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DYNAMICAL STABILITY OF A ROPE WITH SLOW VARIABILITY OF THE PARAMETERS

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Abstract

The longitudinal-transversal vibrations of a rope with varying length are considered here. The analysis of parametric resonances is the primary purpose of this paper. The dynamic state of the investigated system is described by a non-linear set of partial differential equations with boundary conditions varying with time. The physical non-linearity and damping properties of the rope material as well as dry friction between flakes are taken into account. The determination of the unstable regions by balance harmonic method for the main, secondary and combination resonances has been performed. The spatial diagrams of the regions of instability and their cross-sections are presented. The influence of the physical non-linearity and the character of the kinematic excitation are considered. The starting and braking of the winding machine is taken into consideration.

Keywords: dynamic systems, parameter study.

1. Introduction

The problem of stability of a rope has been studied for more then 20 years [3]. The early research on the subject by O. A. GOROSZKO and G. H. SAWIN is discussed in [3]. Considerable progress in this field was made due to S. MARCZYK and J. NIZIOL [2] and P. I. ULSZIN [4]. All works mentioned consider the longitudinal-transversal vibrations of a lifting rope only for a linear physical model and uniform motion of the drum. Studies on the subject connected with stability are concentrated only on determining the unstable region for the main resonance and for the first mode of the transversal oscillations. One can see that the wider analysis is necessary. The primary purpose of the work reported in the presented paper is, thus, the analysis of parametric resonance in the case of a non-linear physical model.

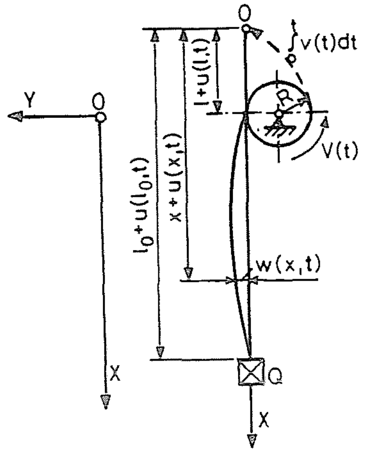

Fig. 1. General system model

2. Formulation of the Problem

The investigated system consists of a drum rotating with a circumferential velocity $v(t)$, on which a steel rope loaded by load Q is reeled, cf. *Fig. 1.*

The analysis of small longitudinal vibrations of such a system was presented in [1].

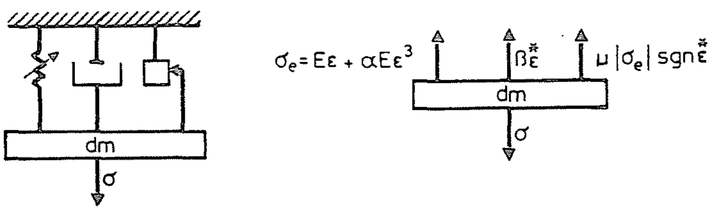

Fig. 2. Physical model

In the present paper, the following simplifications and assumptions are adopted:

- the longitudinal-transversal vibrations are small,
- the rope material is homogeneous,
- the rope material is physically non-linear,
- the internal viscous damping of the material is according to the Voigth–Kelvin model,

- dry friction exists between the particular rope flakes and the stress resulting from dry friction forces is proportional to the absolute value of non-linear elastic stress in the rope,
- the flexural stiffness is disregarded,
- the drum is perfectly rigid,
- the slip of the rope on the drum is neglected,
- load Q is treated as a point load with one degree of freedom which can move along the Ox axis only,
- the kinematic excitation consists of the starting, uniform motion and braking of the winding machine.

All considerations are done in a movable reference system Oxy, which starts from the point where the rope is fixed on the drum. The total length of the undeformed rope is l_0 and its part reeled on the drum is $l(t)$. Functions $u(x,t)$ and $w(x,t)$ describe longitudinal and transversal displacements, respectively.

The assumed physical model is presented in *Fig. 2.*

The stress in the rope is described by the following equation

$$\sigma(x,t) = E\varepsilon + \alpha E\varepsilon^3 + \beta\dot{\varepsilon} + \mu|\sigma_e|\operatorname{sgn}\dot{\varepsilon}\,, \tag{2.1}$$

where

$$\sigma_\varepsilon = E\varepsilon + \alpha E\varepsilon^3\,. \tag{2.2}$$

The internal force has the form

$$P(x,t) = AE\frac{\partial u}{\partial x} + \alpha AE\left(\frac{\partial u}{\partial x}\right)^3 + \beta A\frac{\partial^2 u}{\partial x\partial t}$$

$$+\mu\left|AE\frac{\partial u}{\partial x} + \alpha AE\left(\frac{\partial u}{\partial x}\right)^3\right|\operatorname{sgn}\frac{\partial^2 u}{\partial x\partial t}\,, \tag{2.3}$$

where the following notation is used:

E – Young's modulus,
A – the cross-sectional area,
$\dot{\varepsilon}$ – the strain rate,
AE – the longitudinal rigidity of the rope,
α – the coefficient of physical non-linearity,
β – the coefficient of viscous damping,
μ – the overall coefficient of dry friction due to geometry and structure of the rope,
σ_e – the non-linear elastic stress.

Since damping in the rope is complex and non-linear, we examine the equivalent viscous damping, which was widely discussed in [1].

Introduce dimensionless variables and constants:

$$
\begin{aligned}
&x = \xi l_0 , && \kappa = \frac{v}{l_0 p_0} , && P^* = \frac{P}{q l_0} , && \lambda = \frac{3\alpha g^2 l_0^2}{c^4} , \\
&\tau = \frac{ct}{l_0} , && l^* = l l_0 , && Q^* = \frac{Q}{q l_0} , && T^* = \frac{cT}{l_0} , \\
&b_1 = \frac{EAgl_0}{Qc^2} , && b^2 = \frac{EAg}{qc^2} , && b_3 = \frac{g l_0}{c^2} , && p_0^* = \frac{p_0 l_0}{c} , \\
&u^* = \frac{uc^2}{g l_0^2} , && w^* = \frac{wc^2}{g l_0^2} , && \beta^* = \frac{c\beta}{E l_0} , && \beta_z^* = \frac{c\beta_z}{E l_0} ,
\end{aligned}
\tag{2.4}
$$

where $u^*(\xi,\tau)$ and $\omega^*(\xi,\tau)$ denote the dimensionless longitudinal and transversal displacements respectively, T^* and p_0^* are the period and frequency of the basic mode of longitudinal vibrations, q is the specific weight, c is the velocity of longitudinal wave propagation, g the acceleration of gravity, λ the dimensionless coefficient of physical non-linearity, β_z^* the equivalent viscous damping coefficient, κ the coefficient of slow variability of the length of the rope.

3. Analysis of the Equations of Motion

Applying d'Alembert's law gives the following set of equations of motion

$$
\begin{gathered}
\frac{\partial^2 u^*}{\partial \tau^2} - b^2 \left\{ \left[1 + \lambda \left(\frac{\partial u^*}{\partial \xi} \right)^2 \right] \frac{\partial^2 u^*}{\partial \xi^2} + c_z^* \frac{\partial^3 u^*}{\partial \xi^2 \partial \tau} \right\} = 1 + \frac{\mathrm{d} v^*}{\mathrm{d}\tau} , \\
\frac{\partial^2 \omega^*}{\partial \tau^2} - b_3 \frac{\partial}{\partial \xi} \left(P^* \frac{\partial \omega^*}{\partial \xi} \right) = 0 ,
\end{gathered}
\tag{3.1}
$$

where

$$
\begin{aligned}
c_z^* &= \beta^* + \beta_z^* , \\
P^* &= b^2 \left\{ \left[1 + \frac{\lambda}{3} \left(\frac{\partial u^*}{\partial \xi} \right)^2 \right] \frac{\partial u^*}{\partial \xi} + c_z^* \frac{\partial^2 u}{\partial \xi \partial \tau} \right\} .
\end{aligned}
\tag{3.2}
$$

The boundary conditions are:

for the lower end $\xi = 1$

$$
\frac{\partial^2 u^*}{\partial \tau^2}\Big|_{\xi=1} + b_1 \left\{ \left[1 + \frac{\lambda}{3} \left(\frac{\partial u^*}{\partial \xi} \right)^2 \right] \frac{\partial u^*}{\partial \xi} + c_z^* \frac{\partial^2 u^*}{\partial \xi \partial \tau} \right\}\Big|_{\xi=1} = 1 + \frac{\mathrm{d} v^*}{\mathrm{d}\tau},
$$

$$
w^*(1,\tau) = 0,
$$

and for the upper end $\xi = l^*(\tau)$

$$u^*(l^*,\tau) = \int_0^\tau \frac{\partial u^*(l^*,\tau)}{\partial \xi}\Big|_{\xi=l^*(\tau)} \frac{\mathrm{d}l^*}{\mathrm{d}\tau}\mathrm{d}\tau,$$

$$w^*(l^*,\tau) = 0. \tag{3.3}$$

The initial conditions are written in a general form:

$$\begin{aligned} u^*(\xi,0) &= \varphi_1^*(\xi),\\ w^*(\xi,0) &= \varphi_3^*(\xi),\\ \frac{\partial u^*(\xi,\tau)}{\partial \tau}\Big|_{\tau=0} &= \varphi_2^*(\xi),\\ \frac{\partial w^*(\xi,\tau)}{\partial \tau}\Big|_{\tau=0} &= \varphi_4^*(\xi), \end{aligned} \tag{3.4}$$

where $\varphi_i^*(\xi)$, $i = 1,2,3,4$ are given functions.

The formula of the change of length of the ropes has the form

$$l^*(\tau) = b_3 \int_0^\tau v^*(\tau)\mathrm{d}\tau\,. \tag{3.5}$$

The fact that during one period of oscillations the length change is insignificant is of primary importance.

The solution of the first equation from (3.1), that is the one for the longitudinal vibrations, was discussed and presented in [1].

Because of the slowly-varying character of function $l^*(\tau)$, its solution was obtained by means of the Galerkin and Bogolubow–Krylov–Mitropolski methods.

For the basic mode of oscillations, the longitudinal displacement has the following form

$$u_0^*(\xi,\tau) = a_0^*(\tau)\sin\beta_1\frac{\xi - l^*}{1-l^*}\cos\varphi_0^* + (\xi - l^*)\left(1 + \frac{\mathrm{d}v^*}{\mathrm{d}\tau}\right)\left(\frac{1}{b_1} + \frac{2-\xi-l^*}{2b^2}\right)$$

$$+ \int_0^\tau \left[\beta_1\frac{a_0^*(\tau)}{1-l^*}\cos\varphi_0^* + \left(1+\frac{\mathrm{d}v^*}{\mathrm{d}\tau}\right)\left(\frac{1}{b_1} + \frac{1-l^*}{b^2}\right)\right]\frac{\mathrm{d}l^*}{\mathrm{d}\tau}\mathrm{d}\tau\,, \tag{3.6}$$

where a_0^*, φ_0^* are the slowly-varying amplitudes and phases for the basic mode of longitudinal oscillations, β_1 is the first eigenvalue.

When considering the equation of the transversal vibrations, the solution is sought in the form

$$w^*(\xi,\tau) = \sum_{m=1}^{N} Z_m(\xi,\tau)\Omega_m(\tau)\,, \tag{3.7}$$

where $Z_m(\xi, l^*)$ are slowly-varying eigenmodes of the transversal oscillations. They are selected in the from

$$Z_m(\xi, l^*) = \sin\frac{m\pi(\xi - l^*)}{1 - l^*}\,. \tag{3.8}$$

These functions satisfy the boundary conditions (3.3).

By means of the Galerkin method, one obtains the following set of N equations:

$$\sum_{m=1}^{N} A_{mn}\ddot{\Omega}_m + \sum_{m=1}^{N} B_{mn}\dot{\Omega}_m + \sum_{m=1}^{N} C_{mn}\Omega_m = 0\,, \tag{3.9}$$

where

$$\begin{aligned}
A_{mn} &= \int\limits_{l^*(\tau)}^{1} Z_m(\xi,l^*)Z_n(\xi,l^*)\mathrm{d}\xi\,,\\
B_{mn} &= 2\int\limits_{l^*(\tau)}^{1} \frac{\mathrm{d}l^*}{\mathrm{d}(\tau)}\frac{\partial Z_m(\xi,l^*)}{\partial l^*}Z_n(\xi,l^*)\mathrm{d}\xi\,,\\
C_{mn} &= \int\limits_{l^*(\tau)}^{1}\left\{\frac{\mathrm{d}^2 l^*}{\mathrm{d}\tau^2}\frac{\partial Z_m(\xi,\tau)}{\partial l^*}Z_n(\xi,l^*) + \left(\frac{\mathrm{d}l^*}{\mathrm{d}\tau}\right)^2\frac{\partial^2 Z_m(\xi,\tau)}{\partial l^{*2}}Z_n(\xi,l^*)\right.\\
&\quad\left. - b_3\frac{\partial}{\partial\xi}\left[P^*(\xi,\tau)\frac{\partial Z_m(\xi,\tau)}{\partial\xi}\right]Z_n(\xi,l^*)\right\}d\xi.
\end{aligned} \tag{3.10}$$

Taking into account (3.8), we obtain

$$\begin{aligned}
A_{mn} &= \begin{cases} 0 & \text{for } n \neq m\,,\\ \frac{1-l^*}{2} & \text{for } n = m\,,\end{cases}\\
B_{mn} &= \begin{cases} \frac{-2nm}{n^2-m^2}\frac{\mathrm{d}l^*}{\mathrm{d}\tau} & \text{for } n \neq m,\\ -\frac{1}{2}\frac{\mathrm{d}l^*}{\mathrm{d}\tau} & \text{for } n = m.\end{cases}
\end{aligned} \tag{3.11}$$

Due to complexity of C_{mn}, their values are not explicitly given here. It was already done in [1].

Each of the equations of the set (3.9) can be written in the form of Hill s equation.

$$\ddot{\Omega}_n(\tau) + H_n(\tau)\left[1 + H_{on}\cos\left(\varphi_k^* + \Theta_n\right)\right]\Omega_n(\tau) =$$
$$= \sum_{m\neq n}^{N} G_{mn}(\tau)\left[1 + G_{onm}\cos\left(\varphi_k^* + \delta_{mn}\right)\right]\Omega_m(\tau)$$
$$- \left[\frac{1}{2}h_{2n}\cos 2\varphi_k^* + \frac{1}{4}h_{3n}\cos 3\varphi_k^*\right]\Omega_n(\tau)$$
$$+ \sum_{m\neq n}^{N}\left[\frac{1}{2}g_{2nm}\cos 2\varphi_k^* + \frac{1}{4}g_{3nm}\cos 3\varphi_k^*\right]\Omega_m(\tau)$$
$$+ \frac{\dot{l}^*}{1-l^*}\dot{\Omega}_n(\tau) + \sum_{m\neq n}^{N}\frac{4nm}{n^2-m^2}\frac{\dot{l}^*}{1-l^*}\dot{\Omega}_m(\tau)\,. \tag{3.12}$$

The set of equations (3.9) can be written in the matrix from

$$\ddot{\Omega} + B\dot{\Omega} + C\Omega = 0\,, \tag{3.13}$$

where $\ddot{\Omega}$, $\dot{\Omega}$, Ω are one column matrices.

The analysis of the properties of the elements of the B matrix was done in [1], and it turned out that the equations (3.9) are coupled due to the 'giroscopic' character of the elements of the B matrix for $i \neq j$.

When one analyzes the properties of the elements of matrix C, one comes to the conclusion that the *Eqs.* (3.9) are coupled not only due to the elements of the B matrix but of the C matrix as well.

The most important is that the considerations of the set (3.9) cannot be limited only to the analysis of the nth equation as it has been done till now [4].

In the analyzed problem, the solution of the *Eqs.* (3.9) is not of such importance as the determination of the regions of dynamic instability.

Because of the complexity of the set (3.9), we have chosen the set of only two equations for the analysis that follows.

It can be written in the form:

$$\ddot{\Omega}_1 + \omega_1^2\left[1 + c_{11}\cos\varphi_k^* + s_{11}\sin\varphi_k^* + c_{12}\cos 2\varphi_k^* + c_{13}\cos 3\varphi_k^*\right]\Omega_1$$
$$-\omega_{12}^2\left[1 + \hat{c}_{12}\cos\varphi_k^* + \hat{s}_{21}\sin\varphi_k^* + c_{22}\cos 2\varphi_k^* + c_{23}\cos 3\varphi_k^*\right]\Omega_2$$
$$+b_{11}\dot{\Omega}_1 + b_{12}\dot{\Omega}_2 = 0\,,$$
$$\ddot{\Omega}_2 + \omega_2^2\left[1 + c_{21}\cos\varphi_k^* + s_{21}\sin\varphi_k^* + c_{22}\cos 2\varphi_k^* + c_{23}\cos 3\varphi_k^*\right]\Omega_2$$
$$-\omega_{21}^2\left[1 + \hat{c}_{11}\cos\varphi_k^* + \hat{s}_{11}\sin\varphi_k^* + c_{12}\cos 2\varphi_k^* + c_{13}\cos 3\varphi_k^*\right]\Omega_1$$
$$+b_{11}\dot{\Omega}_2 + b_{12}\dot{\Omega}_1 = 0\,. \tag{3.14}$$

4. Analysis of Dynamical Instability

The value of the internal force P^* resulting from the longitudinal vibrations couples 'parametrically' the transversal and longitudinal vibrations. Physically this means that for some values of parameters of the rope, the longitudinal vibrations can excite the transversal vibrations with an increasing amplitude. From this it follows that there is a probability and a danger of the occurrence of parametric resonance.

In this chapter, we are looking for the answer to the question:

What are the values of frequencies and amplitudes of the longitudinal vibrations for given parameters of the rope which cause the dynamic instability of the system?

Due to the complexity of the considered set (3.14), the harmonic balance method has been chosen for determining the boundaries of the parametric resonances.

For the analysis of this problem, we have adopted the following method: amplitudes a_0^* of the longitudinal vibrations are treated as changeable and for each of their values the interval for frequency p_0^* is sought.

By considering the main parametric resonance, the solution to the boundaries of the unstable regions is sought in the form:

$$\Omega_n = A_{n1} \sin 0.5\varphi_k^* + B_{n1} \cos 0.5\varphi_k^* + A_{n3} \sin 1.5\varphi_k^* + B_{n3} \cos 1.5\varphi_k^* ,$$
$$n = 1, 2. \tag{4.1}$$

where k is the number of the mode of longitudinal vibrations.

We assumed that

$$\frac{\mathrm{d}\varphi_k^*}{\mathrm{d}\tau} = p^*(\tau) , \qquad \frac{\mathrm{d}^2\varphi_k^*}{\mathrm{d}\tau^2} = 0 . \tag{4.2}$$

For the case when each of Ω_n consists of only one harmonic mode, the solution is as follows

$$\Omega_n = A_{n1} \sin 0.5\varphi_k^* + B_{n1} \cos \varphi_k^* , \qquad n = 1, 2. \tag{4.3}$$

In both cases, we obtain a set of homogeneous equations. The condition for the existence of a nonzero solution leads to the determination of the sought regions for the first and the second modes of transversal vibrations. We also examine the possibility of occurrence of the third mode.

If we consider the secondary parametric resonance, the solution sought has the form

$$\Omega_n = 0.5b_{0n} + A_{n1} \sin \varphi_k^* + B_{n1} \cos \varphi_k^* + A_{n2} \sin 2\varphi_k^* + B_{n2} \cos 2\varphi_k^* ,$$

$$n = 1, 2. \tag{4.4}$$

When limiting the analysis to the first approximation

$$\Omega_n = 0.5b_{0n} + A_{n1} \sin \varphi_k^* + B_{n1} \cos \varphi_k^* , \qquad n = 1, 2, \tag{4.5}$$

the boundaries of the instability zone are described by the following condition

$$p^{*2} = \omega_i^2 - 0.5 \left[0.5\omega_i^2 \left(c_{i1}^2 + s_{i1}^2 \right) + b_{11}^2 \right] \pm 0.5\sqrt{\Delta_i} , \tag{4.6}$$

where

$$\Delta_i = \left[o.5\omega_i^2 \left(c_{i1}^2 + s_{i1}^2 \right) + b_{11}^2 \right] - 4 \left[0.25\omega_i^4 \left(c_{i1}^2 - s_{i1}^2 - c_{i2} \right) c_{i2} + b_{11}^2 \omega_i^2 \right] > 0 \tag{4.7}$$

for $i = 1, 2$, as we consider only the first and the second modes.

A vibrating non-linear system with many degrees of freedom is rich in many kinds of resonances. Because of the particular type of coupling taken into account here, periodic combination resonance is particularly interesting.

In our considerations, the following assumption is adopted:

$$p^* = p_1^* + p_2^* , \tag{4.8}$$

where p_1^*, p_2^* are the frequencies of the two components of the solution sought

$$\Omega_n = A_{n1} \sin \psi_1^* + B_{n1} \cos \psi_1^* + A_{n3} \sin \psi_2^* + B_{n3} \cos \psi_2^* , \qquad n = 1, 2. \tag{4.9}$$

The following notation is used

$$\frac{\mathrm{d}\psi_1^*}{\mathrm{d}\tau} = p_1^* , \qquad \frac{\mathrm{d}\psi_2^*}{\mathrm{d}\tau} = p_2^* , \tag{4.10}$$

and the following assumptions are adopted

$$\frac{\mathrm{d}^2\psi_1^*}{\mathrm{d}\tau^2} = 0 , \qquad \frac{\mathrm{d}^2\psi_2^*}{\mathrm{d}\tau^2} = 0 , \tag{4.11}$$

The analysis was done for both, the case when two components are excited and for the case as follows

$$\Omega_n = A_{n1} \sin \psi_n^* + B_{n1} \cos \psi_n^* , \qquad n = 1, 2. \tag{4.12}$$

By using the balance harmonic method, the set of equations was obtained

$$\begin{gathered} 4 \left[\left(-p_1^{*2} + \omega_1^2 \right) \left(-p_2^{*2} + \omega_2^2 \right) + p_1^* p_2^* b_{11}^2 \right] = \omega_{12}^2 \omega_{21}^2 \sqrt{\left(\hat{c}_{11}^2 + \hat{s}_{11}^2 \right) \left(\hat{c}_{21}^2 + \hat{s}_{21}^2 \right)} , \\ \left(-p_1^{*2} + \omega_1^2 \right) p_2^* = \left(-p_2^{*2} + \omega_2^2 \right) p_1^* . \end{gathered} \tag{4.13}$$

The condition (4.8) and the equations (4.13) determine the regions of the combination resonance.

5. Computations

For the numerical calculations, the following values of parameters have been chosen:

$$Q = 2 \cdot 10^5\,[\mathrm{N}]\,, \qquad EA = 2 \cdot 10^8\,[\mathrm{N}]\,,$$
$$\frac{\beta}{E} = 0.005\,[\mathrm{s}]\,, \qquad l_0 = 10^3\,[\mathrm{m}]\,,$$
$$\frac{\alpha}{E} = 0.043 \cdot 10^{-16}\,[\mathrm{m}^4/\mathrm{N}^2]\,, \qquad q = 10^2\,[\mathrm{N/m}]\,,$$
$$c = 5 \cdot 10^3\,[\mathrm{m/s}]\,, \qquad \mu = 0.11\,.$$

The starting, the uniform motion and the braking of the winding machine have been taken into account as based on the diagrams given in *Fig. 3*.

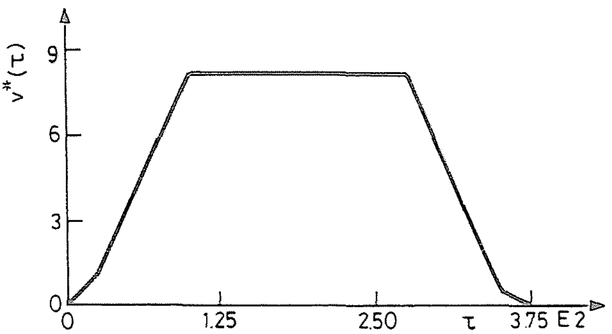

Fig. 3. Diagram of the velocity $v^*(\tau)$

The diagrams of frequency $p^*(\tau)$ and amplitude $a^*(\tau)$ of the longitudinal vibrations are presented in *Figs. 4* and *5.*

Spatial diagrams of the regions of the main, secondary and combination parametric resonances for the first and second modes of the transversal vibrations are presented in *Figs. 6, 7, 8, 9, 10.*

For the arbitrarily chosen time-points $\tau = 0$ (the moment of the starting of the machine), $\tau = 22.5$ (from the starting interval), $\tau = 187.5$ (from the uniform motion interval), the cross-sections are marked in the diagrams.

In the graphs presented in *Figs. 11, 12, 13,* all the types of considered resonances in the selected moments in time are shown.

The influence of the decrease in the stiffness of the system on the character of instability regions is considered. We also examine the effect of

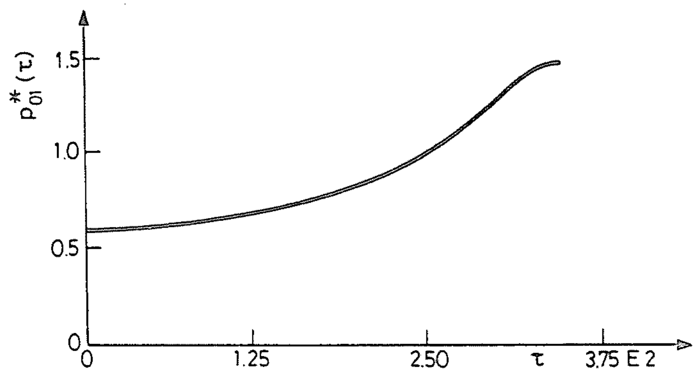

Fig. 4. Diagram of the frequency of the longitudinal vibrations

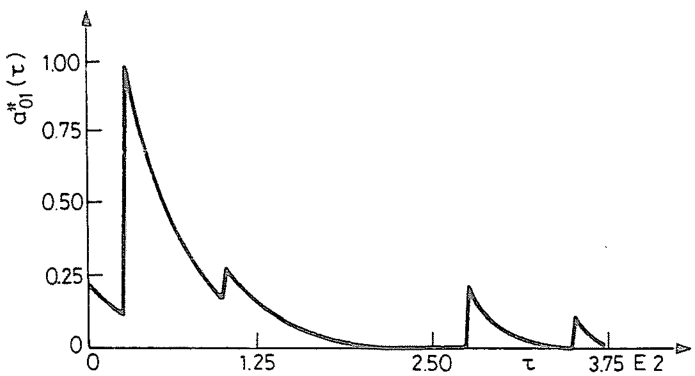

Fig. 5. Diagram of the amplitude of the longitudinal vibrations

the decrease in the value of weight Q^*. The results obtained are presented in *Fig. 14* only for the first mode of the transversal vibrations and for $\tau = 22.5$. The change of the physical non-linearity has its influence on the location of the instability regions. The graph in *Fig. 15* shows this effect for the first mode of the transversal vibrations.

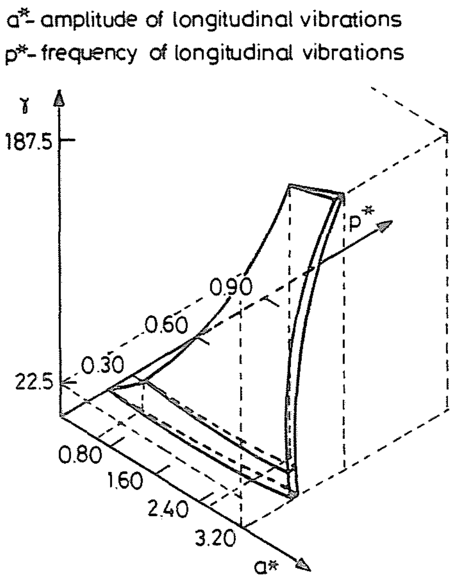

Fig. 6. Region of the main parametric resonance for the first mode of transversal vibrations

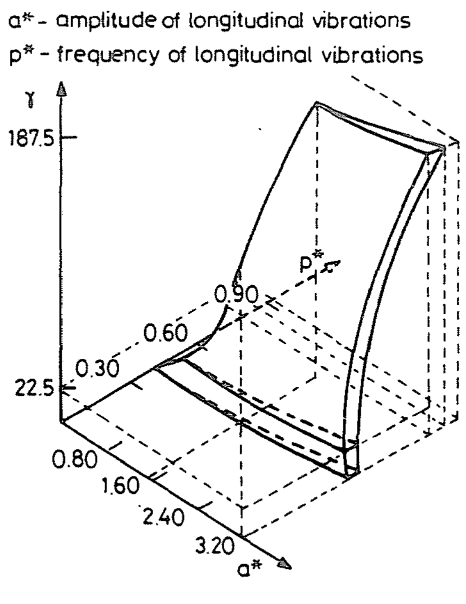

Fig. 7. Region of the main parametric resonance for the second mode of transversal vibrations

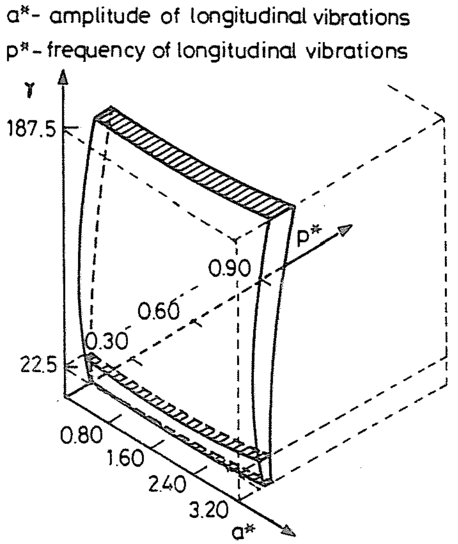

Fig. 8. Region of the secondary parametric resonance for the first mode of transversal vibrations

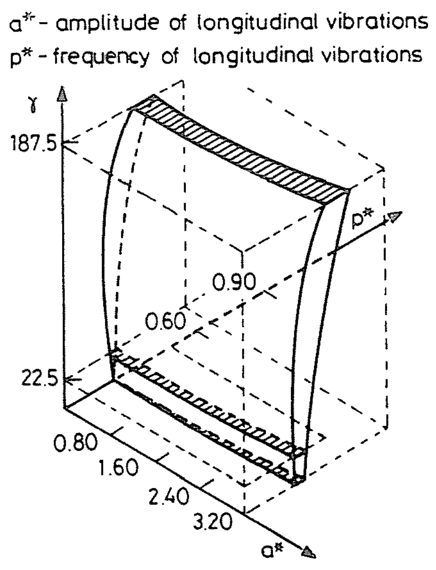

Fig. 9. Region of the secondary parametric resonance for the second mode of transversal vibrations

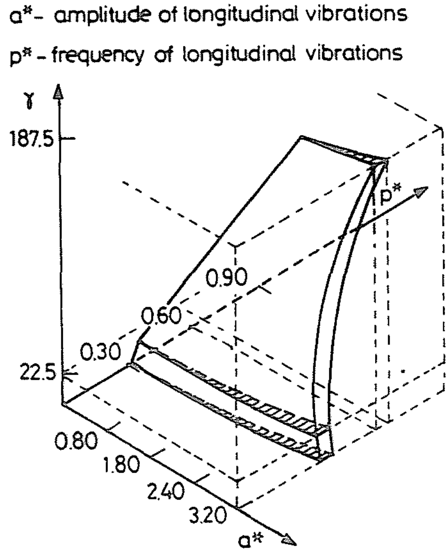

Fig. 10. Region of the combination parametric resonance

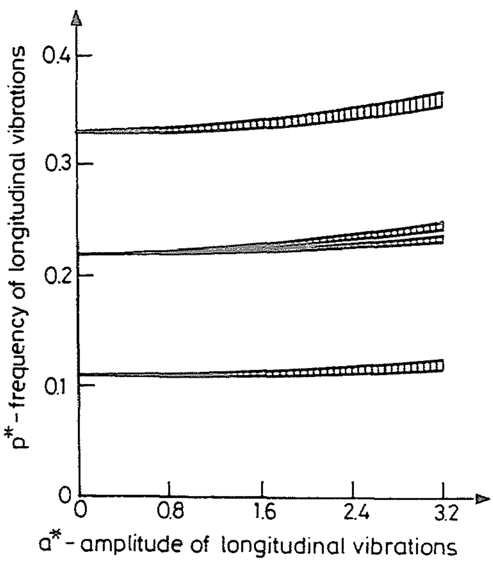

Fig. 11. Regions of: main, secondary and combination parametric resonances for the first and second modes of transversal vibration for $\tau = 0$. Regions of resonances are indicated as follows: white – for main resonances, shadow – for secondary resonances, double shadow – for combination resonance

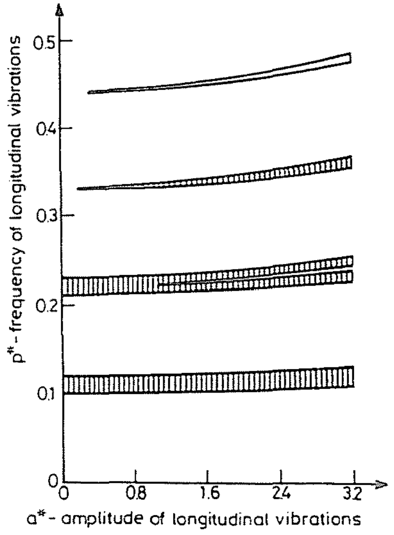

Fig. 12. Regions of: main, secondary and combination parametric resonances for the first and second modes of transversal vibration for $\tau = 22.5$. Region of resonances are indicated as follows: white – for main resonances, shadow – for secondary resonances, double shadow – for combination resonance

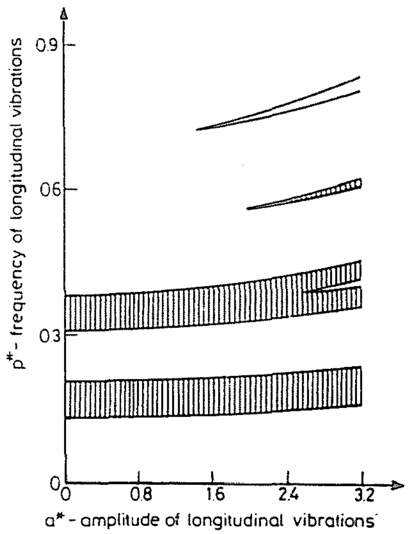

Fig. 13. Regions of: main, secondary and combination parametric resonances for the first and second modes of transversal vibration for $\tau = 187.5$. Regions of resonances are indicated as follows: white – for main resonances, shadow – for secondary resonances, double shadow – for combination resonance

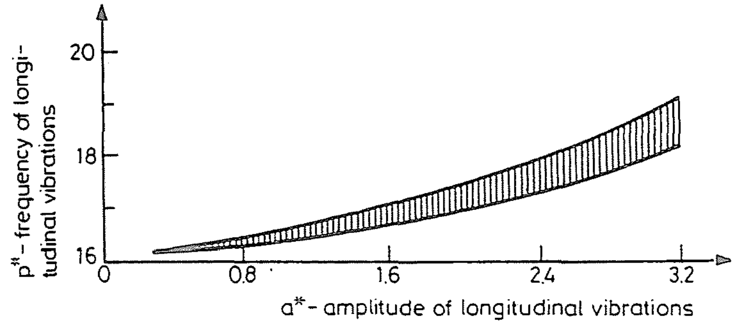

Fig. 14. Region of the main parametric resonance for the first mode of transversal vibrations for $\tau = 22.5$, for load $0.5Q^*$

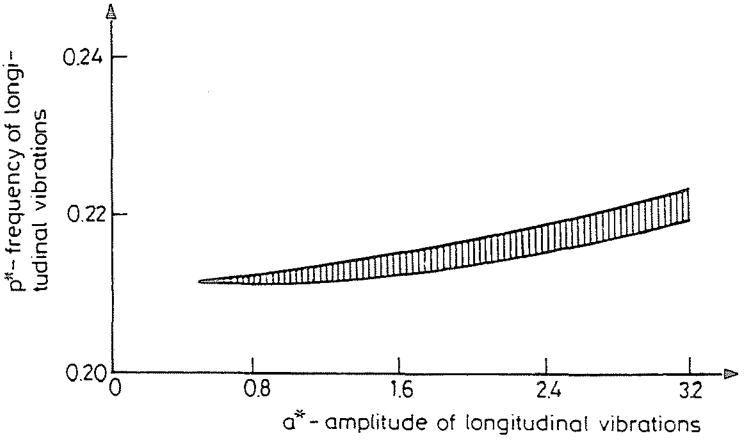

Fig. 15. Region of the main parametric resonance for the first mode of transversal vibrations for $\tau = 22.5$, for load Q^* and for coefficient of physical non-linearity 0.5λ

6. Conclusion

The results presented in the paper are of qualitative and quantitative character. The system considered serves as a model of a realistic system with varying parameters. The non-linear model which was adopted includes many features occurring during the work of ropes as load carrying and winding elements. The set of differential motion equations based on a physical model describes almost precisely the occurring process.

The analysis of the dynamic instability leads us to the following conclusions:

- For some values of the frequencies and amplitudes of the longitudinal vibrations, the motion of the considered system can be unstable. The longitudinal vibrations bring about the increase amplitudes of the transversal vibrations;
- For the chosen parameters of the rope, only the first or the second modes can be parametrically excited, but it requires high amplitude of the longitudinal vibrations;
- The lowest value of the frequency of longitudinal vibrations, for which the third parametrically excited transversal mode occurs, is higher than the real value of the frequency for longitudinal vibrations. Having chosen the parameters of the rope as in the present paper, we cannot excite the transversal oscillations even for high amplitudes of longitudinal vibrations;
- The higher the mode of the transversal vibrations is, the wider the regions of instability will be;
- The value of the peak coordinate of an unstable region along the amplitude axis depends on the coupling and it increases with the increase of velocity $v^*(\tau)$;
- For velocity $v^* = 0$, the regions for the main, secondary and combination resonances have a similar width and they occur even with the amplitudes tending to zero;
- For small changes in velocity v^*, the unstable regions for main resonance are in the neighbourhood of doubled frequencies of the transversal vibrations for the considered first and second modes. The regions of the secondary resonances are placed in the neighborhood of frequencies which are twice as low as the main resonances. The combination resonance occurs in the region of the arithmetical mean frequencies corresponding to the main resonances;
- As a result of increasing $v^*(\tau)$, the broadening of the unstable regions for the secondary resonances is observable. It is due to the existence of the 'giroscopic' in character elements of the B matrix. The regions for the main and the combination resonances become narrower, and the instability regions shift towards the increasing values of amplitudes a^*;
- As a results of the changes in the frequencies, the instability regions shift upwards the p^* axis;
- The decrease in load Q^* shifts the unstable regions downwards the frequency axis;

- The decrease of the physical non-linearity has the effect of negligible narrowing of the zones of instability and it moves them down the frequency axis.

We end up with the most important conclusion:

As a results of the coupling between the longitudinal and transversal vibrations, the parametric resonance is theoretically possible but its occurrence requires high amplitudes of the longitudinal vibrations. It can occur only in emergencies (i.e. impulses resulting in high increase of the longitudinal amplitude a^*).

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