

# IDENTIFICATION OF STRUCTURAL DAMPING OF A HELICOPTER ROTOR BLADE

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Received: Nov. 10, 1992

## Abstract

In the elastic motion of helicopter rotor blades the structural damping also has an important role. The purpose of this article is the investigation of the mathematical model of the structural damping and the estimation of the parameters of this model. The parameter estimation is based on measurings (the first three natural oscillations were measured).

*Keywords:* structural damping, identification, rotor blade.

## 1. Investigation of the Mathematical Model

The differential equation of the elastic motion of a rotor blade (according to [3], [7] and [10]):

$$M_i \ddot{\xi}_i + c M_i \dot{\xi}_i + \frac{\theta}{\omega} K \dot{\xi}_i + M_i \omega_i^2 \xi_i = Q_i, \quad i = 1, 2, \dots, n,$$

where:

$M_i = \int_0^R \eta_i^2(r) m(r) dr$ ; the generalised mass;

$Q_i = \int_0^R p(r, t) \eta_i(r) dr$ ; the generalised force;

$c$  - viscous damping coefficient;

$\theta$  - structural damping coefficient;

$\eta_i(r)$  - the  $i$ -th normal mode;

$\omega_i$  - the  $i$ -th natural frequency.

The  $\eta_i(r)$  functions and the  $\omega_i$  values are known from another work, they are ground dates in this estimation.

For the determination of the generalised force load distribution of the rotor blade  $\{p(r, t)\}$  is necessary.

At the measuring of the normal modes the rotor blade was nearly vertical, so the gravity is negligible. So it is enough to take into account only the aerodynamic and the structural damping.

### *The Aerodynamical Damping*

The maximal value of the vibration of the rotor blade was about 20 cm, and the motion is sinusoidal, so we can calculate the mean velocity of the vibration and also the mean Reynolds number; the minimal mean value is greater than 7000 - so the airflow about the rotor blade is definitely turbulent. So we can choose the well-known aerodynamical drag expression:

$$p(r, t) = \frac{\rho}{2} V c_D h = \frac{\rho}{2} (\dot{\xi}_i \eta_i) c_D h_i,$$

where:

- $\rho$  - the density of the air;
- $(\dot{\xi}_i \eta_i)$  - the velocity of a section;
- $c_D$  - the drag coefficient;
- $h$  - the chord length.

The connected mass is the air mass in the blade-connected ellipsoid and this mass is less than 1 per cent of the mass of the rotor blade. It is negligible.

### *The Structural Damping*

According to [7] the structural damping is proportional to the bending-potential energy of the rotor blade:

$$U_b = \int_0^R IE[\eta_i''(r)]^2 dr,$$

where:

$IE$  - the bending stiffness of the rotor blade.

So the differential equation of the free vibration may be written:

$$M_i \ddot{\xi}_i + c M_i \dot{\xi}_i + \frac{\theta}{\omega} \left\{ \int IE[\eta_i''(r)]^2 dr \right\} \dot{\xi}_i + M_i \omega_i^2 \xi_i = Q_i,$$

where:

$$Q_i = -\text{sgn}(\dot{\xi}_i) \frac{\rho}{2} \dot{\xi}_i^2 c_D \left( \int \eta_i^3(r) h dr \right).$$

## 2. The Boundaries of the Parameters

It is possible to determine the boundaries of the parameters  $c$ ,  $\theta$  and  $c_D$ . (We will look for the parameter values only between these boundaries.)

The boundaries of the drag coefficient: in [5] there is a measuring of the drag coefficient of the NACA 0012 profile in a region of 0–360 degrees. The profile of the investigated rotor blade is NACA 0015, so we can write, that the absolute value of the drag coefficient is less than 2.1, so:

$$0 > c_D > -2.1.$$

The boundaries of the structural damping coefficient: according to [8] we can write:

$$-0.001 < \theta < -0.01.$$

The boundaries of the viscous damping coefficient: from the measuring we know a set of the  $\{\xi_i\}$  values. We suppose that

$$\xi_{im}(t = 0) = \xi_i(0)$$

and we will choose the  $\alpha$  and  $\beta$  values so that

$$\xi_{im}(t) \leq \xi_i(0)e^{\alpha t} \cos(\beta t).$$

With  $\alpha$  and  $\beta$  values we can calculate the boundaries of the viscous damping coefficient, which is between:

$$0 < c < -0.05.$$

### 3. The Parameter Estimation

For the identification the next functional was defined:

$$F = \sum_{i=1}^3 \sum_{j=1}^N (\xi_{ijm} - \xi_{ijc})^2.$$

We are looking for the  $\{c, \theta, c_D\}$  values so that  $F$  must be minimum. The values of  $\xi_{ijc}$  were calculated by numerical integration.

The mathematical model is nonlinear so we can't find a direct way for the calculation of parameter values.

The parameter values were found by using the Rosenbrock minimum finding method. This method is applicable very well to our mathematical model, since the boundaries of the estimated parameters must be given. The results:

$$\begin{aligned} c &= -0.023, \\ \theta &= -0.00175, \\ c_D &= -0.578. \end{aligned}$$

#### 4. The Discussion of the Results

The aerodynamic drag coefficient is very good, because the air around the rotor blade is in turbulent motion. At the motion in one direction, behind the rotor blade is a low-pressure region; but the direction of the movement changes and then this region supports the motion of the blade.

The loss of the total bending energy – due to the structural damping – approximately 1.1 is given, but the strength of the rotor blade is low and the structure is inhomogeneous.

The viscous damping has a part which is characteristic of the damping. The equivalent critical damping coefficient was 17.44, so these dampings are slight.

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