ASSESSMENT OF STRESS STATISTICS FOR COMMERCIAL VEHICLE FRAMES

P. Michelberger, L. Szeidl and A. Keresztes

Department of Transport Engineering of Mechanics,
Technical University, H-1521 Budapest

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Abstract

The paper presents the statistical problems (unbias, consistency, etc.) of a nonlinearity degree for vehicle system dynamics. Using this nonlinearity degree estimated from measured input/output data one can separate the linear behaviour of the vehicle vibrating phenomena from the nonlinear one.

Introduction

Recently, research on the dimensioning of vehicle frames has come to the foreground of interest. Actually, a primary goal of research has been to calculate dynamic assessment of vehicle frames. This is conditioned, of course, by the analysis of the so-called "permanent" vehicle operation by relating traffic and vehicle design processes, by establishing the fundamental load and stress statistics of the vehicle. This fundamental stress arises from the stochastic road excitation of the vehicle driven on a rough roadway.

For determining two-dimensional stress distribution function [1] and level intersection numbers typical of the expected stress [2], stress statistics apply the spectral method, easy to handle, assuming linearity of road excitation/frame stress models. This assumption generally provides for a close approximation of real processes in cases on high-quality road types and medium travel speeds. For poorer roads and generally higher travel speeds, however, nonlinearities due to wheel bouncing, to progressive spring characteristics, and to asymmetric vibration damping effects prevent the linear model from being considered as correct.

It is therefore essential to determine "ranges" (as a function of road profile standard deviation and of speed values) where the linear model is either correct or can be considered as a fair approximation. There more so since, if the linear of linearized model is useless, the estimation process may become extremely complex, increasing the volume of computations by orders of magnitude.

Coefficients have been published for the characterization of closeness of static or dynamic relations between data sets or signal pairs, or even its linearity in no-noise cases. This paper will apply the coefficient deduced from a dispersion and correlation function for describing the road excitation/frame stress model linearity [3] and the analyses will involve confidence of the assessment of this coefficient.

Stating the problem

The autobus is put to vibration by the stochastic input excitation process acting on the four wheels at time t: $x_t = (x_t^1, \ldots, x_t^4)$ corresponding to the road profile/speed process to be represented. Let stress processes y_t^j (nearer, the strain process in linear correlation) in the bus frame be measured at points $j = 1, 2, \ldots, M$.

Random road profile process $w_s = (w_s^1, \ldots, w_s^4)$ *vs.* travelled road — generally accepted in publications in spite of its boundedness — is taken at a fair approximation as a steady Gaussian process of expected value [4] — $m_w = 0$ —, with a continuous, integrable power spectrum $\Phi_w(\omega)$, thereby, at an arbitrary fixed speed v, the random excitation process vs time x_t is fairly approximated by a steady Gaussian process with e.g. a power spectrum

$$\Phi_x^v(\omega) = \frac{1}{v}\,\Phi_w(\omega) \tag{1}$$

in the sufficiently wide, finite range of expected value $m_x = 0$.

Analyses assumed an orthothropic road and the effect $x_t^1 = x_t^2$ at time t on front wheels to attain rear wheels after time $t^* = \frac{l}{v}$ (being the axle spacing), hence $x_t^3 = x_t^4 = x_{t-t^*}^1$. Thereby excitation process x_t becomes:

$$x_t = (x_t, x_t, x_{t-t^*}, x_{t-t^*}).$$

This analysis refers to a vehicle in service operation — after decay of transient effects — where the process pair (y_t, x_t), $t \geq T_0$ can be considered as steady (of distribution) in a restricted meaning. For the sake of simplicity, in the following, $T_0 = 0$ is assumed. None of the ssumptions that x_t and y_t have finite standard deviation matrices is to the detriment of general validity. To decide linearity, linearity degree

$$\alpha_{y}j_x = \frac{\int_0^\infty r_y^2 j_x(u)\, du}{\int_0^\infty \eta_y^2 j_x(u)\, du}$$

defined by normed cross-dispersion function [3]

$$[Dy_0^j = (E(y_0^j - Ey_0^j)^2)^{\frac{1}{2}}, \quad Dx_0 = (E(x_0 - Ex_0)^2)]$$

and

$$\eta_y j_x(u) = \frac{[E\{E(y_u^j \mid x_0) - E\, y_n^j\}^2]^{\frac{1}{2}}}{D\, y_0^j}$$

cross correlation function

$$r_y j_x(u) = \frac{E(y_u^j - E\, y_u^j)(x_0 - E\, x_0)}{Dy_0^j\, Dx_0}, \quad u \geq 0$$

(where E is the operator of expected value formation) will be applied. The system may be considered as about linear if the result is $L_y j_x \sim 1$, $j = 1, 2, \ldots, M$.

Linearity degrees $L_x j_y$ will be assessed from statistics $\hat{\alpha}_y j_x$ (see later) deduced from process $(y_t x_t)$ observed in interval [O, T]. To draw the final conclusion requires to know confidence of assessment $\hat{\alpha}_y j_x$, therefore absolute general deviation of $\hat{\alpha}_y j_x$ from $\alpha_y j_x$, hence magnitude will be assessed. Let us first introduce some symbols.

Let N be a natural number, and for real numbers S_i, $i = 1, \ldots, N-1$ let

$$-\infty < S_1 < S_2 \qquad \ldots < S_{N-1} < +\infty$$

be met, number series S_i be symmetric about the origin, that is, let $S_1 =, -\, - S_{N-1}$, $S_2 = -\, S_{N-2}, \ldots$ (for even N, $S_{N/2} = 0$).

Let $\Delta_1 = (-\infty, S_1)$, $\Delta_2 = [S_1, S_2) \ldots, \Delta_N = [S_{N-1}, +\infty)$,

$$Z_1 = S_1;\ Z_N = S_{N-1};\ Z_i = \frac{s_{i-1} + S_i}{2};\ i = 2, \ldots, N-1.$$

Let process $\tilde{x}_t$, $t \geq 0$ of discrete value be defined, starting from process x_t, as:

$$\tilde{x}_t = Z_i, \text{ if } x_t \in \Delta i$$

hence let

$$\tilde{x}_t = \sum_{i=1}^{N} Z_i\, I(x_t \in \Delta i),$$

where for indicator function I — :

$$I\,(x_t \in \Delta i) = \begin{cases} 1 \text{ if } x_t \in \Delta i \\ 0 \text{ if } x_t \notin \Delta i. \end{cases}$$

In connection with process x_t, let us comment;

— Vector process (y_t, x_t), $t \geq 0$ being steady in a restricted meaning, obviously, so will be vector process

$$(y_t, \tilde{x}_t) = [y_t(\tilde{x}_t, \tilde{x}_t, \tilde{x}_{t-t}, \tilde{x}_{t-t})]$$

— From the complex of condition $E\,x_t = 0$, specially selected Δ_i, and symmetry of the Gaussian distribution, it follows:

$$E\tilde{x}_t = E\,x_t = 0, \quad t \geq 0.$$

— Simple calculation of the variance of process x_i yields:

$$D^2\tilde{x}_t = E(\tilde{x}_t - E\tilde{x}_t)^2 = E\sum_{i=1}^{N} Z^2 I(x_t \in \Delta_i) = \sum_{i=1}^{N} p(\Delta_i) Z_i^2$$

where $p(\Delta_i) = P(x_t \in \Delta_i)$ $\qquad i = 1, 2, \ldots, N.$

Practical assumptions for road profile process w_s, $s \geq 0$ and stress process y_t, $t \geq 0$ may be:

C1. Values recorded at spots distant by at least S^*

$$\{w_{s'}; s' \leq S\} \text{ and } \{w_{s''}; s'' \geq S + S^*\}$$

as random variables are independent.

This assumption implies independence of

$$\{x_{t'} : t' \leq t\} \text{ and } \left\{x_{t''} : t'' \geq t + \frac{S^*}{v}\right\}$$

accordingly, for a vector process x_t, also

$$\{x_{t'} ; t' \leq t\} \text{ and } \left\{x_{t''}, t'' \geq t + t^* + \frac{S^*}{v}\right\}$$

are independent.

C2. For a vibration cycle U belonging to the lowest natural frequency of the bus as a vibrating system, because of inherent damping effects of the system, taking

$$\tau = \max\left(\frac{S^* + l|}{v}, 10\,\mathrm{U}\right)$$

random variables

$$\{y_{t'} t' \leq t\} \text{ and } \{y_{t''}, t'' \geq t + \tau\}$$

and, according to the above,

$$\{(y_{t'}, x_{t'}) t' \leq t\} \text{ and } \{(y_{t''}, x_{t''}), t'' \geq t + \tau\}$$

will be independent.

C3. Process y_s is bounded at a probability 1, hence there are constants $-\infty < Y_1 < Y_2 < \infty$ where

$$P(Y_1 \leq y_s \leq Y_2) = 1.$$

For values Y_1 and Y_2 in *C3*, theoretical bounds may be indicated in the knowledge of the bus frame characteristics.

Stresses at single points of the vehicle in normal operation may be limited by stress values beyond or below that the given point of the structure would undergo permanent strain (yield), involving distorsion of the vehicle. (Actual measurements refer to a structure considered as stable.)

Let $Y_* = |Y_1| + |Y_2|$, then for arbitrary s, t inequality $E(y_s - Ey_0)^2(x_t - E\,x_0)^2 \leq K = Y^2 D^2 x_0$ holds.

C4. Combined probability density function of random variables

$$y_s \text{ and } x_t\colon f_{s,t}(r, z) = f_{y_s x_t}(r, z)$$

in continuous, it can be partially differentiated with respect to r in either variable, furthermore, a number $R \leq 4Dx_0$ exists such that for any $0 \leq u \leq \tau$, $Y_1 \leq r \leq Y_2$ and $-4Dx_0 < z < 4\,Dx_0$, inequality

$$\left|\frac{\partial}{\partial z} f_{u,0}(r, z)\right| \leq R \cdot f_{u,0}(r, z) \text{ holds.}$$

C1 and *C2* directly yield that

$$r_y j_x(u) = 0\,,\ |u| \geq \tau$$

and

$$y_x^j(u) = 0\,,\ |u| \geq \tau$$

in the actual case yielding quotient of integrals over finite intervals

$$\varkappa_y j_x = \frac{\int\limits_0^\tau r_y^2 j_x(u)\,du}{\int\limits_0^\tau \eta_y^2 j_x(u)\,du} = \frac{(D^2x_0)^{-1}\int\limits_0^\tau R_y^2 j_x(u)\,du}{\int\limits_0^\tau \Theta_y^2 j_x(u)\,du}$$

as degree of linearity, where $R_y j_x(u)$ is the cross covariance function, and $\Theta_y j_x(u)$ the cross dipersion function;

$$R_y j_x(u) = E(y_{t+u}^j - E\,y_{t+u}^j)(x_t - E\,x_t);$$
$$\Theta_y j_x(u) = [E\{E(y_{t+u}^j - E\,y_{t+u}^j \mid x)\}^2]^{\frac{1}{2}}$$

Let us assess now the index $\varkappa_y j_x$ from observed values (y_t, x_t), $0 \leq t \leq T$. Since this assessment has to be made for every j, $j=1, 2, \ldots, M$ by the same assessment method, for the sake of simplicity, subscript j will be omitted.

Assessment of functions $R_{yx}(u)$ and $\Theta_{yx}^2(u)$ will apply statistics below (reminding that $E\,\tilde{x}_t = 0$ and $p/\Delta_i)$, $D^2\tilde{x}_0$ are known, modifying accordingly the assessment for function $\Theta_{yx}^2(u)$ in [5]:

$$\breve{R}_{y\tilde{x}}^2(u) = \frac{1}{T-u}\int\limits_0^{t-u}(y_{t+u} - \hat{y})\tilde{x}_t\,dt, \quad 0 \leq u \leq \tau$$

where

$$\bar{y} = \frac{1}{T}\int_0^T y_t\,dt$$

and

$$\widehat{\Theta}^2_{yx}(u) = \sum_{i=1}^{N}\frac{1}{p(\Delta_i)}\left[\frac{1}{T-u}\int_0^{T-u}(y_{t+u}-\bar{y})\;I(x_t\in\Delta_i)\;dt\right]^2,\; 0\le u\le\tau$$

Let us see now, how to assess the absolute mean deviation of statistics

$$\widehat{\alpha}_{y\tilde{x}} = \frac{(D^2x_0)^{-1}\int_0^\tau \widehat{R}^2_{y\tilde{x}}(u)\,du}{\int_0^\tau \widehat{\Theta}^2_{yx}(u)\,du}$$

from magnitude α_{yx}.

Making use of triangle in equality:

$$E\mid\alpha_{yx}-\widehat{\alpha}_{y\tilde{x}}\mid = E\left|\frac{(D^2x_0)^{-1}\int_0^\tau R^2_{yx}(u)du}{\int_0^\tau \Theta^2_{yx}(u)\,du}-\frac{(D^2x_0)^{-1}\int_0^\tau \widehat{R}^2_{y\tilde{x}}(u)du}{\int_0^\tau \widehat{\Theta}^2_{yx}(u)du}\right|\le$$

$$\le (D^2x_0\int_0^\tau \Theta^2_{yx}(u)du)^{-1}\{\mid\int_0^\tau R^2_{yx}(x)-R^2_{y\tilde{x}}(u)\}du\mid +$$

$$+E\left|\int_0^\tau (R^2_{y\tilde{x}}(u)-R^2_{y\tilde{x}}(u))du\right| + E\frac{\int_0^\tau \widehat{R}^2_{y\tilde{x}}(u)du}{\int_0^\tau \widehat{\Theta}^2_{yx}(u)\,du}\left|\int_0^\tau\{\Theta^2_{yx}(u)-\widehat{\Theta}^2_{y\tilde{x}}(u)\}du\right|\}$$

$$\text{Since } \tilde{x}_t = \sum_{i=1}^{N} Z_i I(x_t\in\Delta_i) \text{ and } D^2\tilde{x}_t = \sum_{i=1}^{N} p\,(\Delta_i)\;Z_i^2$$

utilizing Cauchy's inequality yields:

$$\widehat{R}^2_{yx}(u) = \left(\frac{1}{T-u}\int_0^{T-u}(y_{t+u}-\bar{y})x_t dt\right)^2 = \left(\sum_{i=1}^{N} Z_i\frac{1}{T-u}\int_0^{T-u}(y_{t+u}-\bar{y})\;xI(x_t\in\Delta_i)dt\right)^2 =$$

$$= \left(\sum_{i=1}^{N}(\sqrt{p(\Delta_i)}\,Z_i)\right)\left(\frac{1}{\sqrt{p(\Delta_i)}}\cdot\frac{1}{T-u}\int_0^{T-u}(y_{t+u}-\bar{y})\;I(x_t\in\Delta_i)dt\right)^2\le$$

$$\le\left(\sum_{i=1}^{N}p(\Delta_i)Z_i^2\right)\left[\sum_{i=1}^{N}\frac{1}{p(\Delta_i)}\left(\frac{1}{T-u}\int_0^{T-u}(y_{t+u}-\bar{y})\,I(x_t\in\Delta_i)dt\right)^2\right] = D^2\tilde{x}_0\,\widehat{\Theta}^2_{yx}(u),$$

hence

$$\frac{\int_0^\tau \hat{R}^2_{y\tilde{x}}(u)\,du}{\int_0^\tau \hat{\Theta}^2_{y\tilde{x}}(u)\,du} \leq D^2\tilde{x}_0$$

accordingly:

$$E \mid \alpha_{yx} - \alpha_{y\tilde{x}} \mid \leq (D^2x_0 \int_0^\tau \Theta^2_{yx}(u)\,du)^{-1} \{ \mid \int_0^\tau R^2_{yx}(u) - R^2_{y\tilde{x}}(u)\,du \mid +$$

$$+ E \mid \int_0^\tau (R^2_{y\tilde{x}}(u) - \hat{R}^2_{y\tilde{x}}(u))\,du \mid + D^2\tilde{x}_0\, E \mid \int_0^\tau \left(\Theta^2_{yx}(u) - \hat{\Theta}^2_{y\tilde{x}}(u)\right) du \mid \} \quad (3)$$

First two terms in figure brackets in the right-hand side of inequality (3) may be assessed as:

Assessment of term 1

$$\mid \int_0^\tau (R^2_{yx}(u) - R^2_{y\tilde{x}}(u))\,du \mid = \mid \int_0^\tau (R_{yx}(u) - R_{\tilde{x}}(u)(R_{yx}(u) + R_{y\tilde{x}}(u)du \mid \leq$$

$$\leq \int_0^\tau \mid R_{yx}(u) - R_{y\tilde{x}}(u) \mid du\; Dy_0(Dx_0 + D\tilde{x}_0).$$

Obviously:

$$\mid R_{yx}(u) - R_{y\tilde{x}}(u) \mid = \mid E\,(y_u - Ey_0)x_0 - E(y_u - Ey_0)\,\tilde{x} \mid =$$

$$= \mid E(y_u - Ey_0)(x_0 - \tilde{x}_0) \mid \leq Dy_0 \left(E \left(x_0 - \sum_{i=1}^{N} Z_i\, I(x_0 \in \Delta_i) \right)^2 \right)^{\frac{1}{2}} =$$

$$= Dy_0 \left[\sum_{i=1}^{N} E(x_0 - Z_i)^2\, I(x_0 \in \Delta_i) \right]^{\frac{1}{2}} = Dy_0\, H(S_1, \ldots, S_{n-1}, Dx_0),$$

where, for given values $S_1, \ldots, S_{N-1}$ and Dx_0, magnitude

$$H(S_1, \ldots, S_{N-1}, Dx_0) = \left[2 \int_{S_{N-1}}^{\infty} (x - S_{N-1})^2 \frac{1}{\sqrt{2\pi\, Dx_0}} e^{-\frac{x^2}{2Dx_0}}\, dx + \right.$$

$$\left. + \sum_{i=1}^{N-1} \int_{S_{i-1}}^{S_i} \left(x - \frac{S_{i-1} + S_i}{2} \right)^2 \frac{1}{\sqrt{2\pi\, Dx_0}} e^{-\frac{x^2}{2Dx_0}}\, dx \right]^{\frac{1}{2}} \quad (4)$$

can be determined at arbitrary accuracy. Accordingly, it holds:

$$\mid \int_0^\tau (R^2_{yx}(u) - R^2_{y\tilde{x}}(u))du \mid \leq \tau\, D^2y_0(Dx_0 + D\tilde{x}_0)H(S_1, \ldots, S_{n-1}, Dx_0) \quad (5a)$$

It should be noted that a simple assessment can be given for magnitudes $H(S_1, \ldots, S_{N-1}, Dx_0)$:

$$H(S_1, \ldots, S_{N-1}, Dx_0) \leq \left[2 \int_{S_{N-1}}^{\infty} (x - S_{N-1})^2 \frac{1}{\sqrt{2\pi Dx_0}} e^{-\frac{x^2}{2Dx_0}} dx + \right.$$

$$\left. + \frac{\Delta_*^2}{4} \int_{S_1}^{S_{N-1}} \frac{1}{\sqrt{2\pi Dx_0}} e^{-\frac{x^2}{2Dx_0}} dx \right]^{\frac{1}{2}} \leq$$

$$\leq \left[2 \int_{S_{N-1}}^{\infty} (x - S_{N-1})^2 \frac{1}{\sqrt{2\pi Dx_0}} e^{-\frac{x^2}{2Dx_0}} dx \right]^{\frac{1}{2}} + \frac{\Delta_*}{2} \tag{5b}$$

where

$$\Delta_* = \max_{2 \leq i \leq N-1} |\Delta_i|, \qquad \Delta_i = S_i - S_{i-1}$$

Assessment of term 2

Obviously

$$E\left|\int_0^{\tau} (R_{y\tilde{x}}^2(u) - \hat{R}_{y\tilde{x}}^2(u))du\right| \leq E\int_0^{\tau} |R_{y\tilde{x}}(u) - \hat{R}_{y\tilde{x}}(u)|\,|R_{y\tilde{x}}(u)|\,du +$$

$$+ E\int_0^{\tau} |R_{y\tilde{x}}(u) - \hat{R}_{y\tilde{x}}(u)|\,|\hat{R}_{y\tilde{x}}(u)|\,du \leq$$

$$+ Dy_0\, D\tilde{x}_0 \int_0^{\tau} [E(R_{y\tilde{x}}(u) - \hat{R}_{y\tilde{x}}(u))^2]^{\frac{1}{2}}\,du +$$

$$+ \int_0^{\tau} \{E(R_{y\tilde{x}}(u) - \hat{R}_{y\tilde{x}}(u))^2\; E(\hat{R}_{y\tilde{x}}(u))^2\}^{\frac{1}{2}} \leq$$

$$\leq \tau\,\{\sup_{0 \leq u \leq \tau} E(R_{y\tilde{x}}(u) - \hat{R}_{y\tilde{x}}(u))^2\}^{\frac{1}{2}} \cdot \{Dy_0 D\tilde{x}_0 + \sup_{0 \leq u \leq \tau} [E(\hat{R}_{y\tilde{x}}(u)^2]^{\frac{1}{2}}\} \tag{6}$$

For arbitrary $0 \leq u \leq \tau$, simply:

$$E[R_{y\tilde{x}}(u) - \hat{R}_{y\tilde{x}}(u)]^2 \leq 2\,E\left\{\left[\frac{1}{T-u}\int_0^{T-u} (E(y_u - Ey_0)\tilde{x}_0 - \right.\right.$$

$$\left.\left. - (y_{t+u} - Ey_0)\,\tilde{x}_t)dt\right]^2 + \left[\frac{1}{T-u}\int_0^{T-u} Ey_0 - \hat{y})\tilde{x}_t\,dt\right]^2\right\} \tag{7}$$

Evidently:

$$E\left\{\frac{1}{T-u}\int_0^{T-u} [E(y_u - Ey_0)\tilde{x}_0 - (y_{t+u} - Ey_0)\,\tilde{x}_t]\,dt\right\}^2 =$$

$$= \frac{1}{(T-u)^2} \iint\limits_0^{T-u} E\{[E(y_u - Ey_0)\,\tilde{x}_0 - (y_{t+u} - Ey_0)\tilde{x}_t][E(y_u - Ey_0)\,\tilde{x}_0 -$$

$$- (y_{s+u} - Ey_0)\tilde{x}_s]\}\, dt\, ds \leq \frac{2\tau}{T-u} E[(y_u - Ey_0)\tilde{x}_0 - E\,(y_u - Ey_0)\tilde{x}_0]^2 \leq$$

$$\leq \frac{2\tau}{T-\tau} E[(y_u - Ey_0)^2\, \tilde{x}_0^2] \leq \frac{2\tau}{T-\tau}\, \widetilde{K} \tag{8}$$

where $\widetilde{K} = Y_*^2\, D^2\tilde{x}_0$ and

$$E\left[\frac{1}{T-u}\int\limits_0^{T-u} (Ey_0 - \widehat{y})\tilde{x}_t dt\right]^2 = E\left\{\frac{1}{(T-u)^2}\iint\limits_0^{T-u} (Ey_0 - \widehat{y})^2\, \tilde{x}_t\tilde{x}_s dt\, ds\right\} \leq$$

$$\frac{1}{(T-u)^2}\iint\limits_0^{T-u} E\left\{\frac{1}{T}\int\limits_0^{T} (y_z - Ey_0)^2 \cdot dz\right\} \tilde{x}_t\, \tilde{x}_s\, dt\, ds =$$

$$= \frac{1}{(T-u)^2}\iint\limits_0^{T-u} \left\{\frac{1}{T}\int\limits_0^{T} [E(y_z - Ey_0)^2\, \tilde{x}_t\, \tilde{x}_s]\, dz\right\} dt\, ds \tag{9}$$

Clearly, for any $t,\ s,\ z \geq 0$:

$$|\, E\,(y_z - Ey_0)^2\tilde{x}_t\tilde{x}_s \,| \leq [E(y_z - Ey_0)^2\, \tilde{x}_t^2]^{\frac{1}{2}}\, [E(y_z - Ey_0)^2\, \tilde{x}_s^2]^{\frac{1}{2}} \leq \widetilde{K} \tag{10}$$

On the other hand, for $|t-s| \geq \tau$ and $|t-z| \geq \tau$ or $|t-s| \geq \tau$ and $|\, s-z \,| \geq$, equality

$$E(y_z - Ey_0)^2\tilde{x}_t\tilde{x}_s = 0 \tag{11}$$

holds. (9) is easy to assess from inequalities (10) and (11):

$$E\left(\frac{1}{T-u}\int\limits_0^{T-u} (Ey_0 - \widehat{y})\tilde{x}_t dt\right)^2 \leq \frac{\widetilde{K}}{(T-u)^2}\left\{\iint\limits_0^{T-u} I(\,|\, t-s\,| \geq \tau)\,\frac{2\tau}{T}\, \mathrm{dt}\, ds +\right.$$

$$\left. + \iint\limits_0^{T-u} I(\,|\, t-s\,| < \tau)\, dt\, ds\right\} \leq \frac{2\tau}{T}\widetilde{K} + \frac{2\tau}{T-u}\widetilde{K} = \frac{2\tau}{T-u}\widetilde{K}\left(1 + \frac{T-u}{T}\right) \leq$$

$$\leq 4\,\frac{\tau}{T-\tau}\widetilde{K} \tag{12}$$

Since

$$E(\widehat{R}_{y\tilde{x}}(u))^2 = E\left(\frac{1}{T-u}\int\limits_0^{T-u} (y_{t+u} - \widehat{y})\tilde{x}_t\, dt\right)^2 \leq$$

$$\leq \frac{1}{T-u}\int_0^{T-u} E(y_{t-u}-\hat{y})^2\tilde{x}_t^2\,dt \leq Y_*^2\frac{1}{T-u}\int_0^{T-u} E\tilde{x}_t^2\,dt = Y_*^2 D^2\tilde{x}_0 = \tilde{K} \quad (13)$$

using (6), (7), (8) and (13) yields:

$$E\left|\int_0^{\tau}[R_{y\tilde{x}}^2(u)-\hat{R}_{y\tilde{x}}^2(u)]\,du\right| \leq \tau\left[\frac{4\tau}{T-\tau}\tilde{K}+\frac{8\tau}{T-\tau}\tilde{K}\right]^{\frac{1}{2}}\times$$

$$\times[Dy_0 Dx_0 + K^{\frac{1}{2}}] \leq 4\sqrt{3}\,\tilde{K}\frac{\tau^{\frac{1}{2}}}{(T-\tau)^{\frac{1}{2}}} \quad (14)$$

Assessment of term 3

Utilizing the triangle inequality:

$$E\,|\,\Theta_{yx}^2(u)-\Theta_{y\tilde{x}}^2(u)\,| \leq |\,\Theta_{yx}^2{}^2(u)-\Theta_{y\tilde{x}}^2(u)\,| + E\,|\,\Theta_{y\tilde{x}}^2(u)-\hat{\Theta}_{y\tilde{x}}^2(u)\,| \quad (15)$$

The lengthy derivation of the assessment of two terms in the right-hand side of inequality (15) will be omitted, only the final result will be quoted. Remind, however, that assessment of term 1 in the right-hand side was obtained by using assumptions *C3* and *C4*, while assessment of term 2 was obtained by the same assessment procedure as that for terms 1 and 2 in the right-hand side of inequality (3).

Results are:

$$|\,\Theta_{yx}^2(u)-\Theta_{y\tilde{x}}^2(u)\,| \leq 2\,Y_*^2[p\Delta_1) + p(\Delta_n)] + Y_*(1+Y_*)\,R\Delta_* \quad (16)$$

and

$$E\,|\Theta_{y\tilde{x}}^2(u)-\Theta_{y\tilde{x}}^2(u)\,| \leq 4\,Y_*^2\left[p(\Delta_1)+(p\Delta_N)+\frac{2\tau}{T-\tau}\right]+$$

$$+2\sqrt{6}\;Y_*^2\frac{1}{p^*}\frac{\tau^{\frac{1}{2}}}{(T-\tau)^{\frac{1}{2}}} \quad (17)$$

where $p^* = \min\limits_{2\leq i\leq N-1} p(\Delta_i)$

Thereby from inequalities (15), (16) and (17):

$$E\,|\,\Theta_{y\tilde{x}}^2(u)-\hat{\Theta}_{y\tilde{x}}^2(u)\,| \leq 2\,Y_*^2\,[p(\Delta_1)+p(\Delta_N)] + Y_*(1+Y_*)\,R\Delta_* +$$

$$+4Y_*^2\left[p(\Delta_1)+p(\Delta_N)+\frac{2\tau}{T-\tau}\right]+2\sqrt{6}\;Y_*^2\frac{1}{p^*}\frac{\tau^{\frac{1}{2}}}{(T-\tau)^{\frac{1}{2}}} \quad (18)$$

After having separately assessed three terms in the right-hand side of inequality (3), inequalities (3), (5), (14) and (18) permit to directly indicate assessment of error

$$E\,|\,\alpha_{yx}-\hat{\alpha}_{y\tilde{x}}\,|:$$

$$E \mid \alpha_{yx} - \hat{\alpha}_{y\tilde{x}} \mid \leq [D^2 x_0 \int_0^T \Theta_{yx}^2(u)\, du]^{-1} \{\tau Y_*^2 (Dx_0 + D\tilde{x}_0) \times$$

$$\times H(S_1, \ldots, S_{N-1}, Dx_0) + 4\sqrt{3}\, \tilde{K} \frac{\tau^{3/2}}{(T-\tau)^{\frac{1}{2}}} + D^2 \tilde{x}_0 \tau \Big[2 Y_*^2 (p(\Delta_1) +$$

$$+ p(\Delta_N)) + Y_*(1 + Y_*) R\Delta_* + 4 Y_*^2 \left(p(\Delta_1) + p(\Delta_N) + \frac{2\tau}{T-\tau} \right) +$$

$$+ \frac{1}{p^*} 2\sqrt{6}\, Y_*^2 \frac{\tau^{\frac{1}{2}}}{(T-\tau)^{\frac{1}{2}}} \Big] \Big\} \tag{19}$$

Since

$$\Gamma_{yx}^2(u) = \frac{\Theta_{yx}^2(u)}{D^2 y_0}$$

therefore

$$D^2 x_0 \int_0^\tau \Theta_{yx}^2(u)\, du \geq D^2 y_0 D^2 x_0 \int_0^\tau r_{yx}^2(u)\, du = \int_0^\tau R_{yx}^2(u)\, du \tag{20}$$

and since $\int_0^\tau R_{yx}^2(u)\, du$ is estimable from the given observation, it is advisably considered:

$$E \mid \alpha_{yx} - \hat{\alpha}_{y\tilde{x}} \mid \leq \frac{D^2 \tilde{x}_0 Y_*^2 \tau}{\int_0^T R_{yx}^2(u) du} \left\{ \frac{\mid D^2 x_0 - D^2 \tilde{x}_0 \mid}{D^2 \tilde{x}_0} + 4\sqrt{3} \frac{\tau^{\frac{1}{2}}}{(T-\tau)^{\frac{1}{2}}} + \right.$$

$$\left. + 6[p(\Delta_1) + p(\Delta_N)] + \frac{8\tau}{T-\tau} + \left(\frac{1}{Y_*} + 1 \right) R\Delta_* + \frac{1}{p^*} 2\sqrt{6} \frac{\tau^{\frac{1}{2}}}{(T-\tau)^{\frac{1}{2}}} \right\} \tag{21}$$

Conclusion

According to the Tchebysheff inequality, for any $\lambda > 0$:

$$P(\mid \alpha_{yx} - \hat{\alpha}_{y\tilde{x}} \mid \geq \lambda) \leq \frac{E \mid \alpha_{yx} - \hat{\alpha}_{y\tilde{x}} \mid}{\lambda}$$

Hence, taking $\lambda = 0.1$:

$$P(\hat{\alpha}_{y\tilde{x}} - \lambda < \alpha_{yx} < \hat{\alpha}_{y\tilde{x}} + \lambda) \geq 1 - \frac{1}{\lambda} E \mid \alpha_{yx} - \hat{\alpha}_{y\tilde{x}} \mid$$

that is, at confidence level $1 - \frac{1}{\lambda} \mathrm{E} \mid \alpha_{yx} - \alpha_{y\tilde{x}} \mid$ inequality $\alpha_{y\tilde{x}} - \lambda < \alpha_{yx} < \alpha_{y\tilde{x}} + \lambda$ holds.

For instance, to have inequality

$$\hat{\alpha}_{y\tilde{x}} - 0.1 < \alpha_{yx} < \hat{\alpha}_{y\tilde{x}} + 0.1$$

hold at a confidence level of 0.9 (confidence interval for α_{xy}, values N, T, $S_1, \ldots, S_{N-1}$ have to be selected to have)

$$10.\ E \mid \alpha_{yx} - \hat{\alpha}_{yx} \mid < 0.1$$

That is:

$$E \mid \alpha_{yx} - \hat{\alpha}_{y\tilde{x}} \mid < 0.01$$

to hold.

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Prof. Dr. Pál MICHELBERGER
Dr. László SZEIDL } H-1521 Budapest
Dr. Albert KERESZTES