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OPTIMAL DYNAMIC PARAMETER SELECTION FOR A STOCHASTIC WHEEL SUSPENSION MODEL UPON PASSANGER'S COMFORT

György RICHLIK, Ferenc TAKÁCS and György TÓTH

Department of Mechanics
Faculty of Transportation Engineering
Technical University of Budapest
H-1521 Budapest, Hungary

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Abstract

The object of the paper is to provide a method which is able to design optimal dynamic parameters for car suspension systems.

The mechanical model is a two-degree-of-freedom damped system which is excited by stochastic geometrical loading. The type of loading is a band limited (pink) noise generated by a realistic road surface. The model represents a wheel suspension system of a vehicle. The aim of the optimization is to define the road type-dependent damping and stiffness parameters of the suspension system which may provide the maximal passanger's comfort. The method is illustrated by a numerical example.

Keywords: optimal parameters, wheel suspension, passanger's comfort.

1. Introduction

During the last decade there was a considerable change in the methods of bus design. The requirements were getting more and more strict concerning the traffic safety, efficiency and passanger's comfort as well. For obtaining the higher passanger's comfort, one of the most known methods is the control of the elements of the wheel suspension system. For the moment, the passive control is widespread [1], which consists of an air bag and damper, both of them can have either linear or non-linear characteristic. The passive suspension system may be optimized for a certain road type but in the case of new road type, the passanger's comfort can become disadvantageous. In the so-called semi-active suspension system, the stiffness and damping parameters can be controlled as a function of the road surface. As the realization of the active control — which implies an active force between the vehicle body and the wheel system — is quite expensive at present, therefore we tried to optimize the parameters of the suspension system in order to provide the maximal passanger's comfort by using several possible objective functions.

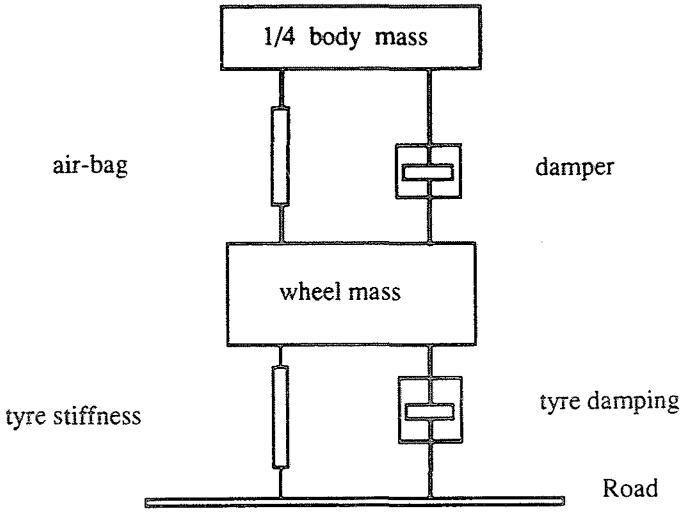

Fig. 1. Two-degree-of-freedom model of a wheel suspension

2. Description of the Selected Model

As the object of our analysis, we have selected a simplified suspension system of a 1/4 vehicle. It can be modelled by a two-degree-of-freedom mechanical system which is generally used in the literature [5] (*Fig. 1*).

In the selected model, the damping, stiffness and mass of the wheel and the 1/4 mass of the vehicle body were considered as constants. The stiffness of the air bag and the damping factor of the damper of the suspension system will be our design variables. Our numerical analysis was done by the SYSTUS general purpose finite element system using its dynamic stochastic module [2, 3]. The input parameters (road type) are selected from the literature which is often characterizing the road surface by a pink noise (*Fig. 2*) [4].

3. Stochastic Module of SYSTUS Finite Element System and Parameters of Passanger's Comfort

As we have already mentioned, we have several ways for the characterization of passanger's comfort. The selected stochastic model is based on a Gaussian ergodic stationary process, therefore it is possible to characterize the passenger's comfort by two objective functions:

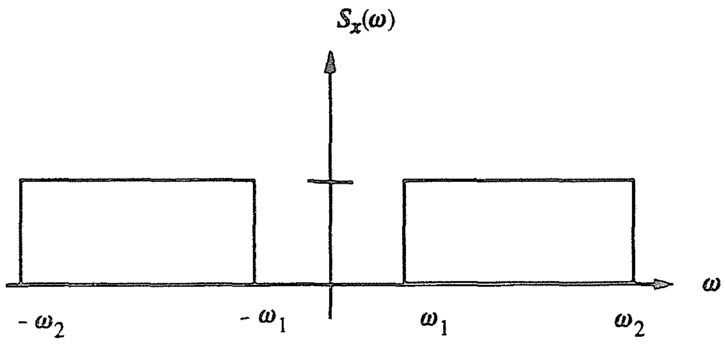

Fig. 2. Input power spectral density function

- $n(u)$ is the average number of overcrossing a given level u per unit time,
- $E_{\max}(T)$ is the average value of the maximum of the process of duration T.

These objective functions can be computed by Systus stochastic module. Prior to the objective functions, we have to compute:
- undamped eigenmodes and eigenfrequencies (Systus Dynamic Module),
- stationary transfer function of the system
- power spectral density function of response (Systus Harmonic Response).

From these functions we can define the statistical moments of the dynamic model as follows:

$$M_i = 2 \int_{-\infty}^{\infty} S(\omega)\omega^i \mathrm{d}\omega\,, \qquad \text{if} \quad i \quad \text{is even}\,,$$

$$M_i = 0\,, \qquad \text{if} \quad i \quad \text{is odd}\,,$$

where M_i is the moment of order i of power spectrum $S(\omega)$ of the response
M_0 is the mean value of the response.

By using the statistical moments, we can compute the objective functions as follows:

$$n(u) = \frac{1}{\pi}\left(\frac{M_2}{M_0}\right)^{1/2} \exp\left(\frac{-u^2}{2M_0}\right)\,,$$

$$E_{\max}(T) = (M_0)^{1/2}\left((2\log(n(0)T))^{1/2} + \frac{Y}{(2\log(n(0)T))^{1/2}}\right),$$

where Y is the Euler constant: 0.5772 [3].

4. Numerical Example and Conclusions

As an illustration of the above described method, we created a numerical example. The model is relating to an Ikarus bus wheel suspension which has the following data:

1/4 body mass = 2500 kg,
wheel mass = 400 kgs,
tyre stiffness = 840 kN/m,
tyre damper rate = 800 kNs/m.

We have selected

a spring rate $19.6 < k < 840$ kN/m
and a damper rate $1.71 < \beta < 25$ kNs/m.

The road quality was characterized by a unit power spectral density between 1 Hz and 40 Hz. In the expression of $n(u)$, we have substituted $u = 0.05$, 0.1, and 0.2 m while for $E_{\max}$ we used $T = 10$, 60 and 3600 s. The results of computations are illustrated by a set of curves.

Fig. 3 is representing a set of curves where for a selected damper rate and fixed overcrossing level we regard the overcrossing number as a function of spring rate. For better understanding, we have used logarithmic scale on the abscissa. *Fig. 4* is showing the same functions for another damper rate. Observing the above mentioned two figures, we can have the following conclusions:

- Apparently for low overcrossing levels, the overcrossing number n is increasing with the increase of spring rate, while for higher levels it is decreasing. Therefore the optimal spring rate cannot be defined.
- In the case of different u levels prescribing upper limit, we shall have an optimal domain for the values of k (*Fig. 4*).
- For the same upper limit in the case of another damper rate, it can occur that we cannot find the above optimal domain (*Fig. 3*).
- For different road types, we can achieve the same analysis with new power spectral density and new frequency bandwith.

Regarding $E_{\max}$ (average value of the maximum of the process of duration T; *Fig. 5*) as our other objective function and the previously

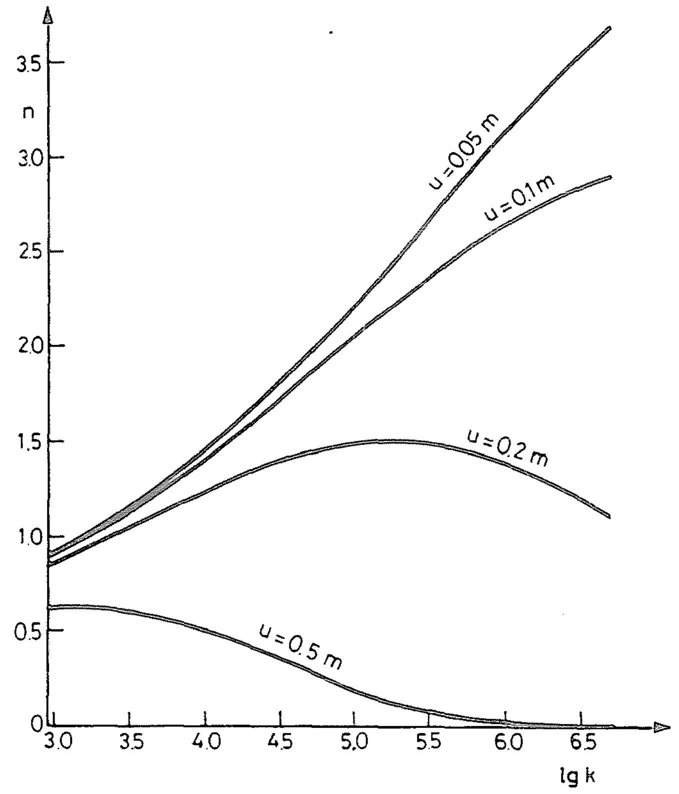

Fig. 3. Average number of overcrossing of given levels versus spring rate

selected optimal domain, we can further restrict the optimal domain, for instance imposing new limits on $E_{\max}$.

Summarizing the above results, we can state that for this very simple model it is already possible to find an optimal domain for the values of parameters of the wheel suspension system with the described method hereby. Concerning the further development of our method, we could select more realistic models. For instance, we would like to increase the numbers of the

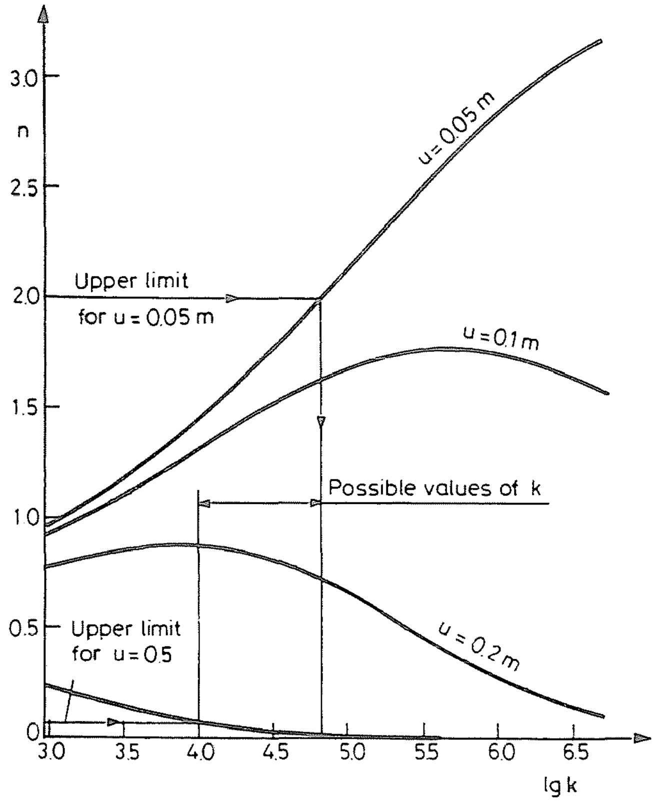

Fig. 4. Definition of the optimal spring rate

degrees of freedom and we would like to take into account the correlation between all the wheels. We could introduce new objective functions for the passanger's comfort as well.

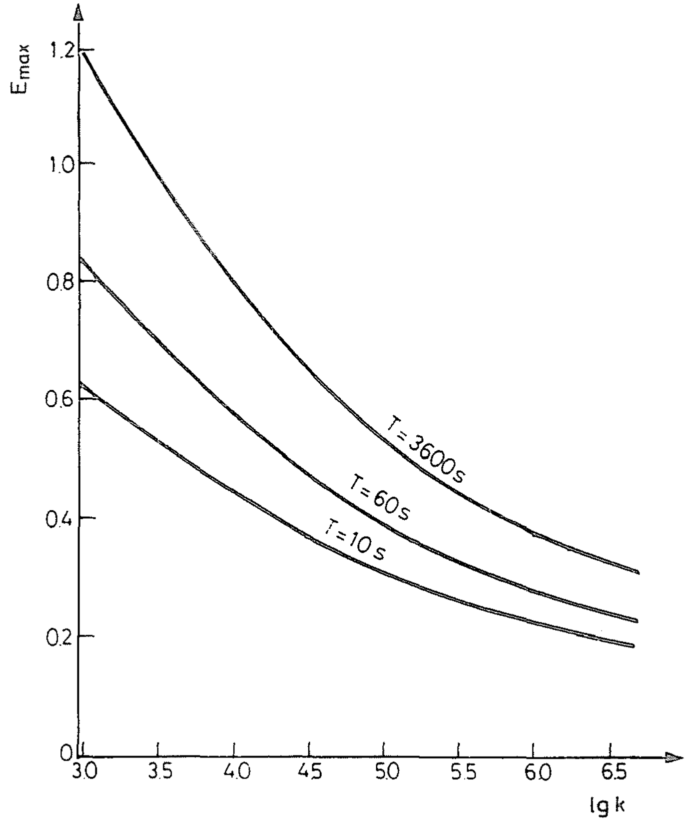

Fig. 5. Average values of the maximum versus spring rate

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