DYNAMIC LOADS IN THE DRIVE SYSTEM OF RAILWAY TRACTION VEHICLES DUE TO TRACK UNEVENNESSES

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Received December 20, 1985
Presented by Prof. Dr. K. Horváth

Abstract

Railway traction vehicles moving along tracks go through excited vertical and pitching vibrations due to the vertical unevennesses in the permanent way. These vibrations give rise to axle-load variations, and dynamic loads in each element of the drive system are created by the variations in the creep- and axle-load-dependent tractive effort transmitted through wheel-rail connections. The dynamic loads can be correctly described by the examination of an integrated "track — vehicle — drive-system" model. This paper deals with the dynamic analysis of the load-processes developing in the drive system. The analysis is based upon the results of digital simulation.

Introduction

Traction vehicle moving along the track goes through vertical and pitching vibrations due to the always existing track unevennesses. These vibrations give rise to changes in axle-load, and so dynamic excess loads arise in each element of the drive system, since the track direction forces transmitted through the wheel-rail connection are basically influenced by the axle-load changes mentioned above. The operational reliability and life of the most valuable traction-vehicle-stock is significantly influenced by the variation in the dynamic loading conditions of the drive systems. The exact description of the dynamic excess loads mentioned above, and the solution of the problems raised by vehicle design and operation can be implemented by setting up the dynamic model of the entire track — vehicle — drive system and the digital simulation based upon the former, on an acceptable cost level.

Dynamic model used for the examination

The dynamic model elaborated at the Chair of Railway Vehicles within the Institute of Vehicle Engineering at Technical University Budapest renders it possible that the dynamic processes brought about in the drive system by track unevennesses can be analyzed as embedded in a complex dynamic environment with respect to the properties of the vehicle and track.

The steady-state motion of the drive system can be interpreted in the case of operational speed v_0 of the traction vehicle as considered to be constant. This state of motion is related with the ideal case when the track is perfectly even, the vehicle has no parasitic motion, and the track directional forces transmitted through the wheel-tread are time-independent. In this state, the elastic elements of the traction vehicle suspension system experience deformation due to the time-independent force actions required to overcome tractive resistances, and consequently, the steady-state vertical axle-loads of the traction vehicle will develop. Let T_{0i} be the steady-state axle-load developed on the i^{th} driven wheel-set of the traction vehicle. Then steady-state tractive effort $Z_{0i} = \mu_{0i}\, T_{0i}$ arises on the i^{th} driven wheel-set where μ_{0i} is the steady-state value of the track directional force-connection coefficient between the wheel and rail. If the rolling radius of the i^{th} wheel-set is symbolized by R_i, then it is evident that — in the steady-state motion of the drive system — steady-state driving torque M_{0i} transmitted to the considered wheel-set from the mechanism and reduced by the resistance torques is held in equilibrium by the torque $Z_{0i}R_i$ of the tractive effort.

But the steady-state motion of the drive system outlined above can almost never develop at the operational speed v_0 of the traction vehicle as considered to be constant. On the one hand, due to the always existing track unevennesses, the vehicle elements suspended on the springing of the vehicle and the elastically supported wheel-sets are imparted an exciting effect from the contact area of wheel-rail connections, and as a consequence, the axle-loads become time-dependent according to relationship: $T_i(t) = T_{0i} + \tilde{T}_i(t)$. Here $\tilde{T}_i(t)$ symbolizes the time-dependent partial axle-load modulating the steady-state axle-load T_{0i}; [1], [2].

Tractive effort transmitted through the i^{th} wheel-set will also be time-dependent due to the variation of the axle-load with time: $Z_i(t) = \mu_i\, T_i(t)$. In this way, the driving torque transmitted to the examined wheel-set from the mechanism is counter-acted by tractive effort torque $Z_i(t)\, R_i$ varying with time. And this, in turn, will result in the formation of the time-dependent accelerating torque acting upon the wheel-set and the angular acceleration brought about by the former. Consequently, it can be stated that the formation of torsional vibrations in the drive system should be reckoned with owing to the drive elements functioning as elastic and inertial energy storages. On the other hand, it also follows from the foregoing that the angular velocity of the driven wheel-set will also be time-dependent ($\omega_i(t)$), and hence, creepage/slipping speed $\Delta v_i = R_i\, \omega_i - v_0$ interpreted as the difference between vehicle speed v_0 and rolling-circle peripheral speed $R_i\, \omega_i$ of the wheel-set will also be time-dependent.

It is known from the rolling contact theory of elastic bodies [3], and the experiences of experiments [4], that force-connection coefficient μ is of paro-

mount importance with respect to the track directional force transmitted through the wheel-rail connection, and this coefficient μ is a function of the vehicle speed and the creepage/slipping speed: $\mu = \mu(v, \Delta v)$. Since, according to the stated above, the creepage/slipping speeds related to the single wheel-sets are time-dependent, therefore the tractive effort variation with time as transmitted through the wheel-rail connection to a driven wheel-set at a constant speed v_0 can be written according to relationship:
$Z_i(t) = \mu_i(v_0, \Delta v_i(t)) \cdot T_i(t)$. Consequently, the variation with time of tractive effort torques $Z_i(t)R_i$ determining the loading conditions of the drive system can be traced back, on the one hand, to axle-load time-functions $T_i(t)$,and on the other hand, to the creepage/slipping speeds time-functions $\Delta v_i(t)$; [2].

It follows from the foregoing that the dynamic model mapping the operational loading conditions of the drive system can be divided into two sub-systems interdependent dynamically from each other [6]:

a) the vehicle-track sub-system mapping the vertical and pitching vibrations of the vehicle as excited by the track unevennesses to determine axle-load time-functions $T_i(t)$.
b) The drive-system — vehicle sub-system mapping the torsional vibrations of the drive system as excited by the tractive effort torques to determine the dynamic loading conditions of the drive system.

The system-model developed in this way is a planar dynamic model as far as its basic construction is concerned, in which the inertial, elastic and dissipative characteristics of the track, the structural parts of the vehicle and the drive system are considered as reduced to the vertical medium plane of the track [6].

The stiffness- and damping characteristics of the track, and the effective track masses, resp., placed under the wheel-sets are built into sub-system a). Furthermore, here are built in the inertial, elastic and dissipative elements mapping the structural elements of the vehicle according to their dominant properties. The elements mapping the longitudinal dynamics reaction of the hauled train are also included in this sub-system.

The dynamic model of a finite degree of freedom of the entire track-vehicle sub-system is yielded in a way usual with the examination of mechanical systems. In this dynamic model the following are contained as free co-ordinates: the vertical displacements of the effective track-masses placed under the wheel-sets of the traction vehicle (z_{pi}; $i = 1, 2, \ldots, n$); the vertical displacements of the wheel-sets (z_{ki}; $i = 1, 2, \ldots, n$); the longitudinal and vertical displacements of the bogies and the body (x_{f1}, x_{f2}, x_s; z_{f1}, z_{f2}, z_s); the angular displacements ($\varphi_{f1}, \varphi_{f2}, \varphi_s$) developing in the vertical plane fitting onto the longitudinal centre of the track; as well as the longitudinal displacement (x_v) of the mass replacing the hauled train. All the displacements (including the angular ones) are measured starting from the state of equilibrium of the traction vehicle, and the train, respectively. It should be noted that in certain cases (e.g. in the case of one driven wheel-set, or one driven bogie) the number of free co-ordinates of the dynamic system is reduced.

The exciting effect of track unevennesses is represented by function $u^*(x)$ given as a function of the longitudinal co-ordinate of the track. In the case of travelling speed v_0 considered as constant, already a time-dependent exciting function is yielded by expression $u^*(v_0\, t)$ where the values for the single wheel-sets are given with a delay depending on the wheel-arrangement of the traction vehicle. If the vertical displacement of the i^{th} wheel-set is $z_{ki}(t)$, and the vertical displacement of the effective track-masses placed under the same wheel-set $z_{pi}(t)$, then relationship $z_{ki}(t) = z_{pi}(t) + u_i(t)$ is in force where $u_i(t) = u^*(v_0(t-t_i))$. In case of distance d_i between the axles of the leading and the considered i^{th} wheel-set taken in the direction of travelling, the occurring time-delay t_i is yielded in the form of $t_i = d_i/v_0$; [6].

Track unevenness function $u^*(x)$ can be a deterministic or a stochastic one. In the case of a deterministic track excitation, the track unevenness function can be a periodic one, or given arbitrarily on a prescribed sequence of points. In the case of periodic excitation, the track unevenness function is approximated by means of a finite Fourier's expansion, while in the case of unevenness values given on a discrete sequence of points, it is approximated by means of spline interpolation. The treatment of the stochastic track unevennesses takes place by means of realization functions to be generated with the knowledge of spectral density functions [2]. Note that in the case of a linearized dynamic model, the spectral density functions of track unevennesses can be transformed directly by means of the complex frequency function-matrix of the model into the spectral density function of the required dynamic characteristics [3].

If the examined traction vehicle is of n-axle, then the track excitation of the entire system is yielded by vector function $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T$, which can be formed from track unevenness time-functions $u_i(t)$; $i = 1, 2, \ldots, n$.

The sub-system corresponding to b) contains the elements of the drive system as performing rotation or angular vibrations. So the respective dynamic model contains the following as free co-ordinates: angular displacement (φ_{ki}; $i = 1, 2, \ldots, n_1$; $n_1 \leq n$) of the driven wheel-sets; angular displacements (φ_{hi}; $i = 1, 2, \ldots, n_2$; $n_2 \leq n$) of the final-drives, or those of the nose-suspended motors, respectively, around the axle of the wheel-set, or else, angular displacements (φ_{2i}; $i = 1, 2, \ldots, n_3$; $n_3 \leq n$) of the output shaft of the hydrodynamic transmission gear, or those of the rotor of the traction motors. (We should like to note that in the case of an electric traction vehicle with bogies of monomotors, the role of the angular displacements of the final-drive and the nose-suspended traction motor, respectively, is taken over by the angular displacement of the gear-case during torque-application.) Here, in the b) sub-system are built in tractive effort torques ($Z_i(t) R_i$; $i = 1, 2, \ldots, n_1$; $n_1 \leq n$) transmitted to the wheel-sets from the track. The autonomous properties of the system are not changed by the characteristic-curve of the driving torque in the case of travelling speed v_0 considered as constant.

The degree of freedom of the entire dynamic model is yielded by the sum of the degrees of freedom for the sub-systems according to a) and b). The numerical value of degree of freedom in question can range from DF = 6 with the two-axle traction rail-car having a single driven wheel-set as far as DF = 34 with the two-bogie locomotive having a wheel-set driven by six nose-suspended motors.

Motion equations and response characteristics of the dynamic model

The motion equations applied to the dynamic planar model outlined above are derived in a synthetic way. For non-linear system elements and at a constant mean travelling speed v_0, the following set of equations have been yielded:

$$(1) \qquad [\mathbf{M}_1 + \mathbf{M}_2(v_0, \dot{\mathbf{X}}(t))]\, \ddot{\mathbf{X}}(t) = \mathbf{f}_{v_0}(\mathbf{X}(t), \dot{\mathbf{X}}(t), \mathbf{u}(t), \dot{\mathbf{u}}(t), \ddot{\mathbf{u}}(t)),$$

where $\mathbf{X}(t)$ is the symbol of the vector-valued time-function yielded from the free co-ordinates of the model, while $\mathbf{u}(t)$ symbolizes the vector-valued time-function describing the exciting effect of the track. Mass-matrix $\mathbf{M}_1$ is constant, while $\mathbf{M}_2$ is the derivative function of v_0 and $\dot{\mathbf{X}}(t)$ due to the state-dependence of the wheel-rail connection force. The five-variable vector-function $\mathbf{f}_{v_0}$ on the right-hand side of the equation is determined by the non-linear structural properties of the vehicle.

With the linearization of the match-point applied, set of equations (1) takes the following form:

$$(2) \qquad [\mathbf{M}_1 + \mathbf{M}_2(v_0)]\, \ddot{\mathbf{X}}(t) + [\mathbf{K}_1 + \mathbf{K}_2(v_0)]\, \dot{\mathbf{X}}(t) + [\mathbf{S}_1 + \mathbf{S}_2(v_0)]\, \mathbf{X}(t) =$$

$$= \mathbf{A}_0 + \mathbf{B}(v_0)\, \mathbf{u}(t) + \mathbf{C}(v_0)\, \dot{\mathbf{u}}(t) + \mathbf{D}(v_0)\, \ddot{\mathbf{u}}(t).$$

A computer programme has been elaborated for the numerical solution of both the non-linear (1) and the linearized (2) differential set of equations. This solution yields the system of values defined on a discrete sequence of time-points of solution-function $\mathbf{X}(t)$ and its first and second derivatives with respect to time. The mechanical characteristics describing the dynamic loading conditions of the drive system can be formed from these motion-state characteristics with the use of proper evaluation function $\mathbf{g}_{v_0}$, according to the following vectorial relationship:

$$\mathbf{V}(t) = \mathbf{g}_{v_0}(\mathbf{X}(t), \dot{\mathbf{X}}(t), \ddot{\mathbf{X}}(t)) \tag{3}$$

The torque-arm-support forces, cardan-torques, etc. can enter the co-ordinates of vector-function $\mathbf{V}(t)$. With linearization applied, the following simpler expression is yielded from relationship (3):

$$\mathbf{V}(t) = \mathbf{V}_0(v_0) + \mathbf{V}_1\, \mathbf{X}(t) + \mathbf{V}_2\, \dot{\mathbf{X}}(t) + \mathbf{V}_3\, \ddot{\mathbf{X}}(t), \tag{4}$$

where $\mathbf{V}_0(v_0)$ is the value depending on the mean travelling speed as a parameter, while $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$ are constant matrices.

Computer-programme system elaborated for digital simulation

To examine the mechanical processes of the system model introduced above, the numerical determination of the solution-functions of the corresponding linear and non-linear sets of second order differential equations is required. While ensuring the possibility of multilateral parameter-analysis, the following computer-programme of FORTRAN language has been prepared:

a) A programme suitable for the examination of a linear dynamic model under periodic track excitation, by which the vector of the generalized co-ordinates, and the vectors comprising the first and second derivatives of the generalized co-ordinates, as well as the response-vector to be formed from these can be determined for arbitrary time-points by means of complex frequency-functions.

b) A programme suitable for the examination of a linear dynamic model under weakly stationary stochastic track excitation, by which starting from the spectral density-function of track unevennesses, the spectral density-function matrix of the vectorial process of the system response can be derived by means of the complex frequency-function matrix. With the main diagonal elements of the spectrum-matrix of the response process as integrated with respect to the angular frequency, the variance-vector of the response process and the dynamic coefficients of the response process co-ordinate-functions will be determined.

c) A programme suitable for the examination of a non-linear dynamic model enabling the consideration of two types of track excitation. On the one hand, the exciting effects of the periodic track unevennesses

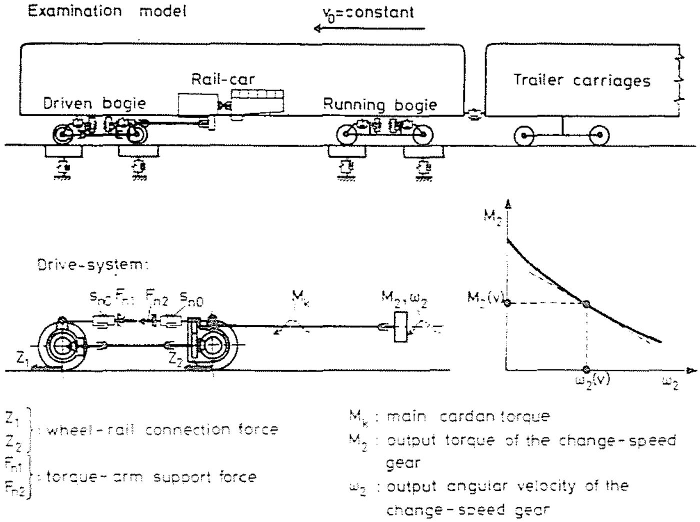

Fig. 1

given by the finite section of Fourier's series, and on the other hand, the exciting effect of the particular track unevenness-function prescribed arbitrarily on a given sequence of points can be taken into consideration. In the latter case, spline interpolation will be carried out. The numerical solution of the set of differential equations of the dynamic model as reduced to a first-order one is given in the programme by means of the fourth-order Runge-Kutta method of varying stepinterval on a prescribed level of accuracy. So the vector of the generalized co-ordinates together with its first and second derivatives will be determined on a prescribed sequence of time-points, while the value-system of the response process can also be derived from those by substituting them into the corresponding vector-function.

The results of examinations

The dynamic models introduced in the foregoing, and the digital simulation based on them are illustrated by load-analysis of the hydrodynamic power-transmission system of a diesel rail-car of 1000 mm gauge (Fig. 1). The rail-car has one driven bogie, hence the degrees of freedom of the associated dynamic model were yielded as DF = 18. The non-linearities of the dynamic model were resulted from the geometrical properties of the bogie, from the non-linear variation of the wheel-rail force-connection coefficient as the function of creepage/slipping speed (Fig. 2), as well as from the non-linear displacement-relationship and dry-friction damping of the connection-force between the traction vehicle and the set of trailer carriages.

The exciting effects acting on the dynamic model were derived from the periodic track unevennesses existing at the fish-plate rail-joints of the short-stretch-rail permanent way, and they were described by a finite sum containing the first 40 harmonics of the Fourier's series. The shape of the track-profile taken into consideration in the neighbourhood of the rail-joints are shown in Fig. 3.

The dynamic loading conditions developing in the drive system of the examined rail-car at a mean travelling speed of $v_0 = 105.91$ km/h are represented by the time-function diagram of three charactaristic quantities. In Fig. 3, time-functions $F_{t1}(t)$ and $F_{t2}(t)$ of the force actions arising due to the periodic exciting effects in the torque-arm supports of the final-drives are plotted. The time-function of the torsional torque arising in the main cardan-shaft is represented in Fig. 4.

It can be seen well from the diagram that during running over a rail-joint, as much as 24—28 times the stationary value occurs in the torque-arm support forces, while as much as about 19 times the stationary value occurs in the cardan-torque as a peak-value, respectively. The level-non-achieving distribution- and density functions of the examined time-functions are plotted in Fig. 5. It is striking that the distribution of the dynamic loads shows a shape deviating significantly from the normal (Gaussian) distribution.

In the course of numerical analysis, the variation of the response-functions yielded as a result of applying non-linear and linearized dynamic models was also examined. The variation of the time-function of force F_{t1} arising in the torque-arm-support of the single-stage final-drive is shown in Fig. 6, in the cases of applying non-linear and linearized models. The variation of the two functions is significantly deviating from each other. It can be stated about the function yielded by the non-linear model that its peak-values are greater by 25—35% than those yielded by the linearized model, and the damping characteristics of the functions obtained with the use of non-linear model are less intensive.

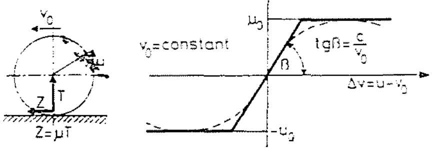

Fig. 2

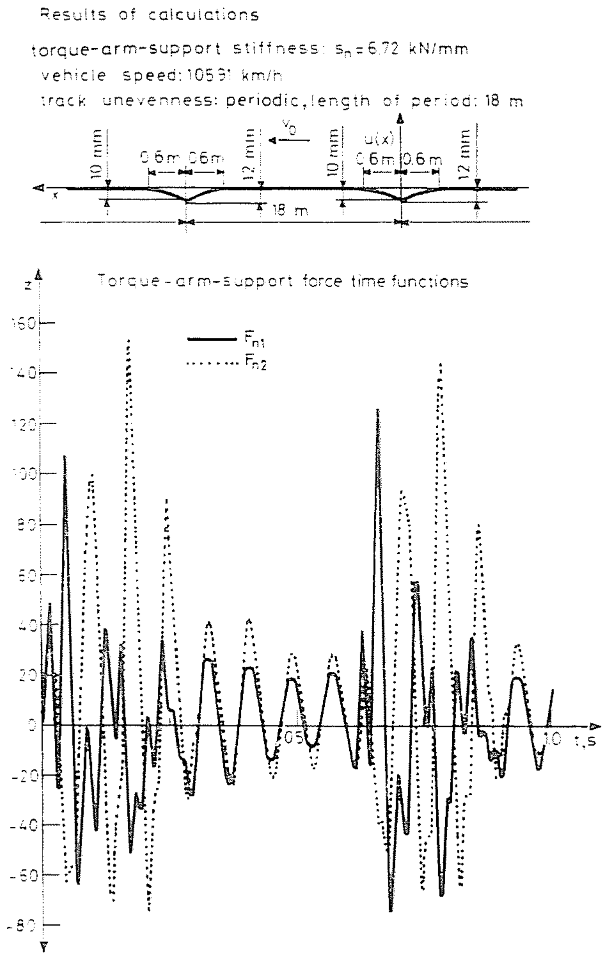
Results of calculations
torque-arm-support stiffness: s_n = 6.72 kN/mm
vehicle speed: 105.91 km/h
track unevenness: periodic, length of period: 18 m
10 mm
0.6m 0.6m
12 mm
v_0
u(x)
18 m
Torque-arm-support force time functions
F_{n1}
F_{n2}
160
140
120
100
80
60
40
20
0
-20
-40
-60
-80
0.5
1.0
t, s

Fig. 3

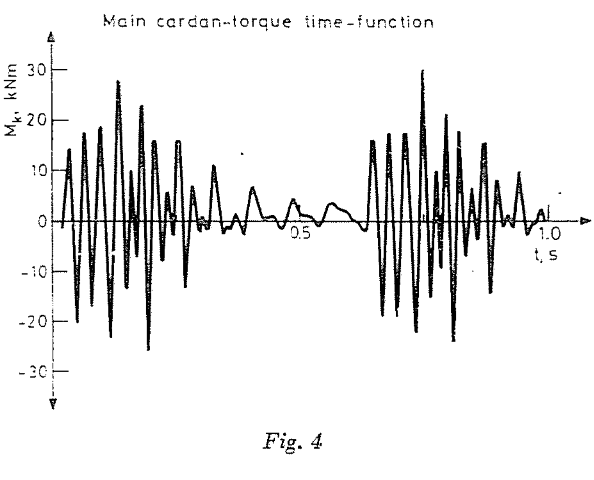
Main cardan-torque time-function
M_k, kNm
30
20
10
0
-10
-20
-30
0.5
1.0
t, s

Fig. 4

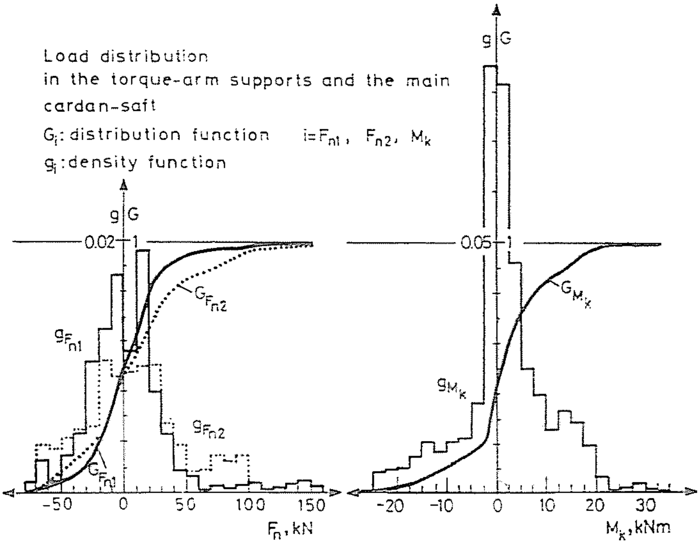
Load distribution
in the torque-arm supports and the main
cardan-saft
Gi: distribution function i=Fn1, Fn2, Mk
gi: density function
g G
0.02
1
GFn2
gFn1
gFn2
GFn1
-50 0 50 100 150
Fn, kN
0.05
GMk
gMk
-20 -10 0 10 20 30
Mk, kNm

Fig. 5

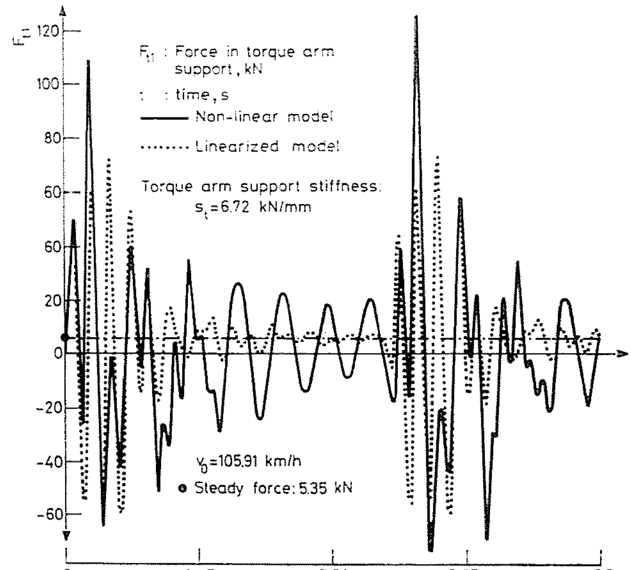
Ft1 : Force in torque arm support, kN
t : time, s
Non-linear model
Linearized model
Torque arm support stiffness: st=6.72 kN/mm
v0=105.91 km/h
Steady force: 5.35 kN
120
100
80
60
60
20
0
-20
-40
-60
0 0.25 0.50 0.75 t 1.00

Fig. 6

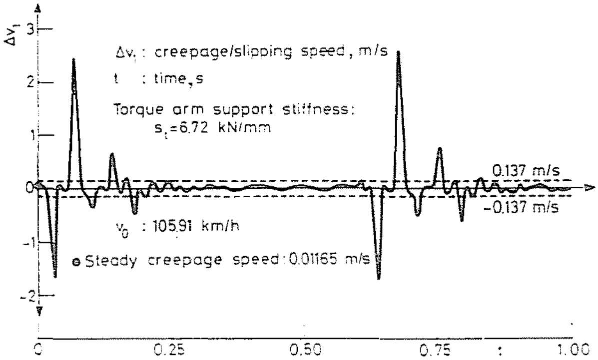

Fig. 7

The reason of the significant deviations in the case of the examined branching drive system can be explained by the significant macroscopic slippings occurring at the rail-joints. The peak-values of the slipping speed can reach even the magnitude of 2.5 m/s as shown in Fig. 7, as opposed to the stationary value 0.01165 m/s of creepage speed. On the contrary, macroscopic slippings can not develop at all with the linearized model.

Concluding remarks

On the basis of the results of actual examinations it can be stated that the dynamic processes developing in the drive system of railway traction vehicles as a result of track unevennesses can be detected by means of model-formation and dynamic simulation. It is revealed from the comparison of the results obtained by using linearized and non-linear models that the application of a non-linear model is required in the case of existing significant track unevennesses.

The level-non-achieving probability-distributions can be used effectively for the evaluation of dynamic processes, and these level-non-achieving probability distributions can be built also into the objective-functions of the optimization problems [7].

Further investigations are required for promoting the development of models, in the course of which, first of all, the parametric exciting effect of the cardan shaft built into the drive system should be taken into consideration, as well as the effects of tooth elasticity, those of the tooth-pitch- and tooth-profile errors of the built-in gear-wheels [8].

In connection with the application of the model even identification problems will arise [9]. On the one hand, identification processes can be used for the determination of system-parameters, and on the other hand, starting from the measurement data related with the single generalised co-ordinates, a possibility is also offered for the identification of the exciting track unevenness-functions through the constructed model.

References

1. BIRJUKOV, I. V.—LVOV, N. V.: Dinamika elektricheskovo lokomotiva i realizácija sceplenyija. Manuscript, 1979.
2. ZOBORY, I.: Influence of Vertical Exciting Effects during Running Transmitted from the Track onto the Drive System of Traction Vehicles.* Material of the 4th symposium of the Transportation Engineering Association, Pécs, 1983. 228—242. p.
3. KALKER, J. J.: On the rolling contact of two elastic bodies in the presence of dry friction. Nederlandsch Drukkerij Bedrijf N. V. Leiden, 1967.
4. PEARCE, T. G.—ROSE, K. A.: Tangential Force-Creepage Relationships in Theory and Practice. Contact Mechanics and Wear of Rail/Wheel Systems. Vancouver, 1982.
5. ZOBORY, I.: On the Dynamics of Drive Systems of Railway Traction Vehicles under Stochastic Track Excitation. Periodica Polytechnica (Transportation Engineering), Budapest, 12. 1984. 1—2. 73—83 p.
6. ZOBORY, I.: Dynamic Processes in the Drive Systems of Railway Traction Vehicles in the Presence of Excitation Caused by Unevennesses in the Track. Vehicle System Dynamics, 14. 1985. 1—3. June, 33—39. p.
7. ZOBORY, I.: On the Optimum Selection of Torque-Arm-Support Stiffnesses for Final Drives of Railway Traction Vehicles. Newsletter, Technical University of Budapest, III. 1985. 4. 17—26. p.
8. Drive-dynamics Examination of Electric Train-set III.* Research report 492018/85/3. BME Járműgépészeti Intézet, Budapest, 1985.

[9] MICHELBERGER, P. — KERESZTES, A. — BOKOR, J. — VÁRLAKI, P.: Non-linearity Analysis for Identification of Commercial Road Vehicle Structure Dynamics. Vehicle System Dynamics, 14. 1985. 1—3. June, 172—176. p.

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