A QUASI-GROUP ASSOCIATED WITH THE WEB FORMED BY THREE PENCILS OF CIRCLES WHICH BELONG TO THE SAME BUNDLE

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Abstract

It is well-known that the 3-web formed by three pencils of straight lines is topologically equivalent to the hexagonal 3-web composed of three pencils of circles which belong to the same bundle. In this work, corresponding to the web under consideration a quasi-group is constructed with the help of the property mentioned.

1. Preliminaries

Three pencils of circles are said to belong to the same bundle, if there exist a real circle or a circle with null sheet or a circle of radius zero [1, p. 64] cutting all the circles of the pencils at right angles.

It is well known that, in the plane, three pencils of straight lines form a hexagonal web [2] and that this web is topologically equivalent to the 3-web formed by three pencils of circles which belong to the same bundle [2], [3]. Blaschke gave a few examples of circle pencils which form a hexagonal 3-web [2]. He, further, stated that it was very difficult to solve this problem in the general case. This latter problem is completely solved in my dissertation [3], [4].

The problem of determining hexagonal n webs ($n > 3$) of circle pencils is studied by myself and presented at the International Conference On Web Geometry which was held in August 1987, Szeged (Hungary).

It seems reasonable to construct quasi-groups corresponding to the hexagonal 3-webs studied in [3].

In this short note, a quasi-group corresponding to the 3-web which is formed by three pencils of circles of the same bundle, is constructed.

2. Quasi-group

A set G is said to be a quasi-group with binary operation ($\cdot$), if the following conditions are satisfied [5], [6], [7]:

$$x \cdot b = c, \tag{2.1}$$

and

$$a \cdot y = d \tag{2.2}$$

has exactly one solution for x and y respectively.

If $x, y \in G$, the ordered pair (x, y) is called a "point".

The set of points (c, y) (c is a constant and y is variable) is called a "1-curve" while the set of points (x, c) (x is variable) is called a "2-curve". On the other hand, if c is a fixed element of G and $x \cdot y = c$, then the set of points (x, y) is called a "3-curve". The set of 1-curves, 2-curves and 3-curves is the web corresponding to the quasi-group.

3. The main problem

We will now try to find a quasi-group corresponding to a web formed by three pencils of circles which belong to the same bundle. To do this, we make use of the fact that the web under consideration ist opologically equivalent to the three-web which is formed by three pencils of straight lines. If $x, y, u \in \mathbf{R}$ (the set of real numbers) and $u \neq 0$, we define the binary operation $(\cdot)$ by the equation

$$x \cdot y = ux + y. \tag{3.1}$$

It is easy to see that this operation satisfies conditions (2.1) and (2.2). In fact, from (2.1) we find $ux + b = c$ which gives $x = \dfrac{c-b}{u}$. From (2.2) we find $ua + y = d$ and consequently $y = d - ua$.

On the other hand,

i) The set of points (u_1, y) is "1-curves", where U_1 is a constant and y is a variable (Fig. 1),

ii) The set of points (x, U_2) is "2-curves", where x is a variable and U_2 is a constant (Fig. 2), and

iii) The set of points $(x, y) = (x, -ux + U_3)$ is "3-curves", where $x \cdot y = U_3$ (U_3 a constant), (Fig. 3).

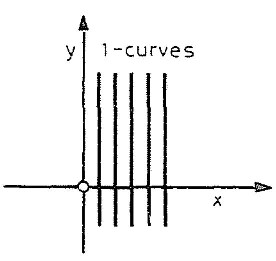

Fig. 1

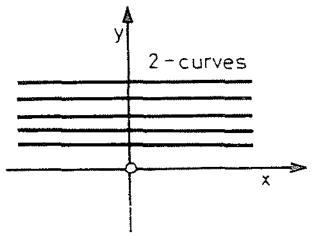

Fig. 2

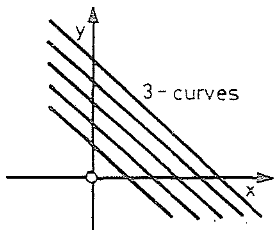

Fig. 3

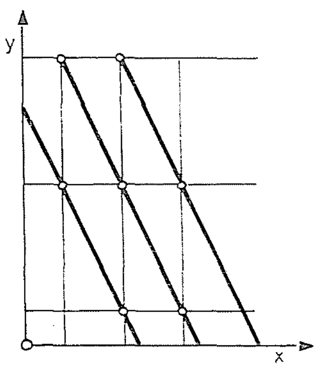

Fig. 4

Therefore, by means of the binary operation (3.1) we have assigned a quasi-group corresponding to the hexagonal 3-web which is formed by three pencils of straight lines, (Fig. 4). Since, by suitable inversion, we can transform the three pencils of straight lines under consideration into three pencils of circles belonging to the same bundle, the quasi-group which we have already found corresponds also to the web formed by the three pencils of circles of the same bundle.

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