THE APPLICATION OF THE NON-FOURIER HEAT CONDUCTION EQUATION

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Abstract

An increasing number of publications proposing various modified forms of the heat conduction equation have appeared in the past decades. Most of them belong to hyperbolic type differential equations (HDE) instead of the traditional parabolic type Fourier equation (PDE). In spite of the hight number of papers, part of which are dealing with the solution of the modified equations, there is no method to decide which form of the heat conduction equation should be used in given engineering applications. There is no definite answer to the question what the validity region of PDE is, where the switch over to modified forms can be reasonable. The situation is confusing partly due to the lack of reliable experimental verification of theoretical conclusions, as well as due to the non-matured technique of handling the hyperbolic and other types of equations for solving practical problems.

In the first part of the paper a short overview of different physical mechanisms resulting in various, non-Fourier type heat conduction equations is given.

Then the most widely known hyperbolic equation with only one additive second order time derivative term (so called single relaxation time model) is discussed in detail.

$$\tau_r \ddot{T} + \dot{T} = aT''_{xx}.$$

Dimensional analysis is applied to clarify the role of the second order term independently of the unknown material properties. After the order-of-magnitude analysis, the numerical solution of both PDE and HDE will be given. The solutions of both equations will be compared for the cases of pulse and jump-like changes in temperature and heat flux at the surfaces of a semi-infinite body as well as a wall of finite thickness.

The problem of giving the initial and boundary conditions will also be discussed. When applying HDE for heat conduction, a second initial condition should be ensured, which sometimes is not adequately known. The equations describing boundary conditions of 2nd and 3rd kind can have different forms, according to the type of constitutive equations, i.e. to the physical model of the relaxation phenomenon.

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