PERIODICA POLYTECHNICA SER. TRANSPORTATION ENG. VOL. 20, NO. 1, PP. 3–12 (1992)

# ON THE EXISTENCE OF STABLE STATIONARY FORCED VERTICAL VIBRATIONS IN RAILWAY VEHICLE SYSTEM DYNAMICS

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Received: October 14, 1988.

## Abstract

Applying some mathematical results on the asymptotic behaviour of solutions of stationary stochastic differential equations elaborated by H. BUNKE [2] and T. V. NHUNG [3–5], we derive explicit conditions ensuring the existence of the stable stationary forced vertical vibrations of the railway vehicle dynamic system model which was constructed by I. ZOBORY [6].

By using the Routh–Hurwitz criterion (see e. g. L. ARNOLD [1], p. 196) our stability conditions are stated in terms of algebraic inequalities involving parameters of the system model. Those inequalities can be easily checked by computers.

## 1. The Model

To describe vertical displacements $z_t$, $z_{kt}$ in the dynamic model examined by I. ZOBORY (see *Fig. 1* in [6]) the following system of two second-order random linear differential equations was used (see (8) in [6]).

$$m\ddot{z}_t = k_v\dot{z}_t - k_v\dot{z}_{kt} + s_v z_t - s_v z_{kt} = 0\,, \tag{1.1}$$

$$(m_p+m_k)\ddot{z}_{kt} - k_v\dot{z}_t + (k_v+k_p)\dot{z}_{kt} - s_v z_t + (s_v+s_p)z_{kt} = s_p u_t + k_p\dot{u}_t + m_p\ddot{u}_t\,,$$

where $m$, $m_p$, $m_k$, $k_v$, $k_p$, $s_v$, $s_p$ are constant parameters characterizing the model, and $u_t$ is the random excitation representing the vertical unevennesses in the track. The stochastic process $u_t$ is assumed to be weakly stationary with spectral density function $g_{uu}(\omega)$. By introducing the notations:

$$\mathbf{Z}_t = \mathrm{col}(z_t, z_{kt}) \quad \text{and} \quad \mathbf{F}_t = \mathrm{col}(0, s_p u_t + k_p\dot{u}_t + m_p\ddot{u}_t)\,,$$

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system (1.1) is now equivalent to the following matrix equation:

$$\mathbf{M}\ddot{\mathbf{Z}}_t + \mathbf{K}\dot{\mathbf{Z}}_t + \mathbf{S}\mathbf{Z}_t = \mathbb{F}_t\,, \tag{1.2}$$

where $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{S}$ are constant $2 \times 2$-matrices (see (9) in [6]). If the spectral density function $g_{uu}(\omega)$ of process $u_t$ is given, then the spectral density matrix $\mathbb{G}_F(\omega)$ of the excitation vector process $\mathbb{F}_t$ is known (see (10) and (11) in [6]). Spectral density matrix $\mathbb{G}_Z(\omega)$ of the output process $\mathbb{Z}_t$ can be constructed by using $\mathbb{G}_F(\omega)$ and the fundamental theorem of statistical dynamics (see (12) and (13) in [6]). Finally, by using $\mathbb{G}_Z(\omega)$, we are able to calculate the real-valued spectral density function $g_{\tilde{T}\tilde{T}}(\omega)$ for the normalized vertical axle force process

$$\tilde{\mathbb{T}}_t = \mathbb{T}_t - \mathbb{T}_0 = m_k\ddot{z}_{kt} + k_v\dot{z}_{kt} + s_v z_{kt} - s_v z_t - k_v\dot{z}_t \tag{1.3}$$

(see (15) in [6]). Thus, in this way, the forced vertical vibrations of the system model can already be analysed.

From mathematical view-point, the following questions should be naturally raised:

1. Will a weakly stationary output process $\mathbb{Z}_t$ of the system (1.2) always exist for any weakly stationary input $\mathbb{F}_t$ (random excitation) and for any parameters $m$, $m_p$, $m_k$, $k_v$, $k_p$, $s_v$ and $s_p$? As we shall see in the next section, these questions should not always be answered affirmatively (cf. [1–3, 5], too).
2. Whether solution process $\mathbb{Z}_t$ is asymptotically stable in Lyapunov sense, i. e. all other solutions of (1.2) tend (exponentially) a.s. or in $\mathbf{L}^1$ to the process $\mathbf{Z}_t$ as $t$ tends to the infinity.
3. Under what conditions of the parameters and $\mathbf{F}_t$ the weakly stationary solution process $\mathbf{Z}_t$ found by I. ZOBORY [6] exists and whether it is asymptotically stable. Such conditions will be set up in the third section. Then we can state that this solution process $\mathbf{Z}_t$ has necessary and 'nice' mechanical properties.

## 2. Some Mathematical Results to be Used

The three questions arisen at the end of the previous section are very closely related to the study of the stability behaviour of the stationary solutions of differential equations with random parameters elaborated by e. g. H. BUNKE [2], T. V. NHUNG [3–5], etc. To answer those questions concerning ZOBORY's model [6] the following mathematical results are required.

It is wellknown, from *Floquet theory,* that the fundamental matrix $\mathbf{Q}(t)$ ($\mathbf{Q}(t_0) = \mathbf{I}$) of a periodic system

$$\dot{\mathbf{q}} = \mathbf{A}(t)\mathbf{q}, \tag{2.1}$$

where $\mathbf{A}(t)(t \in \mathcal{R})$ is a continuous $T$-periodic $n \times n$-matrix function, has the form $\mathbf{Q}(t) = \mathbf{P}(t)\exp(t\mathbf{B})$, where $\mathbf{P}(t)$ is a differentiable $T$-periodic non-singular $n \times n$-matrix function and $\mathbf{B}$ is a constant $n \times n$-matrix. If all Floquet characteristic exponents of $\mathbf{A}(t)$ have negative real parts:

$$\max_i \operatorname{Re}\lambda_i < -r < 0, \tag{2.2}$$

then we have the estimate

$$\|\mathbf{Q}(t)\mathbf{Q}^{-1}(s)\| \le k\exp\left[-r(t-s)\right] \qquad (k = \text{constant},\ t \ge s). \tag{2.3}$$

One of the interesting results of H. Bunke is the following

*Theorem 2.1* (BUNKE [2], p. 51). Suppose that the following conditions of the perturbed system

$$\dot{y}_t = \left[\mathbf{A}(t) + \mathbf{C}(t)\right]\mathbf{y}_t + \mathbf{z}_t, \qquad t \in \mathcal{R}, \tag{2.4}$$

are satisfied:

(i) The deterministic matrix $\mathbf{A}(t)$ holds as in (2.1), and (2.2);

(ii) The deterministic perturbation $n \times n$-matrix $\mathbf{C}(t)$ is continuous on $\mathcal{R}$ and an estimate

$$\|\mathbf{C}(t)\| \le c\,exp\,(-Kt) \tag{2.5}$$

holds for all $t \ge t_1 \ge 0$ where $c = \text{const} \ge 0$ is small enough, e. g. $c < k^{-1}\min(r, K/2)$, and $K = \text{const} > 0$;

(iii) The $n$-dimensional stochastic process $\mathbf{z}_t$ (random input) is a.s. continuous (i. e. it has continuous trajectories a.e.) and $T$-periodic in the wide sense so that

$$\int_0^T \mathrm{E}\|\mathbf{z}_t\|\mathrm{d}t < \infty.$$

Then every solution of the perturbed system (2.4) converges a.e. to the wide sense $T$-periodic solution

$$\mathbf{x}_t^0 = \int_{-\infty}^{t} \mathbf{Q}(t)\mathbf{Q}^{-1}(s)\,\mathbf{z}_s\,\mathrm{d}s \tag{2.6}$$

of the periodic unperturbed system

$$\dot{\mathbf{x}}_t = \mathbf{A}(t)\mathbf{x}_t + \mathbf{z}_t \tag{2.7}$$

as $t \to \infty$, i. e.

$$\lim_{t\to\infty} ||\mathbf{y}_t - \dot{\mathbf{x}}_t^0|| = 0 \qquad \text{(a.e.)}\,. \tag{2.8}$$

Applying the above theorem of Bunke and a theorem of A. STRAUSS and J. A. YORKE [8], we have proved the following result which is a natural generalization of BUNKE's work ([2] p. 51) for the case when the non-homogeneous term (input) $\mathbf{z}_t$ in (2.7) is also disturbed.

*Corollary 2.1* (NHUNG [3, 5]) Consider the 'completely' perturbed system

$$\dot{\mathbf{y}}_t = [\mathbf{A}(t) + \mathbf{C}(t)]\,\mathbf{y}_t + \mathbf{z}_t + \zeta_t\,, \tag{2.9}$$

where $\mathbf{A}(t), \mathbf{C}(t), \mathbf{z}_t$ are as in Theorem 2.1, and $\zeta_t$ is an a.s. continuous random excitation (on the input) having either the property according to which

$$\lim_{t\to\infty} \int_t^{t+1} ||\zeta_\tau||\,\mathrm{d}\tau = 0 \qquad \text{(a.e.)}\,, \tag{2.10}$$

or

$$\lim_{t\to\infty} \int_t^{t+1} \mathrm{E}\,||\zeta_\tau||\,\mathrm{d}\tau = 0\,. \tag{2.11}$$

Then any solution of (2.9) tends to the $T$-periodic solution $\mathbf{x}_t^0$ given in (2.6) of equation (2.7) either a.e. or in the mean, respectively, as $t \to \infty$ i.e. (2.8) holds for any solution of (2.9) or

$$\lim_{t\to\infty} \mathrm{E}\,||\mathbf{y}_t - \mathbf{x}_t^0|| = 0\,. \tag{2.8'}$$

In NHUNG [3–5], Theorem 2.1 of Bunke has been essentially extended to a large class of random perturbations, e. g. to the case of equation:

$$\dot{\mathbf{y}}_t = [\mathbf{A}(t) + \mathbf{C}_t]\,\mathbf{y}_t + \mathbf{z}_t + \zeta_t\,, \tag{2.12}$$

where the disturbance matrix $\mathbf{C}_t$ now may be also random, or to the most general case of the equation

$$\dot{\mathbf{y}}_t + \mathbf{A}(t)\mathbf{y}_t + \mathbf{z}_t + \mathbf{f}(t, \mathbf{y}_t, \omega)\,, \tag{2.13}$$

where the disturbance vector process $\mathbf{f}$ may be random and non-linear in $\mathbf{y}_t$ (cf. Theorem 2.1 and 3.1 in [3]). The following results concern the system (2.12).

*Theorem 2.2* (NHUNG [3, 5]). Assume that matrix $\mathbf{A}(t)$ in (2.12) is as in Theorem 2.1, and the following conditions are satisfied:

(i) Random matrix $\mathbf{C}_t$ is a.s. continuous on $\mathcal{R}$ so that

$$\lim_{t\to\infty} ||\mathbf{C}_t|| = 0 \qquad \text{(a.e.)}; \tag{2.14}$$

(ii) Random process $\mathbf{z}_t$ is a.s. continuous and $T$-periodic with the following property:

$$\int_0^T \mathrm{E}\,||\mathbf{z}_t||\,\mathrm{d}t < \infty\,,$$

and there exist a $\bar{t} \geq 0$ and a random variable $h = h(\omega)$ so that for $t \geq \bar{t}$ we have

$$||\mathbf{z}_t|| < h \qquad \text{(a.e.)}\,. \tag{2.15}$$

(iii) Random excitation $\zeta_t$ on the input is a.s. continuous so that

$$\lim_{t\to\infty} \int_t^{t+1} ||\zeta_\tau||\,\mathrm{d}\tau = 0 \qquad \text{(a.e.)}\,. \tag{2.16}$$

Then each solution of the disturbed equation (2.12) converges a.e. to the $T$-periodic solution $\mathbf{x}_t^0$ of (2.7) as $t \to \infty$, i.e. (2.8) holds.

*Theorem 2.3* (NHUNG [3, 5]). Suppose that conditions (i) and (iii) of Theorem 2.1 and the following ones are satisfied for equation (2.12):

(i) Random matrix $\mathbf{C}_t$ is a.s. continuous and independent of the input $\mathbf{z}_t$ so that (2.14) and

$$\lim_{t\to\infty} \mathrm{E}||\,\mathbf{C}_t|| = 0 \tag{2.17}$$

hold;

(ii) Random excitation $\zeta_t$ on the input is a.s. continuous so that

$$\lim_{t\to\infty} \int_t^{t+1} \mathrm{E}||\,\zeta_\tau||\,\mathrm{d}\tau = 0\,. \tag{2.18}$$

Then each solution $\mathbf{y}_t = \mathbf{y}(t; t_0, y_0(\omega), \omega)$ of equation (2.12) with $t_0$ large enough, and $\mathrm{E}\,||\mathbf{y}_0|| < \infty$ converges in the mean to the $T$-periodic solution $\mathbf{x}_t^0$ of equation (2.7), i. e.

$$\lim_{t\to\infty} \mathrm{E}||\,\mathbf{y}(t; t_0, \mathbf{y}_0(\omega), \omega) - \mathbf{x}_t^0(\omega)|| = 0\,. \tag{2.19}$$

*Remark 2.1* Of course, all results stated in this section are automatically true for the case when $\mathrm{A}(t) \equiv \mathrm{A} =$ constant matrix and $\mathbf{z}_t$ is a stationary input process.

## 3. Applications to the Elementary Model of Railway Vehicles

Let us now return to the system (1.1). Introducing the notations

$$z_t^1 = z_t\,, \qquad z_{kt}^1 = z_{kt}\,,$$

$$z_t^2 = \dot{z}_t\,, \qquad z_{kt}^2 = \dot{z}_{kt}\,,$$

we get the following systém of 4 linear random differential equations of the first order, which is equivalent to the original one (1.1)

$$\begin{bmatrix} \dot{z}_t^1 \\ \dot{z}_t^2 \\ \dot{z}_{kt}^1 \\ \dot{z}_{kt}^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{s_v}{m} & -\dfrac{k_v}{m} & \dfrac{s_v}{m} & \dfrac{k_v}{m} \\ 0 & 0 & 0 & 1 \\ \dfrac{s_v}{m_p + m_k} & \dfrac{k_v}{m_p + m_k} & -\dfrac{s_v + s_p}{m_p + m_k} & -\dfrac{k_v + k_p}{m_p + m_k} \end{bmatrix} \cdot \begin{bmatrix} z_t^1 \\ z_t^2 \\ z_{kt}^1 \\ z_{kt}^2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{m_p + m_k}(s_p u_t + k_p \dot{u}_t + m_p \ddot{u}_t) \end{bmatrix}. \tag{3.1}$$

In the case of system (3.1), the Floquet characteristic exponents of system matrix $\mathbf{A}$ are just the eigenvalues of $\mathbf{A}$, i. e. the solutions of the following characteristic equation

$$\det(\mathbf{A} - \lambda\mathbf{I}) = \begin{bmatrix} -\lambda & 1 & 0 & 0 \\ -\dfrac{s_v}{m} & -\dfrac{k_v}{m} - \lambda & \dfrac{s_v}{m} & \dfrac{k_v}{m} \\ 0 & 0 & -\lambda & -1 \\ \dfrac{s_v}{m_p + m_k} & \dfrac{k_v}{m_p + m_k} & -\dfrac{s_v + s_p}{m_p + m_k} & -\dfrac{k_v + k_p}{m_p + m_k} - \lambda \end{bmatrix} = 0\,. \tag{3.2}$$

From (3.2) we get the following 4th order algebraic equation

$$\lambda^4 + \lambda^3 \left[ \frac{k_v(m_p + m_k) + m(k_v + k_p)}{m(m_p + m_k)} \right]$$

$$+\lambda^2 \left[ \frac{k_v k_p + m(s_v + s_p) + s_v(m_p + m_k)}{m(m_p + m_k)} \right]$$

$$+\lambda \frac{k_v s_p + k_p s_v}{m(m_p + m_k)} + \frac{s_v s_p}{m(m_p + m_k)} = 0 . \tag{3.3}$$

Our question is now the following: Under what conditions imposed on the parameters $m$, $m_p$, $m_k$, $k_v$, $k_p$, $s_v$, $s_p$ will the polynomial on the left-hand side of (3.3) be a Hurwitz one? In other words, when will all 4 solutions of (3.3) have negative real parts

$$\operatorname{Re} \lambda_i < 0 , \qquad i = 1, 2, 3, 4? \tag{3.4}$$

In that case, the essential condition (2.2) ensuring the stability in §2 will be satisfied.

From the Routh–Hurwitz criterion (see e. g. L. ARNOLD [1], p. 196) it is wellknown, that an algebraic polynomial of the fourth order of the form

$$P_4(\lambda) = a_0 \lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 ,$$

where $a_0$, $a_1$, $a_2$, $a_3$ and $a_4$ are real numbers, $a_0 > 0$, is a Hurwitz one if and only if the following inequalities hold:

$$a_1 > 0 , \quad a_3 > 0 , \quad a_4 > 0 , \quad a_3(a_1 a_2 - a_0 a_3) - a_4 a_1^2 > 0 . \tag{3.5}$$

Applying conditions (3.5) to (3.3) and taking into account that the parameters are positive, we get the following inequality[2]:

$$(k_v s_p + k_p s_v) \{ [k_v(m_p + m_k) + m(k_v + k_p)]$$

$$\cdot [k_v k_p + m(s_v + s_p) + s_v(m_p + m_k)]$$

$$-m(m_p + m_k)(k_v s_p + k_p s_v) \} - s_v s_p [k_v(m_p + m_k) + m(k_v + k_p)]^2 > 0 . \tag{3.6}$$

Our aim is to apply the mathematical results formulated in §2 to system (1.1) or, equivalently, to system (3.1). The following assertion is directly obtained from Theorem 2.1 of Bunke.

[2] It was pointed out by E. Zibolen that inequality (3.6) is satisfied in the dynamical model of ZOBORY [6].

*Corollary 3.1* Suppose that

(i) the three inequalities in (3.6) are satisfied;
(ii) the random excitation process $\mathbf{u}_t$ in (1.1) and (3.1) is a.s. differentiable up to the third order and weakly stationary so that

$$\mathrm{E}|\mathbf{u}_t| < \infty\,, \qquad \mathrm{E}|\dot{\mathbf{u}}_t| < \infty\,, \qquad \mathrm{E}|\ddot{\mathbf{u}}_t| < \infty\,, \tag{3.7}$$

then the random process

$$\mathbf{x}_t^0 := \int_{-\infty}^{t} e^{\mathrm{A}(t-s)} v_s \, \mathrm{d}s\,, \tag{3.8}$$

where

$$\mathrm{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{s_v}{m} & -\dfrac{k_v}{m} & \dfrac{k_v}{m} & \dfrac{k_v}{m} \\ 0 & 0 & 0 & 1 \\ \dfrac{s_v}{m_p + m_k} & \dfrac{k_v}{m_p + m_k} & -\dfrac{s_v + s_p}{m_p + m_k} & -\dfrac{k_v + k_p}{m_p + m_k} \end{bmatrix} \tag{3.9}$$

and

$$\mathbf{v}_t = \mathrm{col}\left[0, 0, 0, \frac{1}{m_p + m_k}(s_p \mathbf{u}_t + k_p \dot{\mathbf{u}}_t + \ddot{\mathbf{u}}_t)\right] \tag{3.10}$$

is a weakly stationary solution to (3.1), which is globally stable in the sense that for any other solution $\mathbf{x}_t = \mathrm{col}(z_t^1, z_t^2, z_{kt}^1, z_{kt}^2)$ of (3.1), we have

$$\lim_{t\to\infty} \|\mathbf{x}_t - \mathbf{x}_t^0\| = 0 \qquad \text{(a.s.)}\,. \tag{3.11}$$

*Remark 3.1* By using (3.8), (3.9) and (3.10), we can write the explicit analytic expression for the vertical displacements $z_t$, $z_{kt}$, and thus for the normalized vertical axle force process $\tilde{T}_t$ in (1.3), too. Note that the spectral density matrix of vector process $\mathbf{Z}_t = \mathrm{col}(z_t, z_{kt})$ and the spectral density function $\tilde{T}_t = T_t - T_0$ are given in ZOBORY [6] (see (12) and (15)).

The Corollary 2.1 (NHUNG [3, 5]) applied to (3.1) yields
*Corollary 3.2* Suppose that conditions (i) and (ii) of Corollary 3.1 are satisfied, and moreover that $\zeta_t$ is a one-dimensional a.s. continuous random process (the perturbation of $\mathbf{u}_t$) having either the property:

$$\lim_{t\to\infty} \int_{t}^{t+1} |\zeta_\tau| \, \mathrm{d}\tau = 0 \qquad \text{(a.s.)} \tag{3.12}$$

or

$$\lim_{t\to\infty} \int_t^{t+1} \mathrm{E}|\zeta_\tau| \,\mathrm{d}\tau = 0\,. \tag{3.13}$$

Then any solution to the order perturbed system first

$$\dot{\mathbf{x}}_t = \mathbb{A}\mathbf{x}_t + \mathbf{v}_t + \xi_t\,, \tag{3.14}$$

where $\mathbb{A}$ and $\mathbf{v}_t$ are given in (3.9) and (3.10), and

$$\xi_t = \mathrm{col}\left[0, 0, 0, \frac{1}{m_p + m_k}(s_p\zeta_t + k_p\dot{\zeta}_t + m_p\ddot{\zeta}_t)\right]\,, \tag{3.15}$$

tends to the weakly stationary solution $\mathbf{x}_t^0$ given in (3.8) to the system

$$\dot{\mathbf{x}}_t = A\mathbf{x}_t + \mathbf{v}_t \tag{3.16}$$

either a.s. or in the mean, respectively, as $t \to \infty$.

*Remark 3.2* The conditions (3.12) and (3.13) characterize a large class of perturbations of the random excitation $\mathbf{u}_t$. It is easy to see that perturbations $\zeta_t$ will not necessarily converge to zero a.s. or in the mean as $t \to \infty$, even will not necessarily be bounded a.s. or in the mean on any finite interval of $[0, \infty]$.

In the case when the constant system matrix $\mathbb{A}$ in (3.14) is also perturbed by some random matrix $\mathbf{C}_t$ satisfying conditions (2.14) and (2.17), then Theorems 2.2 and 2.3 in §2 of NHUNG [3, 5] may be used. It should be noted that in ZOBORY [6] and ZOBORY and PÉTER [9], the time-dependent random track excitation $\mathbf{u}_t$ at a constant travelling speed $\dot{x}_0$ usually has the form

$$u_t = \Psi_0 + \sum_{k=1}^{N} 2\sqrt{S(\Omega_k)\Delta\Omega_k} \cdot \cos(\Omega_k \dot{x}_0 t + \Psi_k)\,,$$

where

$\Psi_0 \in \mathcal{N}(0, \sigma^2) = \mathcal{N}(0, S(\Omega_0)\Delta\Omega_0)$,
$\Psi_i \in \xi[-\pi, \pi]$ (the class of the uniform distributions over interval $[-\pi, \pi]$)

$i = 1, 2 \ldots N$, where $S(\Omega)$ stands for the spectral density function of the track unevennesses $u_x^*$ along the track. Subscript $x$ designates the longitudinal coordinate of the track. It is obvious that at a constant travelling speed $\dot{x}_t$ $u_t = u^*_{\dot{x}_0 t}$. Thus, condition (2.15) is satisfied.

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