
| F68 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 73 | 0.0 | 12.0 | 23.8 | 32.2 | 36.5 | 54.8 | 108.8 | 180.4 | 244.8 | 702.0 |
| F73 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $t 75$ | 0.0 | 1.7 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 3.9 | 60.0 |
| F75 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 76 | 0.0 | 2.7 | 4.05 | 5.4 | 6.75 | 8.1 | 9.45 | 10.8 | 12.15 | 13.5 |
| F76 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $\pm 711$ | 0.0 | 18.4 | 24.4 | 27.2 | 30.0 | 32.8 | 35.6 | 41.6 | 50.8 | 60.0 |
| F711 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 83 | 0.0 | 106.6 | 140.2 | 173.6 | 192.0 | 206.4 | 234.9 | 280.9 | 353.6 | 468.0 |
| F83 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 85 | 0.0 | 0.04 | 0.06 | 0.08 | 0.1 | 0.38 | 0.66 | 0.94 | 1.22 | 1.5 |
| F85 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 86 | 0.0 | 2.0 | 2.1 | 2.3 | 2.4 | 3.6 | 6.4 | 10.6 | 89.4 | 183.0 |
| F86 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| $t 87$ | 0.0 | 0.9 | 1.35 | 1.42 | 1.5 | 1.65 | 2.25 | 4.5 | 6.75 | 9.0 |
| F87 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 810 | 0.0 | 12.6 | 18.9 | 29.2 | 41.5 | 53.8 | 65.4 | 75.6 | 85.8 | 96.0 |
| F810 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 811 | 0.0 | 12.8 | 19.2 | 25.6 | 32.0 | 38.4 | 44.8 | 51.2 | 57.6 | 64.0 |
| F811 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |

Table 2/b

| t 96 | 0.0 | 1.2 | 1.51 | 1.52 | 1.53 | 1.54 | 1.55 | 1.6 | 1.8 | 2.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F96 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 97 | 0.0 | 1.8 | 2.01 | 2.02 | 2.03 | 2.04 | 2.05 | 2.5 | 8.45 | 32.0 |
| F97 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 98 | 0.0 | 0.8 | 1.4 | 2.2 | 3.0 | 4.4 | 5.8 | 9.0 | 14.0 | 19.0 |
| F98 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 910 | 0.0 | 6.8 | 10.2 | 13.6 | 17.0 | 20.4 | 23.8 | 27.2 | 30.6 | 34.0 |
| F910 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 911 | 0.0 | 1.0 | 23.0 | 45.0 | 59.5 | 74.0 | 91.5 | 109.0 | 121.5 | 134.0 |
| F911 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 914 | 0.0 | 2.4 | 3.6 | 4.8 | 6.0 | 7.2 | 8.4 | 9.6 | 10.8 | 12.0 |
| F914 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| Ł 107 | 0.0 | 0.3 | 0.45 | 0.6 | 0.75 | 0.9 | 1.05 | 1.2 | 1.35 | 1.5 |
| F107 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| ¢ 108 | 0.0 | 2.0 | 2.1 | 2.2 | 2.25 | 3.2 | 4.9 | 5.0 | 10.0 | 70.0 |
| F108 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| ¢ 109 | 0.0 | 0.6 | 0.9 | 6.7 | 15.25 | 23.8 | 31.95 | 39.3 | 46.65 | 54.0 |
| F109 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 118 | 0.0 | 0.5 | 0.75 | 1.0 | 1.25 | 1.5 | 1.75 | 2.0 | 2.25 | 2.5 |
| F118 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 119 | 0.0 | 1.7 | 2.0 | 2.4 | 3.0 | 4.0 | 19.6 | 41.8 | 87.4 | 123.0 |
| F119 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1110 | 0.0 | 29.6 | 59.2 | 88.8 | 118.4 | 148.0 | 177.6 | 236.8 | 266.4 | 296.0 |
| F1110 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t1112 | 0.0 | 7.8 | 11.7 | 15.6 | 19.5 | 26.4 | 33.3 | 40.2 | 47.1 | 54.0 |
| F1112 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1113 | 0.0 | 8.2 | 12.3 | 16.4 | 20.5 | 24.6 | 28.7 | 32.8 | 36.9 | 41.0 |
| F1113 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1210 | 0.0 | 1.6 | 2.4 | 3.2 | 4.0 | 18.2 | 32.4 | 46.6 | 60.8 | 75.0 |
| F1210 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1213 | 0.0 | 9.0 | 13.5 | 18.0 | 22.5 | 27.0 | 31.5 | 36.0 | 40.5 | 45.0 |
| F1213 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1214 | 0.0 | 19.0 | 28.5 | 38.0 | 47.5 | 57.0 | 66.5 | 76.0 | 85.5 | 95.0 |
| F1214 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1311 | 0.0 | 22.4 | 33.6 | 44.8 | 56.0 | 66.2 | 76.4 | 86.6 | 96.9 | 106.0 |
| F1311 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1312 | 0.0 | 11.8 | 17.7 | 23.6 | 29.5 | 35.4 | 41.3 | 47.2 | 53.1 | 59.0 |
| F1312 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1314 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| F1314 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1412 | 0.0 | 14.2 | 21.3 | 28.4 | 35.5 | 42.6 | 49.7 | 56.8 | 63.9 | 71.0 |
| F1412 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1415 | 0.0 | 22.8 | 3-2 | 45.6 | 57.0 | 64.8 | 72.6 | 80.4 | 88.2 | 96.0 |
| F1415 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| t 1513 | 0.0 | 2.4 | 3.6 | 4.8 | 6.0 | 16.6 | 27.2 | 37.8 | 48.4 | 59.0 |
| F1513 | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |

## 6. Concluding Remarks

The stochastic simulation procedure realised by software STOPSIM gives a powerful tool to the vehicle designers for predicting the operational conditions of the vehicles to be designed. The information required to create the transition probability matrix and the matrix of the conditional probability distribution functions can be achieved either by the evaluation of operational measurements made on similar systems working under similar service conditions or by appropriate deterministic computer procedures, which can simulate the real-time operation of the vehicle from the computer keyboard. Authors are intensively working on the creation of the deterministic real-time simulation procedure mentioned, which will be apt to generate input data for STOPSIM. The software realizing the real-time simulation will be described in a subsequent publication.

## References

1. Strömer, H. (1969): Zur Theorie der Zuverlässigkeit komplexer Systeme. Megbízhatóságelméleti kollokvium. (Colloquium on Reliability Theory) Tihany, 1969. September 16-19.
2. Tyihonov, V.I. - Mironov, M.A. (1977): Markovszkie processzü. Moszkva, 'Szovjetszkoje Ragyio'.
3. Horváth, S. - Keresztes, A. - Michelberger, P. - Szeidl, L. (1981): Preliminary Load Analysis of Commercial Vehicles. Periodica Polytechnica (Transportation Engineering) Vol. 9. No 2. pp. 151-163.
4. Horváth, S. - Keresztes, A. - Michelberger, P. - Szeidl, L. (1982): Mathematical Model of the Load and Stress Statistics of Vehicle Structures. Journal of Applied Mathematical Modelling. Vol. 6, pp. 92-96.
5. Zobory, I. (1993): Stochasticity in Vehicle System Dynamics. Proceedings of the 1 st Mini Conference on Vehicle System Dynamics, Identification and Anomalies. Held at The Technical University of Budapest, 14-16 of November 1988. Issued in 1993 by the TU of Budapest, pp. 32-41.
6. Zobory I. (1990): Hajtott forgóváz üzemi terhelési viszonyainak elörejelzése. Jármüvek, Mezögazdasági Gépek, Vol. 37, No. 10. pp. 373-376.
7. Michelberger,P. - Zobory,I. (1990): Operation Loading Conditions of Ground Vehicles - Analysis of Load History. Proceedings ASME Winter Annual Meeting, Dallas, New York, pp. 175-182.
8. Zobory I. (1992): Vasúti vontatójármüvek lengésmentes fömozgásfolyamatának statisztikai dinamikai vizsgálata a hajtásrendszer kvázistatikus terhelési viszonyainak feltárására. Kutatási Jelentés. BME JGI Vasúti Jármüvek Tanszék. Budapest.
9. Zobory I. (1992): Vasúti vontatójármüvek lengésképes hajtásrendszerében kialakuló terhelési viszonyok elörejelzése dinamikai modellre alapozott sztochasztikus szimulációval. Kutatási Jelentés. BME JGI Vasúti Jármüvek Tanszék. Budapest.
10. Zobory I. (1992): Vasúti vontatójármüvek hajtásdinamikája. Doktori tézises összefoglaló. Budapest.
11. Zobory I. (1993): Vasúti vontatójármüvek mozgás- és terhelésfolyamatának sztochasztikus szimulációja. Közlekedéstudományi Szemle, Vol. XLII. No. 1. pp. 19-29.
