THE MATHEMATICAL PROBLEMS OF A HEAT TRANSFER PHENOMENON

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Abstract

This paper is dealing with the numerical problems of the solution of two-dimensional heat conduction phenomenon. These can be groupped into three almost separated points:

I. dimensional analysis numerical, how to choose the dimensionless parameters;
II. the stability problem of the boundary condition of third kind;
III. advantages of using the Bajcsay method to solve the differential equation.

Introduction

We investigated the distortion effect of the heat flux sensors. The following mathematical model was applied:

The differential equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 F}{\partial z^2} = 0$$

Boundary conditions:

1. $T = T_0 = \text{const}$ or $qn = \text{const}$
2. and 4. $qn = 0$
3. $\alpha \cdot (T - T_\infty) = \frac{\partial T}{\partial z}$,

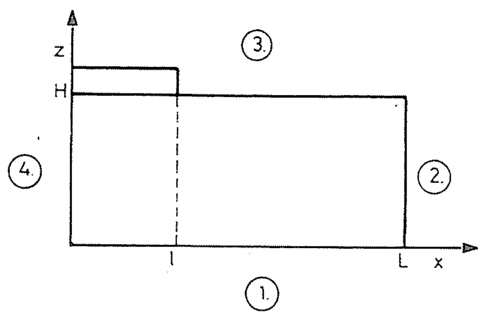

Fig. 1

where

$$\alpha = \begin{cases} \dfrac{1}{\dfrac{1}{\alpha_0} + Rb}, & x \in [0, 1] \\ \alpha_0, & x \in [1, L] \end{cases}$$

I. As there are several sets of independent dimensionless groups, there are two respects:

a) the dimensionless parameters must have clear physical and mathematical meaning;

b) they must help to reduce the numerical difficulties.

So we have chosen the following dimensionless parameters:
the dependent variable:

$$T^* = \frac{T - T_\infty}{T - T_\infty} \cdot \left[\frac{\lambda}{H \cdot \alpha} + 1 \right]$$

the independent variables:

the parameters of the continuum model:

- Heat: $\begin{cases} Bi = \dfrac{\alpha_0 \cdot l}{\lambda} \\ Rb = \alpha_0 \end{cases}$
- Geometrical: $\begin{cases} M = \dfrac{L}{l} \\ P = \dfrac{H}{l} \end{cases}$

the parameter of the discretization: $M_{hfs} = \dfrac{l}{D}$.

If we make such dimensionless parameter T then values of T^* are between 0 and 1. So the errors indicated by the finite representation will be smaller.

II. In the case of third kind boundary condition, sometimes stability problems may occur. We have dealt with the following three cases:

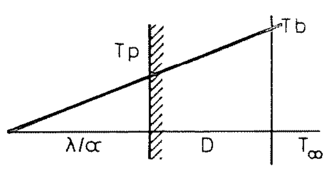

Fig. 2. Differential approximation with inlet points

$$\alpha(T_p - T_\infty) = \frac{Tb - Tp}{D}$$

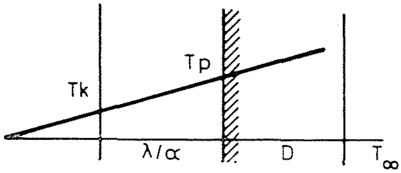

Fig. 3. Differential approximation with outlet points

$$\alpha(T_p - T_\infty) = \frac{Tp - Tk}{D}$$

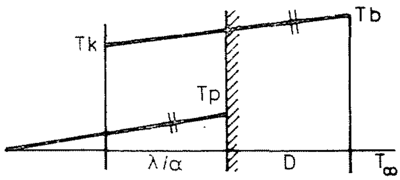

Fig. 4. Central

$$\alpha(T_p - T_\infty) = \frac{Tb - Tk}{2 \cdot D}$$

By a simple model we examined whether it is convergent in function of parameters and α.

These results are:

a) unconditionally convergent

b) convergent if $\frac{\lambda}{\alpha} > \frac{D}{3}$

c) convergent if $\frac{\lambda}{\alpha} > D$

Our calculation was in good correlation with this result.

We can generalize this convergence examination method for every boundary condition of third kind

$$a \cdot f' + b \cdot f = c$$

where a, b, c are rational figures.

III. If we solve the Laplace differential equation with boundary condition of second kind on one side, by the well-known finite difference method, the convergence was very slow, and big mass storage was needed for this method.

This indicated that we used the Bajcsay method based on semidiscretization and consequently all these two problems have been avoided.

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