HEAT TRANSFER IN A CIRCULAR PIPE WITH ARTIFICIALLY PERTURBED LAMINAR AND TURBULENT AIR FLOW

F. KONECSNY—TRINH VAN QUANG

Institute of Vehicle Engineering,
Technical University of Budapest, H-1521

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Abstract

Artificially generated perturbation has been superposed on steady-state air flow before entering a pipe heated by about constant heat flux, to examine heat transfer under these circumstances. Periodic perturbation had a shape of half sine or square wave. Compared to unperturbed flow, unambiguous improvement of heat transfer throughout the tested range of Reynolds numbers was observed. The role of frequency and that of the waveform were manifested by the position and extension of the transient domain between laminar and turbulent flow, while there was no effect on the Nusselt number neither in laminar nor in turbulent domains, where, however, increase of the perturbation intensity entrained a uniform improvement of heat transfer at the same rate. Formulae have been presented for determining the critical Reynolds number and the Nusselt number.

Several studies have been published on heat transfer in pulsating flow. Test reports are often contradictory as concerns the effect of pulsation on heat transfer. For instance, Linke [1] reports of heat transfer to improve by 400%, and by 35%, in laminar, and in turbulent oil flow, respectively, while the respective results by Hübner [2] are as low as 170% and 30%. Havemann and Narayan [3] found a rather uneven improvement by 30% in pulsating air flow, while similar tests on water by West and Taylor [4] showed 60 to 70% improvements. On the other hand, Morris [5], Webb [6], as well as Leistner and Marterstock [7] observed no effect whatsoever of flow on heat transfer. Similar were the observations made by Martinelli & al. [8], and by Marchant [9] in turbulent water flow tests, although Marchant points to a slight increase of the heat transfer coefficient in the laminar domain. On the other hand, Darling found no effect in the laminar domain [10], while in the turbulent domain he found at times a decrease, at other times an increase up to 70%.

Legend

- C — proportionality factor;
- c_p — specific heat of air at temperature T_m (J/kgK);
- d — internal diameter of test pipe (= 0.00945 m);
- f — perturbation frequency (Hz);

g — gravity acceleration ($\mathrm{m/s^2}$);
Gr — Grashof number: $Gr = \dfrac{\beta g \dot{q}_w \alpha^4}{\lambda \nu^2}$
I — heating current intensity (A);
K — factor of perturbation effect (interpreted in the text); $K = K(\sigma)$;
L — test pipe length ($= 0.709$ m);
$\dot{m}$ — air mass flow (kg/s); $\dot{m} = \dot{m}(\tau)$;
$\bar{\dot{m}}$ — timely average of mass flow (kg/s); $\bar{\dot{m}} = \dfrac{1}{B}\int_0^B \dot{m}(\tau)\,d\tau$
n — exponent of the Reynolds number;
Nu — mean Nusselt number: $Nu = \dfrac{\alpha d}{\lambda}$;
P' — heat loss of the test pipe (W);
Pr — Prandtl number of air ($= 0.695$);
$\dot{q}_w$ — heat flux to air ($\mathrm{W/m^2}$); $\dot{q}_w = \dfrac{\dot{m} c_p (T_2 - T_0)}{\pi \cdot d \cdot L}$;
Re — Reynolds number: $Re = \dfrac{v_0 d}{\nu}$;
Re_f — dynamical Reynolds number: $Re_f = \dfrac{f d^2}{\nu}$;
Re_{cr} — critical Reynolds number (interpreted in the text);
Re'_{cr} — upper critical Reynolds number (interpreted in the text);
T_0 — air temperature before entering the pipe (K);
T_2 — outlet air temperature (K);
T_k — surrounding temperature (K);
T_m — air mean temperature in the pipe (K):

$$T_m = (T_0 + T_2)/2;$$

T_{w1} — inlet pipe wall temperature (K);
T_{w2} — outlet pipe wall temperature (K);
T_{wm} — mean pipe wall temperature (K):

$$T_{wm} = (T_{w1} + T_{w2})/2;$$

$\Delta T_{\ln}$ — mean logarithmic temperature difference (K):

$$\Delta T_{\ln} = \frac{(T_{w2} - T_2) - (T_{w1} - T_0)}{\ln \dfrac{T_{w2} - T_2}{T_{w1} - T_0}};$$

U — voltage in the heating coil (V);

v_0 — cross-sectional mean value of timely average velocity at the inlet (m/s): $v_0 = \frac{4\bar{\dot{m}}}{\varrho_0 d^2 \pi}$;

α — mean heat transfer coefficient (W/m²K): $\alpha = \frac{\dot{q}_w}{\Delta T_{ln}}$;

β — isobaric thermal expansion coefficient of air at temperature T_m(1/K);

λ — thermal conductivity coefficient of air at T_m(W/mK);

ν — kinematic viscosity of air at T_m(m²/s);

ϱ_0 — air density at temperature T_m and atmospheric pressure (kg/m³);

σ — fullness degree (interpreted in the text);

τ — time (s).

Here test results for heat transfer in pulsating air flow [11] will be outlined. Before entering the circular pipe heated from outside, deliberately generated periodic perturbations have been superposed on the steady-state flow, and the quasi-steady-state convective heat transfer process arisen under these conditions has been investigated. The test pipe was tightly wound by a resistance wire applying timely constant electric heating to produce a heat flux $\dot{q}_w \approx$ const. along the wall, about evenly distributed along the pipe length. Mass flow fluctuation was produced by means of revolving valves connected to the pipe inlet. The timely course of perturbation had a shape either of a sine half-wave or of a square wave. In this latter case, the ratio of duration of the square wave to the period time could be altered. This ratio is expressed by the fullness degree σ. Obviously, the concept of fullness degree, quotient of the average time $\dot{m}$ by the maximum value $\dot{m}_{max}$ of mass flow, can also be interpreted in the case of sinusoidal perturbation. This interpretation adapts fullness degree to qualitatively describe the amplitude of longitudinal pressure gradient oscillations maintaining mass flow oscillations, hence, the perturbation intensity. A low σ value corresponds to high-intensity perturbation, and vice versa. The maximum value of fullness degree $\sigma = 1$ belongs to unperturbed steady-state flow.

Analyses of the equations of motion and of energy for non-isothermal flow, as well as preliminary considerations showed the mean Nusselt number for convective heat transfer in incompressible perturbed air flow for a given pipe length to depend on the Reynolds number, the Grashof number, the dynamical Reynolds number for the nondimensional frequency of perturbation, on the fullness degree, and the waveform:

$$Nu = f(Re;\ Gr;\ Re_f;\ \sigma;\ \text{waveform}). \tag{1}$$

In our tests, the mean heat transfer coefficient needed for forming Nu was referred to the mean logarithmic temperature difference $\Delta T_{\ln}$. The internal pipe diameter was chosen as characteristic length. Material characteristics λ, ν, c_p and β were reckoned with at mean air temperature T_m. The Reynolds number involved timely average velocity v_0.

To assume relationship (1) in the test, correlated $\bar{m}$, T_0, T_2, T_{w1}, T_{w2} values for every tested waveform were measured at the given fullness degree, at a fixed value of the frequency defined by the number of revolutions of the perturbation generator. In addition, to establish the heat balance of the equipment, simultaneous measurement of surrounding temperature T_k, voltage U connected to the heating coil, and the heating current I value was needed. Measurements were made in steady state set in about 2.5 to 3.5 hours.

Evaluation reckoned only with points where the sum of heat transfer to the air flow $\bar{m} \cdot c_p \cdot (T_2 - T_0)$ and of heat loss P' of the equipment did not differ by more than 5% from the heating power UI. Heat loss P' has been determined by preliminary calibration *vs.* temperature difference $T_{wm} - T_k$. Calibration was done by connecting a low heating power UI to the test pipe closed both ends, and measuring the temperature difference between wall and surroundings after heat equilibrium set in. In this case the heating power equals heat loss P' to the surroundings across the pipe insulation. Calibration curve $P' = f(T_{wm} - T_k)$ was rectilinear at a fair approximation, as anticipated.

Previous to the main test series, preliminary tests on steady-state, unperturbed flow have been made to refer heat transfer results under perturbed flow conditions to. Preliminary tests involved to essential lessons, correctness of which had been checked repetitively in main perturbation tests. First: the usual correction factor expressing the temperature dependence of viscosity has led to no modification over 1.53%, even under extreme temperature conditions occurring in the tests at all — rather than for a single measurement point. Therefore temperature dependence of viscosity may be omitted in data processing. Second: in course of the tests, no effect hinting to the existence of free convection has been observed. Hence, the Nusselt number may be considered as independent of the Grashof number.

Preliminary tests yielded formulae for reference values of Nusselt numbers to be measured under perturbed flow conditions:

$$Nu = 0.468 \ Re^{1/3} \quad \text{(laminar)} \tag{2}$$

$$Nu = 0.0154 \ Re^{0.83} \quad \text{(turbulent)} \tag{3}$$

practically agreeing with published results [12], [13], [14] generally accepted for computing heat transfer in steady-state flow.

Main tests were made in ranges $750 \leq Re \leq 32500$; $35 \leq Re_f \leq 490$; $0.2 \leq \sigma \leq 1$; $100 \text{ W/m}^2 \leq \dot{q}_w \leq 3400 \text{ W/m}^2$. (Sinusoidal perturbation was only

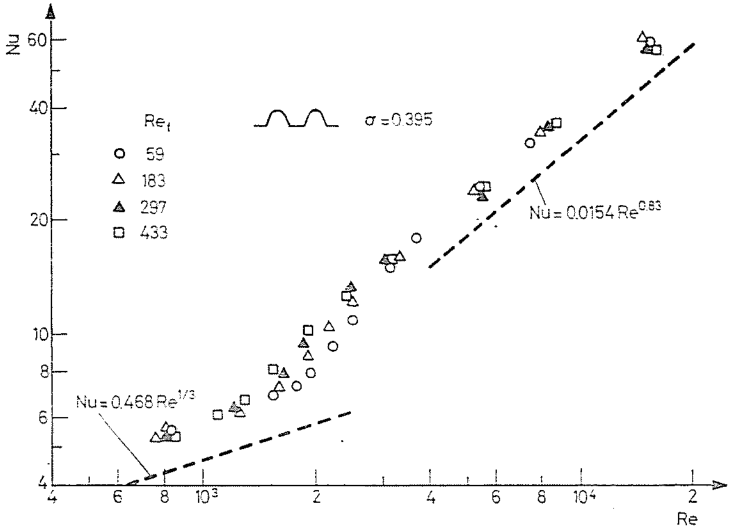

Fig. 1

investigated at a fullness degree $\sigma = 0.395$.) Results have been processed as $Nu = f(Re)$, the other variables are involved as parameters. A typical test result is seen in Fig. 1 showing points taken in flow sinusoidally perturbed at four different frequencies, at a constant fullness degree $\sigma = 0.395$, but points taken for square wave perturbed flow at a constant fullness degree would look essentially similar. Hence, general conclusions drawn from the diagram are valid to all test results, irrespective of the waveform. Reference curves for heat transfer in unperturbed flow according to [2] and [3] have been traced in dashed lines.

The position of measured points first of all leads to the conclusion that upon perturbation, heat transfer unambiguously increases throughout the tested range of Reynolds numbers for both laminar and turbulent flow, compared to steady-state values. For constant values of other flow parameters, the Nusselt number monotonously increased vs. Reynolds number, but for low Reynolds numbers the increase was slower, and in the domain of high Reynolds numbers it was faster. In these domains relationship $Nu = f(Re)$ can be approximated by a power function of the form $Nu = CRe^n$, and the function straight is about parallel to reference lines given by Eqs (1) and (2) in a coordinate system of logarithmic scale. As a consequence, in these domains, perturbation little affects exponent values.

The domain where Reynolds number exponent n is about 1/3, or 0.83, typical of heat transfers in unperturbed laminar, or turbulent flow, is called laminar, and turbulent domain, respectively, of perturbed flow. The two domains are not directly adjacent. Upon exceeding the critical Reynolds number $Re = Re_{cr}$, stability limit for laminar flow, the flow exhibits turbulent nodes, and at an arbitrary place in the pipe, the duration of turbulent state increasingly grows compared to that of the laminar one, until at the upper critical Reynolds number $Re = Re'_{cr}$ the transition to turbulent flow is complete [15]. In the transient domain $Re_{cr} < Re < Re'_{cr}$ the marked scattering of measurement points hints to the dependence of the Nusselt number for a constant value on the perturbation frequency, rather than on the Reynolds number alone. According to [15], the critical Reynolds numbers separating these three typical domains of perturbed flow are independent of the fullness degree, and their values vs. perturbation frequency are obtained, in case of sinusoidal perturbation, from:

$$Re_{cr} = 3007\ Re_f^{-0.129} \tag{4}$$

$$Re'_{cr} = 10090\ Re_f^{-0.284} \tag{5}$$

and in case of square wave perturbation from:

$$Re_{cr} = 2920\ Re_f^{-0.144} \tag{6}$$

$$Re'_{cr} = 4627\ Re_f^{-0.214} \tag{7}$$

Numerical values of the Reynolds number exponent n obtained by detailed analysis of measurement results have been tabulated as:

	Laminar	Turbulent
Sinusoidal perturbation	$n = 0.3375$	$n = 0.8288$
Square wave perturbation	$n = 0.3345$	$n = 0.8263$

These values agree with exponents 1/3 and 0.83 for unperturbed flow with a maximum error of $+1.25\%$ and -0.45%, respectively. Neglecting deviations, exponents $n = 1/3$, and $n = 0.83$ are uniformly accepted for laminar, and for turbulent flow, respectively.

Analysis of the measurement results unambiguously attributes the abnormally high standard deviation in the transient domain to the perturbation frequency. This is supported by Fig. 2 showing Nusselt numbers vs. Reynolds numbers, measured in flow perturbed by square waves of four different frequencies, with fullness degrees of $\sigma = 0.4$, separated according to frequencies, as an example for our measurements. In this presentation the high standard deviation has already vanished but it is manifest that in the transient domain the $Nu - Re$ relationship is of a more complex nature. The situation of points measured under sinusoidal perturbation at a frequency $Re_f = 189$ and a fullness degree $\sigma = 0.395$ shows that in the transient domain, the form of rela-

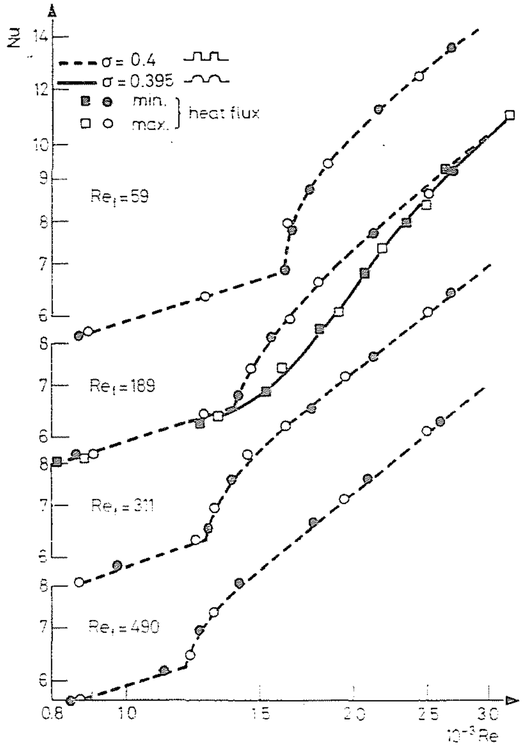

Fig. 2

tionship is also affected by the waveform. Therefore in the range $Re_{cr} < Re \leq < Re'_{cr}$, variation of the Nusselt number has to be indicated by the more general function $Nu = f$ (Re, Re_f, σ, waveform) rather than by the simple power function $Nu = CRe^n$. To clear the actual function form requires further, detailed experimental and theoretical investigations on the micro-structure of non-isothermal flows, and on the intermittence factor of turbulence. Actually, presentation of results obtained in laminar $Re \leq Re_{cr}$ and turbulent $Re \geq Re'_{cr}$ domains will be restricted to. Since no direct effect of frequency and waveform on heat transfer outside the transient domain has been observed, the following will focus on the effect of fullness degree expressing the perturbation intensity.

Earlier tests on unsteady-state isothermal flows have detected several phenomena hinting to the fact that for a mass flow of the same timely average as that of steady-state flow, in perturbed non-isothermal flows heat transfer is improved compared to the unperturbed state. One of these phenomena is the Richardson's ring effect, namely that in pulsating flow the velocity maximum in the pipe cross section increases and is shifted from the midline toward the wall, hence the velocity profile becomes steeper near the wall than in steady-state flow. Uchida [16] has demonstrated that this phenomenon, observable both at low and at high frequencies, exists for timely average velocities, rather than for instantaneous ones alone. On the other hand, at the beginning of the acceleration period of pulsating flow — similar to the beginning section of the transient boost of flows — the flow core is accelerated at a uniform spatial distribution almost like a rigid body. Thereby the boundary layer becomes relatively thinner, its thermal resistance decreases. Another interesting feature is that, according to Schultz and Grunow [17], the spatial variation of velocity, i.e. convective acceleration, and its timely variation, i.e. local acceleration, exert equivalent dynamic effects on flow. Accordingly, by analogy to the segregation related to marked convective decelerations in divergent channels, because of the negative local acceleration high enough in the deceleration period of pulsating flow, segregation an inversion of the flow along the pipe wall may come about. Periodical repetition of the process in the layers near the wall may much increase the mixing intensity, improving thereby the convenctive heat transfer inward the flow.

The order of magnitude of local accelerations, of importance for the heat transfer increase upon perturbance, is determined by the fullness degree. Within a perturbance cycle, the mass flow has to grow from the timely average value to a maximum defined by the fullness degree. For a square wave this is theoretically zero, but also in fact, it comes about in a negligible time compared to a perturbation cycle, independent of the frequency. The same is true for sine half-wave perturbation, if not so explicitly. In the light of this consideration, the observation that frequency does not directly affect heat transfer becomes obvious. The same consideration points to the physical background of that increasing local accelerations, that is, decreasing values of the fullness degree, boost heat transfer.

The effect of fullness degree on laminar and on turbulent heat transfer is seen in Fig. 3, where measurement results for flows perturbed by square waves of four different fullness degrees have been plotted, together with straight lines according to Eqs (2) and (3) and $\sigma = 1$ for unperturbed flow for the sake of comparison. According to the diagram, the perturbation intensity increases the Nusselt number at a fairly equal proportion in both the laminar and the turbulent domains. Situation of the dashed-line curve or the $Nu - Re$ relationship in sinusoidal perturbation at a fullness degree $\sigma = 0.395$ indicates that the increase rate is independent of the waveform.

Analysis of measured points taken at different fullness degrees for constant Reynolds numbers showed the ratio of Nusselt numbers for heat transfers in perturbed and in unperturbed flow to depend only on the fullness degree, and can be quantified as:

$$K(\sigma) = 1 + 0.42\,[1 - \sin 112.5\,(\sigma - 0.2)]. \qquad (8)$$

$$(0.2 \leq \sigma \leq 1)$$

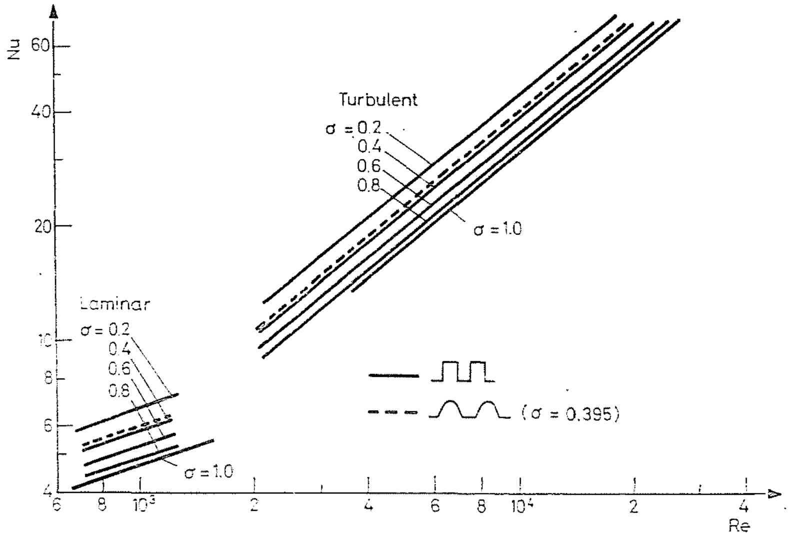

Fig. 3

Equation (8) is valid both in laminar and in turbulent domains, hence, for a given Re number, the Nu number under perturbed flow conditions is obtained as product of factor $K(\sigma)$ by Nusselt number in unperturbed flow:

$$Nu = 0.468\ K(\sigma)\ Re^{1/3} \quad (Re \leq Re_{cr}) \tag{9}$$

$$Nu = 0.0154\ K(\sigma)\ Re^{0.83} \quad (Re \geq Re'_{cr}). \tag{10}$$

The Re_{cr} and Re'_{cr} values are to be reckoned with according to Eqs (4) through (7).

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Dr. Ferenc KONECSNY H-1521 Budapest
Trinh VAN QUANG, Polytechnical Institute Hanoi, Viet-Nam