EXTRAPOLATION IN THE EXPERIMENTAL RESULTS OF C.I.E.

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Based on the main rules of colour mixtures, we can infer as a consequence that:

Adjusting the power of three lamps in a proper way, we can synthesize the white colour in the projection zone of the screen.

In the same way, varying those powers, we are able to obtain a very great range of colours.

Really, the basis of this process is as follows.

"Each one of the three coloured lights strikes the screen, is totally reflexed and finally, arrives at the observer's eye.

The eye receives three radiations, but notices just one colour; I mean, the eye adds the effects. In other words, the human eye works by adding the luminous radiations."

By justifying this internal composition law, we are going to report our research results.

1) We have demonstrated that the electromagnetic spectrum radiations as a whole have the structure of a vectorial space.

Besides that, we prove that the family of three colours: red, green, and blue violet is a "total family" of vectors. Experience shows that none of them can be obtained by adding the others (free family). So we have a vectorial space defined on R, third dimension, and isomorphus with the R^3 vectorial space.

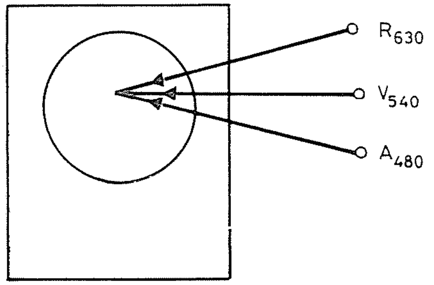

Fig. 1

2) Otherwise, the unit of the electromagnetic spectrum radiation is a dote interacting with R^3 vectorial space.

3) Moreover, in that unit an equivalent relation is defined, because terms which have their coordinates in proportion represent the same radiation.

So, the basis of all the equivalence classes is a surface with a positive and unalterable curvature this surface being limitless but finite.

In the same way, we demonstrate that this geodesical surface is limitless and finite without parallel straight lines.

4) A representative mathematical pattern is created from all spectrum radiations. This pattern has representation in Rienman space, where we establish a non-Euclidean geometry of elliptic type, with a spherical surface (radio one) base.

Finally, two plain figures are planned.

The first one is based on a stereographic projection of the electomagnetic spectrum as a whole.

In the second one and with the use of trilineal coordinates which particularized seem ideal, it is possible to draw on the plain all the visible radiations for the human eye.

Let us examine on the abscissa axis: the wavelength λ(nm) corresponds to different electromagnetic spectrum radiations; and on the ordinate, luminous stimuli E_λ are produced by the radiations on the human eye detector, which we measure in stimulus units, defined by us and in relation with lumen and watt.

Radiations visible to the human eye (360—760 nm interval) are well known; that bright yellow colour ($\lambda = 580$ nm) stimulates much more sight sense than, for example, dark violet colour ($\lambda = 440$ nm); in our reference we show such a thing by yellow (A) and violet (V) points.

The topic of stimulus was studied experimentally by C.I.E. with experiences with human observers, and analyzing their reactions to different colours, they took as an average result, a curve called:

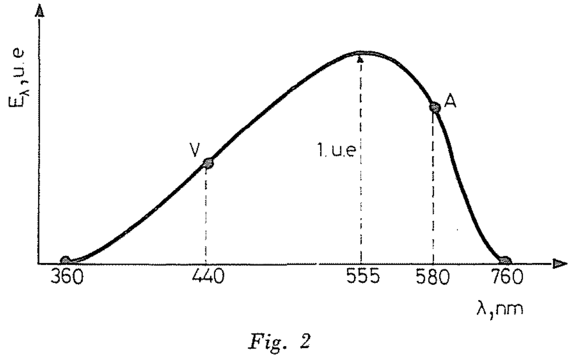

Fig. 2

"Luminous stimulus pattern human observer curve", drawn in this figure.

In the figure it is observed that the most stimulating colour to sight sense (and with a stimulus unit) has a wavelength of about 555 nm, in correspondence with the "bright green yellow" colour.

As C.I.E. worked only with human observers, their curve will be defined only in the 360—760 nm interval, but as a result of our studies about the other radiations, we ourselves propose to find the mathematical equation $f(\lambda)$, of a curve with a proper accommodation to C.I.E. results on the 360—760 nm interval.

Mathematical equation calculation

We observed the bell-shaped curve from C.I.E. and compared it with the density distribution curves: Normal, Student, etc. . .

The Normal distribution curve was the closest one, but the C.I.E. curve has some asymmetry.

So, we rejected the other curves and our work was concerned with the density function of the normal distribution curve in the way indicated:

First, we developed up to the fourth grade term, the density function of Normal distribution (0,1), the equation of which is

$$\frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

the result being:

$$\frac{1}{\sqrt{2\pi}} e^{-t^2/2} \simeq \frac{1}{\sqrt{2\pi}\,(1 + t^2/2 + t^4/8)}$$

from this formula we took the expression with four parameters:

$$\frac{K}{a_0 + a_2 t^2 + a_4 t^4}$$

and it is seen as a pair function, so it is also symmetrical with respect to $t = 0$.

Taking into consideration that although the C.I.E. curve is bell shaped, it shows a certain symmetry, it is clear that just with pair terms we would not obtain the desired approximation so we introduce the t, t^3, t^5 terms multiplied by a_1, a_3, a_5, and finally doing an origin transfer to the bright green yellow point ($\lambda = 555$ nm), replacing t for $\dfrac{\lambda - 555}{\sigma}$.

With all of this, we finally arrived at the following mathematical equation which depends on 8 parameters.

$$f(\lambda) = \frac{K}{\sum_{n=0}^{n=5} a_n \left(\dfrac{\lambda - 555}{\sigma}\right)^n}.$$

So, our work has been to obtain the best value of those 8 parameters, observing that for each wavelength λ inside the spectrum visible to humans, the absolute value $(E_\lambda - f(\lambda))$ of the difference between the stimulus calculated by C.I.E. and our mathematical equation was less than one hundredth of the unit of stimulus.

Therefore, taking 60 points from the C.I.E. curve, with similar intervals between them, we propose to simplify the expression.

$$\sum_{i=1}^{60} |E_{\lambda i} - f(\lambda_i)| = \emptyset (K, \sigma, a_0, a_1, \ldots, a_5).$$

This means that we tried to simplify an eight parameter function.

Conclusions

As there were 8 parameters to obtain, it was difficult to find an easy procedure to simplify their calculation, because of the time the machine needed; so we tried the method of least squares. This method, allowed us to calculate the optimum values of the 8 parameters with five decimals, by a computer.

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