MASS MATRIX ESTIMATION FOR THE DISCRETIZED DYNAMIC MODEL OF RAILWAY CAR BODIES

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Abstract

A method has been developed for estimating the mass matrix for the dynamical model with discretized masses of railway car bodies. This method relies on the comparison between vibrations of real beams and of those with discretized, section-wise uniform mass distribution. The objective function relying on the least squares method has been minimized by the SUMT method. Implementation of the computation method is illustrated on hand of analyzing a railway luggage truck body.

1. Introduction

This scope is strictly related to algorithmic modelling replacing heuristic modelling in the strength analysis of railway car bodies exposed to vertical, symmetric, dynamic loads. The continuum model will be replaced by a discretized model relying on data of the continuum model, and the algorithm is expected to suit mass and stiffness matrices [4, 5].

The method presented here lends itself to estimate the mass matrix of the discretized model of railway car bodies.

2. Method for estimating the discretized mass distribution

This method relies on the comparison between vibrations of real beams and of those with discretized masses, namely:

In the analysis of bending vibrations, the car body is considered as a beam conform to [3]. In estimating the mass distribution m, the car body is considered as a beam of mass sections [1], differing from the former beams with continuous mass distribution by its sectional mass distribution. Masses assigned to beam sections are expected to exhibit as small deviations between vibration patterns of real and discretized-mass beams during test period T as possible. Comparison refers to the case where both beams are hinged at both ends.

In estimating the discrete mass distribution, in compliance with the car body symmetry, the model beam is assumed to suffer linear bending alone. Bending axes of every cross section are normal to the drawing plane. The beam is only exposed to forces in the drawing plane, normal to the axis of the noload beam and applied in the principal plane normal to the bending axis. In addition, angular rotation of the displacing bar section is assumed to be small enough, so that components normal to the axis of the beam at rest of the shear forces tilted together with the beam section are equal to the shear forces themselves. Cross sections of bar sections are supposed to be constant, and within bar sections, no external forces are assumed to act.

In applying the method, regularities of the free vibrations of the continuum beam are assumed to be known. For that, knowledge of continuum beam data E_i, I_i, A_i, H_i, ϱ_i, x_i and F_i is sufficient.

Here: E_i — elasticity constant of bar section i of the continuum beam $(i = 1, 2, \dots, N)$;

I_i — second-order moment of inertia of bar section i of the continuum beam $(i = 1, 2, \dots, N)$;

A_i — cross section of bar section i of the continuum beam $(i = 1, 2, \dots, N)$;

H_i — length of bar section i of the continuum beam $(i = 1, 2, \dots, N)$;

ϱ_i — specific density of bar section i of the continuum beam $(i = 1, 2, \dots, N)$;

x_i — abscissa of point i of the continuum beam $(i = 1, 2, \dots, p)$;

F_i — deflection of the continuum beam at x_i at time $t = 0$ $(i = 1, 2, \dots, p)$.

With these notations and under these conditions, the problem of estimating discrete mass distribution is as follows:

Let us find $m = (m_1, m_2, \dots, m_{N^*})$ such as to minimize scalar vector function

$$\varphi(m) = \sum_{i \in I, I^*} \int_0^T \left(z(S_i^*, t) - z(m)(S_i^*, t)\right)^2 dt \tag{1}$$

under conditions

$$0 \le m_i \le K \quad (\forall_i \in I), \qquad m_i = 0 \Longleftrightarrow i \in I^* \tag{2}$$

$$|\alpha_i - \alpha_i(m)| \le \varepsilon \qquad (i = 1, 2, \dots, NS) \tag{3}$$

$$\left|M - \sum_{i \in I / I^*} m_i\right| \le \delta \tag{4}$$

where

$Z(S_i^*, t)$ — flexural displacement of the cross-sectional cintroid of the continuum beam at a fixed spot S_i^* at time t. S_i^* is the abscissa of the centroid of bar section i of the beam of sectional mass $(i \in I / I^*)$;

$Z^m(S_i^*, t)$ — flexural displacement of the cross-sectional centroid — again at S_i^* — of the beam of discretized mass distribution m at time t $(i \in J/I^*)$;

α_i — i-th circular eigenfrequency of the continuum beam $(i = 1, 2, \ldots, NS)$;

$\alpha_i^{(m)}$ — i-th circular eigenfrequency of the beam of mass distribution m $(i = 1, 2, \ldots, NS)$;

NS — number of circular eigenfrequencies reckoned with;

N^* — numeral of the discretized beam section;

ε, K and δ — fixed positive constants;

M — total mass of the continuum beam;

$I = \{1, 2, \ldots, N^*\}$

$I^* = \{i \in I$ / no-mass, elastic bar section i of the discretized beam.$\}$

Problems (1) to (4) have been solved by optimization method SUMT [2].

3. Practical computation outcomes

This practical computation method has been applied for the railway luggage truck body discribed in (3). The SUMIT method leads to the optimum mass distribution:

i	S_i^* (cm)	TM opt (dNs²/cm)
1	10	6.011
4	352.66	3.209
7	708.86	5.539
9	1064.66	6.885
11	1420.66	6.303
13	1446.66	6.909
16	2132.66	2.011
19	2484.4	2.057

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* In Hungarian.