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ON MAGNETIC PULLING FORCE IN LINEAR EDDY-CURRENT BRAKE

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Abstract

In the paper, the mathematical model of the phenomena occurring in the rail eddy-current brake of a passenger train has been worked out. Braking forces between a moving brake shoe and motionless rail are investigated. The analysis is based on the solution to one-dimensional vector potential in the inductor, brake air-gap and railway rail (rotor).

1. Introduction

The interest in setting up new braking systems for passenger trains results from an increase of travelling speed of trains in which self-acting, friction, air brakes are not efficient enough. Introducing an additional brake becomes inevitable in this situation. The additional brake performs as a service brake within the range of high speed. Afterwards this task is fulfilled by the pneumatic brake only when travelling speed is reducer below certain speed value. A rail-type eddy-current brake may serve for this purpose. The rail-type eddy-current brake is constructed like the rail brake. The brake inductor, parallel to rail brake shoe, does not press on the rail, but it is installed at a short distance from it (several millimeters). In recent works [1 – 5] and [7] on eddy current brakes, only horizontal component of magnetic force acting on brake shoe has been considered, without taking into account vertical magnetic pulling force, treating it as inner force carried by the wheel sets. This force is so big, however, that below 50 km/h to the full stop of a train, an eddy current brake must be switched off because of the possibility of brake damage. Thus the knowledge of the value of this force has crucial meaning for eddy-current brake design.

List of principal symbols:

$A_{x,y,z}$ – magnetic vector potential,
B – magnetic flux density,
F_y – magnetic pulling force,
F_x – braking force,
δ – half-air gap,
μ – magnetic permeability,
γ – electrical conductivity,
Θ – linear current,
v – speed,
$t.$ – time,
τ – pole pitch,
$*$ – denotes complex conjugate of the vector quantity,
S – surface of the poles.

2. Mathematical Model and Assumptions

Phenomena in an eddy-current brake are described by Maxwell equations, which combined into one partial differential equation obtain the following form:

$$\nabla^2 \mathbb{A} = \mu\gamma \left(\frac{\partial \mathbb{A}}{\partial t} - v \times \nabla \times \mathbb{A} \right) . \tag{1}$$

There is no analytical solution of equation (1). For an analysis, the one-dimensional model shown in *Fig. 1* was used. This model is being resolved under the following assumptions:

a) the stator yoke is made of a material of magnetic permeability $\mu_1 =$ const. and electrical conductivity $\gamma_1 = 0$,
b) the model is of unlimited width in the direction of the z axis,
c) all currents have the direction of the z axis,
d) the surface of the stator is penetrated by the linear current produced by the current flowing in the infinitesimal layer and defined as:

$$\Theta = \mathrm{Re}\; i \sum_{n=1}^{\infty} \Theta_{2n-1} e^{i(2n-1)\alpha x} , \tag{2}$$

where:

$$\alpha = \frac{\Pi}{\tau}$$

e) the real air gap is substituted by the effective gap using Carter coefficients,

f) the rotor yoke is made of a material of magnetic permeability $\mu_2 =$ const. and electrical conductivity $\gamma_2 = 0$.

In the range where the current density equals zero, equation (1) is reduced to Laplace equation:

$$\nabla^2 \mathbf{A} = 0 . \tag{3}$$

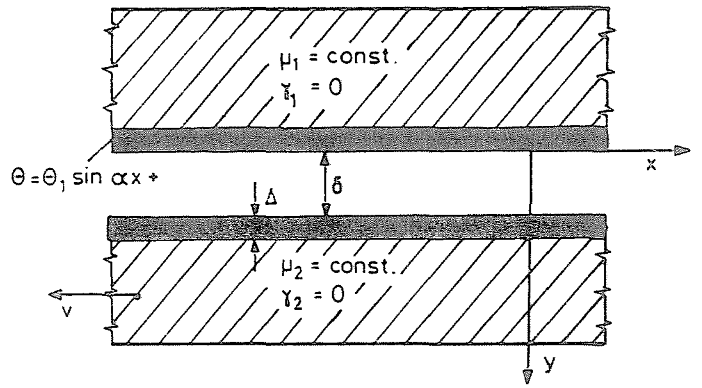

Fig. 1. One-dimensional quasi-static brake model used for analysis

In order to solve this equation, apart from the assumptions as for the basic model it was assumed in addition that the surface of the rotor is covered with the infinitesimal elementary layer of thickness $\Delta \to 0$ and electrical conductivity $\gamma \to \infty$. The final results are as follows:

$$\mathbf{A}_z = \mathbf{A}_z^p + \mathbf{A}_z^w , \tag{4}$$

where:

$$\mathbf{A}_z^p = i \sum_{i=1}^{\infty} \left\{ \frac{\mu_0 \Theta_{2n-1}}{\alpha k_{2n-1}} \cosh(2n-1)\alpha y - \frac{\mu_0 \Theta_{2n-1}}{\alpha} \left[1 - \frac{\mu_0}{\mu_1 k_{2n-1}} \right] \right.$$

$$\left. \sinh(2n-1)\alpha y \right\} \cdot e^{i(2n-1)\alpha x} , \tag{5}$$

$$\mathbf{A}_z^w = i \sum_{i=1}^{\infty} \left\{ \frac{\mu_0 \Theta'_{2n-1}}{\alpha k'_{2n-1}} \cosh \alpha(2n-1)(y-\delta) + \frac{\mu_0 \Theta'_{2n-1}}{\alpha} \left[1 - \frac{\mu_0}{\mu_2 k'_{2n-1}} \right] \right.$$

$$\left. \sinh \alpha(2n-1)(y-\delta) \right\} \cdot e^{i(2n-1)\alpha x} , \tag{6}$$

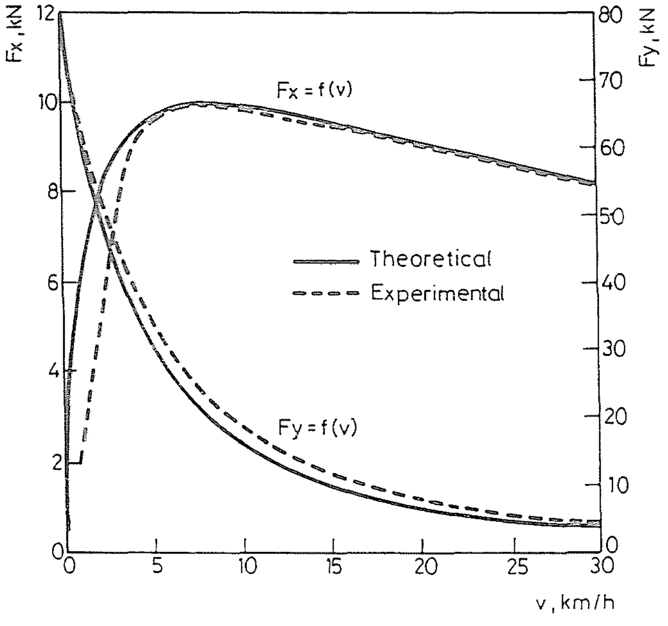

Fig. 2. Components of magnetic pulling force as a function of speed

$$k'_{2n-1} = \frac{\mu_0}{\mu_2} + \frac{\mu_1 \sinh(2n-1)\alpha\delta + \mu_0 \cosh(2n-1)\alpha\delta}{\mu_1 \cosh(2n-1)\alpha\delta + \mu_0 \sinh(2n-1)\alpha\delta}, \tag{7}$$

$$\Theta'_{2n-1} = -\sum_{n=1}^{\infty} v\mu_0\Theta_{2n-1} \frac{\left\{\frac{\cosh(2n-1)\alpha\delta}{k_{2n-1}} - \left[1 - \frac{\mu_0}{\mu_1 k_{2n-1}}\right] \cdot \sinh(2n-1)\alpha\delta\right\}}{\frac{\mu_0 v}{k'_{2n-1}} - ir}, \tag{8}$$

where:

$$r = \lim_{\substack{\gamma\to\infty \\ \Delta\to 0}} \frac{1}{\gamma\Delta}, \tag{9}$$

$$k_{2n-1} = \frac{\mu_0}{\mu_1} + \frac{\mu_2 \sinh(2n-1)\alpha\delta + \mu_0 \cosh(2n-1)\alpha\delta}{\mu_2 \cosh(2n-1)\alpha\delta + \mu_0 \sinh(2n-1)\alpha\delta}. \tag{10}$$

3. Magnetic Forces

For given distribution of magnetic vector potential in the air gap, the components of magnetic force can be determined by using Amper equation and Maxwell formula.

– Amper force

$$\mathbb{F}_x = - \int_{-\tau}^{\tau} \mathrm{Re}\left(\Theta \frac{\partial \mathbb{A}_z^{w*}}{\partial x}\right) \mathrm{d}x \,, \tag{11}$$

hence:

$$\mathbb{F}_x = -\mu_0 \tau \sum_{n=1}^{\infty} k'_{2n-1} \Theta_{2n-1}^2 \frac{\left\{\frac{\cosh(2n-1)\alpha\delta}{k_{2n-1}} - \left[1 - \frac{\mu_0}{\mu_1 k_{2n-1}}\right] \cdot \sinh(2n-1)\alpha\delta\right\}}{\frac{v}{v_{2n-1}} + \frac{v_{2n-1}}{v}} \cdot \left\{\frac{\cosh(2n-1)\alpha\delta}{k'_{2n-1}} - \left[1 - \frac{\mu_0}{\mu_1 k'_{2n-1}}\right] \cdot \sinh(2n-1)\alpha\delta\right\} , \tag{12}$$

where:

$$v_{2n-1} = \frac{k_{2n-1} r'}{\mu_0} \,. \tag{13}$$

– Maxwell force

$$\mathbb{F}_y = -\frac{1}{2\mu_0} \int_{-\tau}^{\tau} (\mathrm{Re\ rot\ } \mathbb{A}_z)^2 \, \mathrm{d}x \,, \tag{14}$$

where:

$$\mathrm{rot}\ _{y=\delta} \mathbb{A}_z = \sum_{n=1}^{\infty} \mu_0 \Theta_{2n-1} \left[\frac{\cosh(2n-1)\alpha\delta}{k_{2n-1}} - \left(1 - \frac{\mu_0}{\mu_1 k_{2n-1}}\right) \cdot \sinh(2n-1)\alpha\delta\right] \cdot \left(1 - \frac{v\mu_0}{v\mu_0 - i r k'_{2n-1}}\right) \cdot e^{i(2n-1)\alpha x} , \tag{15}$$

hence:

$$\mathbb{F}_y = -\frac{\tau}{2\mu_0} \sum_{n=1}^{\infty} \left\{\mu_0 \Theta_{2n-1} \left[\frac{\cosh(2n-1)\alpha\delta}{k_{2n-1}} - \left(1 - \frac{\mu_0}{\mu_1 k_{2n-1}}\right) \cdot \sinh(2n-1)\alpha\delta\right] \cdot \left[1 - \frac{v^2}{v^2 + v_{2n-1}^2}\right]\right\}^2 . \tag{16}$$

In the model under consideration, it has been assumed that the eddy-current induced in the rotor flows in an infinitely thin film but as a matter of fact the thickness of this film may be approximated as a penetration (effective depth) depth of electromagnetic wave into the metal.

$$\Delta = (\mu\gamma v\alpha)^{-0.5}, \tag{17}$$

hence using (9), it can be obtained:

$$\tau = \left(\frac{\mu v a}{\gamma}\right)^{0.5}. \tag{18}$$

For such assumptions, equations (12) and (16) will have the following form:

$$\mathbb{F}_x = -\mu_0\tau\sum_{n=1}^{\infty} k'_{2n-1}\Theta^2_{2n-1}\frac{\left\{\frac{\cosh(2n-1)\alpha\delta}{k_{2n-1}} - \left[1-\frac{\mu_0}{\mu_1 k_{2n-1}}\right]\cdot\sinh(2n-1)\alpha\delta\right\}}{\left(\frac{v}{v'_{2n-1}}\right)^{0.5}+\left(\frac{v'_{2n-1}}{v}\right)^{0.5}}$$

$$\cdot\left\{\frac{\cosh(2n-1)\alpha\delta}{k'_{2n-1}} - \left[1-\frac{\mu_0}{\mu_1 k'_{2n-1}}\right]\cdot\sinh(2n-1)\alpha\delta\right\}, \tag{19}$$

where:

$$v'_{2n-1} = \frac{k'^2_{2n-1}\mu\alpha}{\mu_0^2\gamma}. \tag{20}$$

and

$$\mathbb{F}_y = -\frac{\tau}{2\mu_0}\sum_{n=1}^{\infty}\left\{\mu_0\Theta_{2n-1}\left[\frac{\cosh(2n-1)\alpha\delta}{k_{2n-1}} - \right.\right.$$

$$\left.\left. - \left(1-\frac{\mu_0}{\mu_1 k_{2n-1}}\right)\cdot\sinh(2n-1)\alpha\delta\right]\cdot\left[1-\frac{v}{v+v'_{2n-1}}\right]\right\}^2. \tag{21}$$

The validity of the model and deduction can be proved by assuming $\mu_1 = \mu_2 = \infty$, $n = 1$ and $v = 0$, thus (20) takes known from publications form on dependency of magnetic pulling force:

$$F_{ymax} = -\frac{2\mu_0\tau\Theta_1^2}{4\sinh^2\alpha\delta} \cong -\frac{\mathbf{B}^2 S}{2\mu_0}, \tag{22}$$

hence

$$F_{xmax} = F_{ymax}\tanh\alpha\delta. \tag{23}$$

Furthermore, for thorough verification of the model, calculations of magnetic pulling force components have been carried out (assuming data for Knorr brake [8]) By using the magnetization characteristics of steel, $\mu = \mu_1 = \mu_2$ have been defined, and then by using (19), (20), the searched characteristics have been determined. Theoretical and experimental results are in a good accordance.

4. Final Remarks

On the basis of the mathematical model of the eddy-current brake, which was partially verified by experiments, the following conclusions can be formulated:

1. Proposed method of analysis of electromagnetic phenomena in linear eddy-current brake enables the determination of basic brake parameters.
2. Assumed simplifications do not have significant influence on error values and simultaneously the method enables analytical solutions, what is particularly vital in design.

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