INTEGRAL TRANSFORMS TO GET THE ROAD PROFILE EXCITATION SPECTRAL MATRIX FOR VARIABLE VEHICLE SPEEDS IN CASE OF TWO-INPUT SYSTEMS

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Abstract

Operation at variable speed, in many cases with a certain periodicity, is a characteristic feature of urban traffic. A method is presented for the mathematical derivation of distorted road profile spectra in the case of varying travel speeds. The resulting spectrum is proposed for the dynamic design of vehicles, it is calculated by an integral transformation.

Introduction

Taking into account the variable travel speed in dimensioning for dynamic stresses is particularly important for vehicles in urban traffic throughout their service lives. Such vehicle types are urban buses and trolleybuses, taxis and urban tracked vehicles. [1, 3, 5]

In case of uniform travel speeds, the stochastic road profile processes have spectra well-known from the literature. It is known that these processes can be considered up to second order as stationary, Gaussian processes. [4, 6]

Throughout the paper we assume that the travel speed processes are also stationary and are independent of the road profile processes. [2, 7, 8, 9]

Integral transformation of the road profile spectra
Two-input systems

In our paper we investigate the cross spectra of excitations affecting the wheels of a vehicle travelling at variable speeds. The unevennes of the road profile is described as a stationary stochastic process $U(s)$, where s is the distance from the origin, and U is the level of the road (Fig. 1). Let K denote the covariance function of U, that is

$$K(x) = \mathbf{E}[U(s+x)U(s)] \tag{1}$$

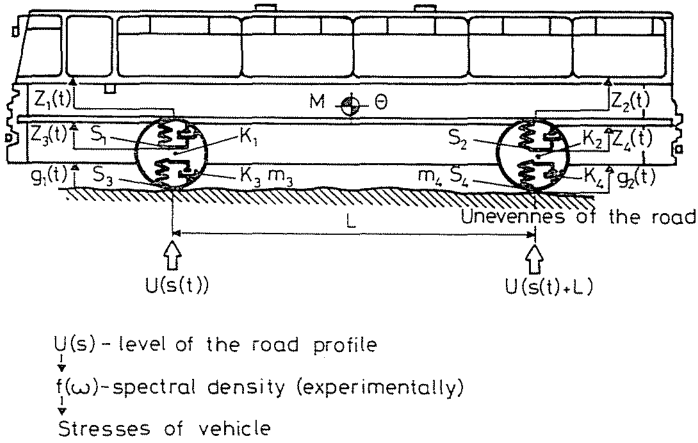

Fig. 1

and consider its inverse Fourier transform

$$f(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} K(x) e^{-i\omega x} dx \tag{2}$$

The function f is the spectral density function of the road profile U and it can experimentally be determined. Thus we assume that the functions f and K are given. In a more systematic study of vibrations of the vehicle one has to take into account that the same stresses appear at the front and also at the rear wheels. The actual profile at the rear and at the front wheels are $U(S(t))$ and $U(S(t) + L)$, respectively, where L is the distance between the wheels and $S(t)$ denotes the distance covered in time t, thus

$$S(t) = \int_0^t V(\tau) d\tau. \tag{3}$$

It is quite natural to assume that the velocity process $V(t)$ is a stationary stochastic process of second order, and $V(t)$ is independent of $U(s)$. The covariance matrix is the following

$$\begin{bmatrix} \operatorname{cov}[U(S(\tau)), & U(S(\tau + t))] & \operatorname{cov}[U(S(\tau)), & U(S(t + \tau) + L)] \\ \operatorname{cov}[U(S(\tau) + L), & U(S(\tau + t))] & \operatorname{cov}[U(S(\tau) + L), & U(S(t + \tau) + L)] \end{bmatrix} =$$

$$= \begin{bmatrix} C_0(t) & C_L(t) \\ C_{-L}(t) & C_0(t) \end{bmatrix} \tag{4}$$

The elements of the matrix (4) do not depend on τ, because the composed function $U(S(t) + L)$, $L \in \mathbf{R}$ is stationary again. Obviously $C_0(t)$ and $C_{-L}(t)$

can be obtained from $C_L(t)$ by inserting $L = 0$ and $-L$ in place of L. Thus it is enough to evaluate $C_L(t)$.

$$\begin{aligned} C_L(t) &= \mathbf{E}\big[U\big(S(\tau)\big) \cdot U\big(S(t+\tau)+L\big)\big] = \\ &= \mathbf{E}\big[\mathbf{E}\,[U\big(S(\tau)\big)U\big(S(t+\tau)+L\big)|\big(S(\tau),\ S(t+\tau)\big)]\big] = \\ &= \iint_{\mathbf{R}^2} \mathbf{E}[U(x)U(y+L)|S(\tau)=x,\ S(t+\tau)=y]G_t(dx,dy) = \\ &= \iint_{\mathbf{R}^2} \mathbf{E}[U(x)U(y+L)]G_t(dx,dy) \end{aligned} \tag{5}$$

as U and S are independent processes. Obviously,

$$G_t(x,y) = \mathbf{P}(S(\tau) < x,\ S(t+\tau) < y).$$

Inserting (1) into (5) we have

$$\begin{aligned} C_L(t) &= \iint_{\mathbf{R}^2} K(y+L-x)G_t(dx,dy) = \\ &= \int_{-\infty}^{\infty} K(s+L)G_t(ds) \end{aligned} \tag{6}$$

where

$$G_t(s) = \mathbf{P}(S(t+\tau) - S(\tau) < s) \tag{7}$$

$$\big(\mathbf{P}(S(t) < s\big) = \mathbf{P}\big(\int_0^t V(x)dx < s)\big).$$

The distribution function of $S(t)$ is equal to the distribution function of $-S(-t)$ thus, for $t < 0$, we have:

$$G_t(s) = \begin{cases} \mathbf{P}\big(S(t) < s\big) & \text{if} \quad t \geq 0 \\ 1 - \mathbf{P}\big(S(-t) \leq -s\big) & \text{if} \quad t < 0. \end{cases} \tag{8}$$

The Fourier transform $\tilde{f}_L$ of $C_L(t)$ of (6) reads as

$$f_L(\tilde{\omega}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t}C_L(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t}\int_{-\infty}^{\infty} K(s+L)G_t(ds)dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t+is\omega+iL\omega}f(\omega)d\omega G_t(ds)dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t+is\omega+iL\omega}f(\omega)d\omega G_t(ds)dt +$$

$$+\frac{1}{2\pi}\int_0^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t+is\omega+iL\omega} f(\omega)d\omega G_t(ds)dt =$$

$$=\frac{1}{2\pi}\int_0^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\tilde{\omega}t-is\omega+iL\omega} f(\omega)d\omega G_{-t}(-ds)dt +$$

$$+\frac{1}{2\pi}\int_0^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t+is\omega+iL\omega} f(\omega)d\omega G_t(ds)dt =$$

$$=\frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{iL\omega}\cos(\tilde{\omega}t - s\omega) f(\omega)d\omega G_t(ds)dt \tag{9}$$

Introducing an integral operator **T**:

$$\mathbf{T}\{f(\omega)\}_{\tilde{\omega}} = \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cos(\tilde{\omega}t - s\omega) f(\omega)d\omega G_t(ds)dt \tag{10}$$

we have:

$$\tilde{f}_L(\tilde{\omega}) = \mathbf{T}\{e^{i\omega L}\cdot f(\omega)\}_{\tilde{\omega}} \tag{11}$$

The cross spectra matrix can be obtained by Fourier transformation from (4), thus in terms of the integral operator **T** we have:

$$\begin{bmatrix} \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t}C_0(t)dt, & \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t}C_L(t)dt \\ \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t}C_{-L}(t)dt, & \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\tilde{\omega}t}C_0(t)dt \end{bmatrix} =$$

$$=\begin{bmatrix} \mathbf{T}\{f(\omega)\}_{\tilde{\omega}}, & \mathbf{T}\{e^{i\omega L}f(\omega)\}_{\tilde{\omega}} \\ \mathbf{T}\{e^{-i\omega L}f(\omega)\}_{\tilde{\omega}}, & \mathbf{T}\{f(\omega)\}_{\tilde{\omega}} \end{bmatrix} =$$

$$= \mathbf{T}\left\{\begin{bmatrix} f(\omega) & e^{i\omega L}f(\omega) \\ e^{-i\omega L}f(\omega) & f(\omega) \end{bmatrix}\right\}_{\tilde{\omega}} \tag{12}$$

where **T** is the integral transformation defined in (10) and (7). Let us consider the special case in which $V(t) = v$ is a constant.

In this case (7) gives a Dirac measure concentrated at the point vt, thus

$$C_L(t) = K(vt + L),\ \text{if}\ V(t) = v \tag{13}$$

$$\tilde{f}_L(\tilde{\omega}) = \int_{-\infty}^{\infty} e^{-i\tilde{\omega}t}K(vt+L)dt = \int_{-\infty}^{\infty} e^{-i\tilde{\omega}\left(\frac{s}{v}-\frac{L}{v}\right)} K(s)\cdot\frac{1}{v}ds$$

whence

$$\tilde{f}_L(\tilde{\omega}) = \frac{1}{v} e^{i\tilde{\omega}\frac{L}{v}} \cdot f\left(\frac{\tilde{\omega}}{v}\right), \quad \text{if} \quad V(t) = v. \tag{14}$$

The cross spectra matrix for a regime of constant velocity:

$$\begin{bmatrix} \frac{1}{v} f\left(\frac{\tilde{\omega}}{v}\right) & e^{i\tilde{\omega}\frac{L}{v}} \cdot \frac{1}{v} f\left(\frac{\tilde{\omega}}{v}\right) \\ e^{-i\tilde{\omega}\frac{L}{v}} \cdot \frac{1}{v} f\left(\frac{\tilde{\omega}}{v}\right) & \frac{1}{v} f\left(\frac{\tilde{\omega}}{v}\right) \end{bmatrix} \tag{15}$$

which is a special case of (12).

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