EFFECT OF PERTURBATION ON THE CRITICAL REYNOLDS NUMBER IN NON-ISOTHERMAL PIPE FLOW

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Abstract

Heat transfer tests with perturbed flow indicate that the nature of the function $Nu = f(Re)$ for the main Nusselt number permits to distinguish between laminar, transient and turbulent domains of non-isothermal air flow. Values of critical Reynolds numbers Re_{cr} and Re' cr separating laminar from transient, and transient from turbulent domains, resp., have been found to depend on the perturbation frequency. With increasing frequency, Re_{cr} values decrease. A relationship has been given for determining Re_{cr}. Experimental observations agree with conclusions drawn from the Tollmien — Schlichting theory of stability of laminar flows.

There are several publications referring to the phenomenon that in pulsating flow the critical value of Reynolds number involving timely average velocity is lower than it would be in steady state for the same mass flow. For instance, Darling [1] in his heat transfer tests with glycerol found the steady-state $Re_{cr} = 2500$ value to drop to a value in the range from 1200 to 2000 in the frequency range $f \leq 17$ Hz, without having reported frequency dependence of Re_{cr}. Elparin & al. [2] found in isothermal water flow the coherent values:

f	0	4.65	5.74	10.72	17.24	19.92
Re_{cr}	2310	1750	1699	1597	1533	1511

Unfortunately, without additional information, these data suit only informative computations. In the following, effect of the mass flow oscillation on the critical Reynolds number will be discussed, in order to refine computational relationships. Underlying test results have been obtained in perturbed non-isothermal air flow in circular pipes [3]. (For other details and outcomes — irrelevant to this subject — of these heat transfer tests see [4].)

Legend

c_p — isobaric specific heat of air at temperature T_m (J/kgK);
d — internal diameter of the test pipe;
f — perturbation frequency (Hz);
L — test pipe length (m);
$\dot{m}$ — air mass flow (kg/s): $\dot{m} = \dot{m}(\tau)$;

$\overline{\dot{m}}$ — timely average mass flow (kg/s): $\overline{\dot{m}} = \frac{1}{B}\int_0^B \dot{m}(\tau)\mathrm{d}\tau$;

n — exponent of Reynolds number;

Nu — mean Nusselt number: $Nu = \frac{\alpha d}{\lambda}$;

$\dot{q}_w$ — heat flux to air (W/m²): $\dot{q}_w = \frac{\dot{m} c_p (T_2 - T_0)}{\pi d L}$;

Re — Reynolds number: $Re = \frac{v_0 d}{\gamma}$;

Re_f — dynamical Reynolds number; $Re_f = \frac{f d^2}{\gamma}$;

Re_{cr} — critical Reynolds number (interpreted in the text);

T_0 — air temperature at pipe inlet (K);

T_2 — air temperature at pipe outlet (K);

T_m — mean air temperature in the pipe (K): $T_m = (T_0 + T_2/2)$;

T_{w1} — pipe wall temperature at inlet (K);

T_{w2} — pipe wall temperature at outlet (K);

ΔT_{ln} — mean logarithmic temperature difference (K);

$$= \frac{(T_{w2} - T_2) - (T_{w1} - T_0)}{\ln \frac{T_{w2} - T_2}{T_{w1} - T_0}}$$

v_0 — cross-sectional mean value of timely average velocity at inlet (m/s):

$$v_0 = \frac{4\overline{\dot{m}}}{\varrho_0 d^2 \pi};$$

α — mean heat transfer coefficient (W/m²K): $\alpha = \frac{\dot{q}_w}{\Delta T_{ln}}$;

λ — heat conductivity coefficient of air at temperature T_m (W/mK);

ν — kinematic viscosity of air at temperature T_m (M²/s);

ϱ_0 — air density at temperature T_0 and atmospheric pressure (kg/m³);

σ — fullness degree (interpreted in the text);

τ — time (s).

In our tests, mass flow oscillation was produced by superposing periodic perturbation on the steady-state flow by means of revolving valves preconnected to the heated test pipe. Dimensionless frequency f of perturbance has been given in terms of the dynamical Reynolds number Re_f. Timely course of, perturbation was either sine half-wave or square wave in form. For the latter case, to describe the perturbation, in addition to Re_f and waveform, fullness degree σ, ratio of the square wave duration by the full period time, has been introduced as third parameter. (In steady-state flow, $\sigma = 1$.)

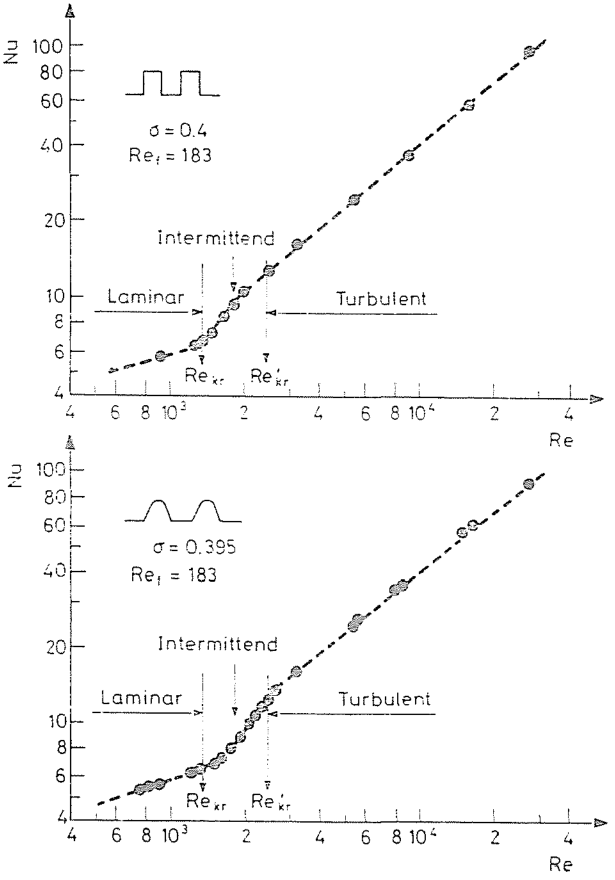

Fig. 1

Obviously, in case of a square wave, fullness degree can be interpreted as quotient of timely average by instantaneous maximum of mass flow. Interpretation $\sigma = \dot{m}/\dot{m}_{\max}$ permits to extend the concept of fullness degree to sinusoidal perturbations. Thereby the fullness degree suits to qualitatively describe the amplitude of the longitudinal pressure gradient oscillation forcing the flow to pulsate, hence, in final account, the perturbation intensity. A low σ value corresponds to a high-intensity perturbation, and a value $\sigma \approx 1$ to a slight perturbation.

Heat transfer tests showed the mean Nussel number in perturbed air flow to depend on the Reynolds number, the dynamical Reynolds number and in certain cases, on the waveform. Two typical measurement results have been plotted as function curves $Nu = f(Re)$ in Fig. 1. These curves have been recorded under pulse-like (top: $\sigma = 0.4$) and sinusoidal (bottom: $\sigma = 0.395$) perturbations at the same frequency ($Re_f = 183$) and about the same fullness degree. The peculiar course of these curves is typical of all the curves recorded throught the ranges of frequencies $35 \leq Re_f \leq 490$, and of fullness degrees $0.2 \leq \sigma \leq 1$ involved in our tests, hence Fig. 1 may underlay general statements on the critical Reynolds number.

Similarly to steady-state flow, also in perturbed flow three distinct ranges of Reynolds numbers appear in the diagram, where variation of the Nusselt number is described by different functions $Nu = f(Re)$. In both extreme ranges these are power functions of the form $Nu = KRe^n$. According to our tests, in the range of low Reynolds numbers, exponent n is rather close to $n = 1/3$ typical of unperturbed laminar flow, and in the range of high Reynolds numbers, to $n = 0.8$ to 0.83, typical of steady-state turbulent flow. In the middle range, relationship $Nu = f(Re)$ cannot be approximated by a power function, it has rather to be given as $Nu = f(Re, Re_f, \sigma,$ waveform$)$.

Existence of the outlined three Re ranges permits to define domains of laminar ($n \approx 1/3$) and turbulent ($n \approx 0.83$) flow in perturbed non-isothermal flows, without a detailed knowledge of their microstructures. Transient flow domain, with mean Reynolds numbers, lays in between. This definition is a symptomatic one, since it relies on the effect: integrated effect on heat transfer intensity — rather than on the change of the flow microstructure (or its completion). Remind that the same is done in hydrodynamics separating the three domains according to the nature of variation of the friction coefficient.

In isothermal steady-state pipe flow, in the range $Re < Re_{cr}$ the laminar flow form is known to be stable against any kind of perturbance. Here the value of the critical Reynolds number is determined by the turbulence degree of the incoming flow, and the pipe wall roughness. In our test equipment, $Re_{cr} = 2264$ for unperturbed flow. Beyond the critical Reynolds number, turbulent nodes appear in the flow, and for a further increase of the Reynolds number, duration of the turbulent state increasingly grows compared to that of laminar at any fixed place in the pipe. At the upper critical Reynolds number $Re = Re'_{cr}$ transition to turbulent flow form has been completed. In our test equipment, $Re'_{cr} = 3700$ was obtained in unperturbed flow. The three typical, distinct domains in perturbed, non-isothermal pipe flow testify of a mechanism analogous to the described one. Accordingly, critical, and upper critical Reynolds numbers will be understood as $Re = Re'_{cr}$, and $Re = Re'_{cr}$ values separating laminar and transient, as well as transient and turbulent domains of flow, respectively.

According to the top diagram in Fig. 1, in case of pulse-like perturbation the critical Reynolds number, stability limit of laminar flow is sharply defined by the inflection point of curve $Nu = f(Re)$. On the other hand, in the bottom diagram, the limit between laminar and transient domains is blurred in sinusoidal perturbation, namely there the curve $Nu = f(Re)$ passes smoothly, with a continuous tangent. To avoid uncertainty of the numerical value of $Re_{c\acute{e}}$, the critical Reynolds number for sinusoidally perturbed flow will be considered by convention, as the Re value where the Nusselt number exceeds by 3% the

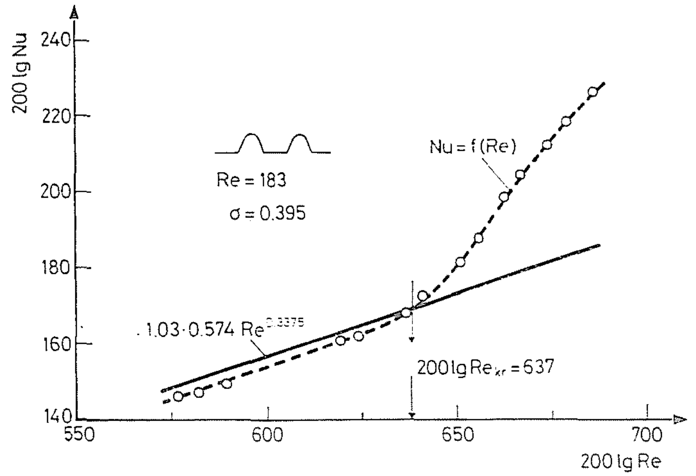

Fig. 2

$Nu = 0{,}574\ Re^{0.3375}$ value valid in the laminar domain. The Re_{cr} value can be graphically determined as exemplified in Fig. 2, yielding $Re_{cr} = 1531$ for the critical Reynolds number of flow sinusoidally perturbed at a frequency $Re_f = 183$.

Since, according to Fig. 1, curve $Nu = f(Re)$ smoothly passes the limit between transient and turbulent domains either under pulse-like or sinusoidal perturbation, the upper critical Reynolds number has to be defined by convention for either type of perturbation. Accordingly, the upper critical Reynolds number Re'_{cr} of perturbed flow is considered to be the Re value where the Nusselt number approximates to 97% the value obtained from the actual expression of the power function $Nu = KRe^n$ valid in the turbulent domain. The numerical Re'_{cr} value can be graphically determined by analogy to the method sketched in Fig. 2.

In agreement with statements for liquid media in (1) and (2), all our measurements in perturbed flow unambiguously showed, also for non-isothermal air flow, that pulsation lowers the critical Reynolds number value. Beyond this general statement, results also showed lowering of the critical Reynolds number to be independent of the fullness degree in perturbation of either waveform, and to depend solely on the perturbation frequency. In conformity with Fig. 3 showing function $Re_{cr} = f(Re_f)$, an increasing frequency involves a decreasing critical Reynolds number. According to a more detailed analysis, critical Reynolds numbers for flow perturbed by sine half-waves, and by square waves, are obtained from:

$$Re_{cr} = 3007\ Re_f^{-0.129} \qquad (Re_f \geq 59)$$

and

$$Re_{cr} = 2920\ Re_f^{-0.144} \qquad (Re_f \geq 35)$$

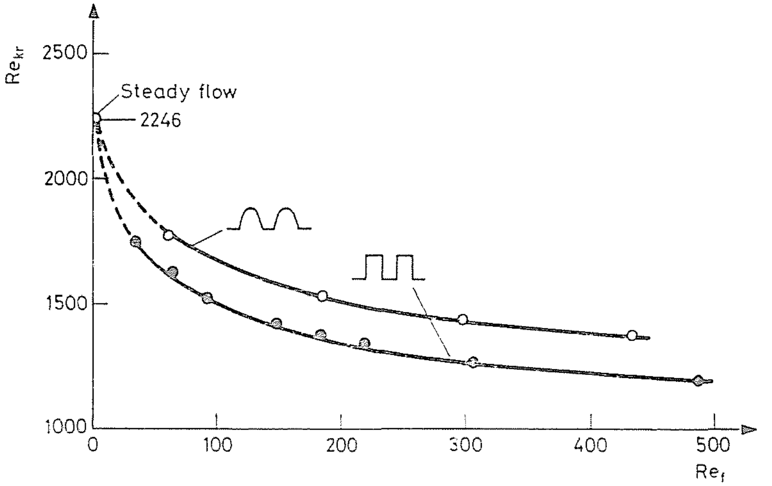

Fig. 3

respectively. By the way, relationships similar to those above can be deduced for relationship $Re'_{cr} = f(Re_f)$.

The relationships above between the critical Reynolds number of perturbed flow, and the perturbation frequency become obvious upon considering that at the essentially unsteady-state flow is at its maximum, the instantaneous value of the Reynolds number exceeds the steady-state value proper to the timely average. Thereby, in fact, the laminar flow becomes unstable at a Reynolds number exceeding the Re_{cr} value formed with the steady-state timely average. While in the phase of pulsation up to the other peak, the arising turbulence has the less time to decay, the higher the perturbation frequency.

Still another parallelism will be pointed out between deductions possible after the Tollmien—Schlichting theory [5] on the destabilization of laminar flow, concerning the critical Reynolds number for flow along a plane, and our statement on the frequency dependence of the critical Reynolds number for pipe flow.

This theory attributes the arise of turbulence to the amplification with time, beyond a certain Reynolds number, of small-amplitude perturbation waves arising in laminar flow from external causes, propagating in the flow direction. In this theory, an arbitrary undulatory motion is produced by Fourier's summation as resultant of fractional oscillations of different frequencies. Such a fractional oscillation for flow in plane $x(x, y)$ is given by flow function:

$$\psi(x, y, \tau) = \varphi(y) \exp [i(\gamma x - \beta\tau)].$$

Here x, and y are respective coordinates along, and normal to, the laminar basic flow at velocity v_0; $\varphi(y)$ is an amplitude function; $i^2 = -1$; γ is

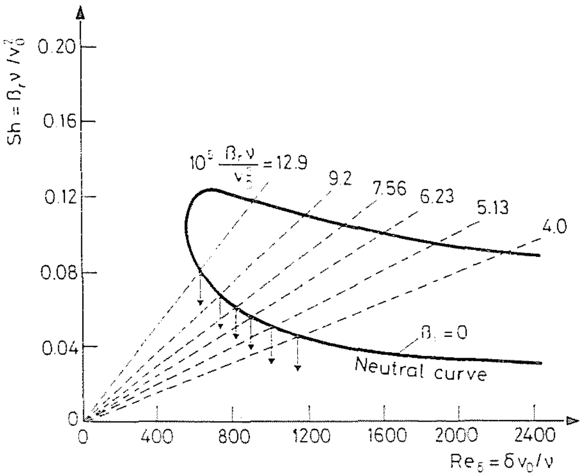

Fig. 4

a real number related to wavelength Λ of the fractional oscillation as $\Lambda = 2\pi/\gamma$. $\beta = \beta_r + i\beta_i$ is a complex magnitude, β_r the circular frequency of fractional oscillation, while numerical value of β^i expresses decay of the fractional oscillation. For $\beta_i < 0$, the fractional oscillation of frequency β_r decays, while for $\beta_i > 0$ the oscillation intensifies and the laminar basic flow becomes unstable. Fractional oscillations with $\beta_i = 0$ define the stability limit. Determination of these so-called neutral oscillations may be reduced to the solution of the eigen value problem related to the Orr — Sommerfeld fourth-order differential equation. For details see e.g. [6] and [7].

Figure 4 is a sketch of neutral curve $\beta_i = 0$ in the coordinate system (Re_δ, Sh), computed by Schlichting for the boundary layer of a plane, where $Re_\delta = v_0\delta/\nu$ and $Sh = \delta/v_0$ are Reynolds, and Strouhal numbers, resp., for boundary layer thickness δ. Each of the straight lines with asymptotes $\beta_r\nu/v_0^2$ starting from the origin can be considered as run diagrams of a fractional oscillation of frequency β_r. A perturbation wave of given frequency — while propagating from its place of origin in flow direction — passes through points of straight line $\beta_r\nu/v_0^2 = \text{const}$ in this diagram. Initially it passes across the stable domain outside the neutral curve, then, crossing the neutral curve at a definite amplitude, it gets to the domain of instability. Here its amplitude will grow. If the perturbance was a priori strong enough, it leads to eddying, and at last, to turbulence of the basic flow. This theory has fairly been supported in tests by Nikuradse [8] applying artificially produced perturbations of given frequency.

What is of interest for us is the intersection between any of the straight lines $\beta_r\nu/v_0^2 = \text{const}$ and the neutral curve, namely abscissae of these points define the Reynolds number $(v_0\delta/\nu)_{cr}$ critical for the given frequency β_r.

Schlichting (7) has obtained the following critical Reynolds numbers $Re_{\delta cr}$ for a plane:

$10^5 \beta_r/v_0^2$	12.9	9.2	7.56	6.23	5.13	4
$Re_{\delta cr}$	635	735	810	895	1000	1150.

These values show the critical Reynolds number of the boundary layer along the plane to decrease with inceasing frequency β_r. Although no numerical agreement may be expected because of different geometries, and different definitions of the Reynolds number, still our statements on the effect of the perturbation frequency of non-isothermal pipe flows on the critical Reynolds number can be stated to agree with the Tollmien — Schlichting theory.

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