LATENT STRUCTURE MODELLING FOR TRIP DISTRIBUTION

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Abstract

This paper outlines the first stage of a research project investigating the application of latent structure modelling techniques to the trip distribution stage of conventional traffic modelling. It has the objective of developing practical computer methods for fitting latent structure models to trip distribution data, and to investigate whether these models give a substantially improved fit to observed matrices of zone to zone flows. Discussion centres around the results of applying the latent approach to four different types of model — negative exponential, negative exponential quadratic, power and Tanner models — and the computing time and resource requirements associated with each. The paper concludes with a summary of future prospects and suggestions for application to real (rather than artificial) trip data matrices.

Introduction

This report outlines the first stage of a research project investigating the application of latent structure modelling techniques to the trip distribution stage of conventional traffic modelling. It has a prime objective of developing practical computer methods for fitting latent; structure models to trip distribution data, and to investigate whether these models give a substantially improved fit to observed matrices of zone to zone flows. In its simplest form, a trip distribution model commonly used takes the form of:

$$T_{ij} = A_i B_j \exp(-\lambda c_{ij})$$

where T_{ij} = the number of trips from zone i to zone j

c_{ij} = the costs of travel from zone i to zone j

A_i, B_j, λ — unknown parameters which need to be estimated

A value for A_i is often approximated by the population size of the zone and attempts to reflect the generation of trips. B_j, similarly, might be taken as the population size of the attracting zone. λ represents the deterrence function and indicates the sensitivity of trip makers and making to the costs involved. A large number of variants of this basic model have been tested. They include the use of alternative deterrence functions — a power function, or Tanner function — or alternative measures of cost (time, money, utility, distance, etc.). In each case, the tendency has been to retain the aggregate nature of the model, using it to produce large-scale predictions of traffic flows.

Disaggregate Modelling

Interest in disaggregate trip distribution modelling has increased in recent years with the realisation of the inaccuracies and inadequacies of conventional aggregate methods. The assumption that the behaviour of large groups of people is predictable on the basis of mathematical probability, with the idiosyncracies of individuals or small groups tending to be cancelled out, has lost much favour. Lee (1973) suggested that the disaggregation of models to take account of differentials in socio-economic characteristics and trip purpose, would result in substantial improvements in their descriptive and forecasting ability. This was re-affirmed by Wilson (1974), and Southworth (1978a, 1978b and 1979) who proceeded to calibrate a production constrained entropy maximising trip distribution model for a variety of trip purposes and income groups. This included the use of origin-specific time delay functions.

The trend towards disaggregation has been typified in the work of the Transport Studies Unit at Oxford University and the development of travel time budget models (Oxford University T. S. U., 1980.). However, in keeping with other efforts to disaggregate trip distribution modelling, the demands for data and analysis increase alarmingly, detracting from the improved analysis which it makes possible. It is the objective of this work to assess the ability of a new approach to trip distribution modelling which makes full use of traditional modelling procedures whilst at the same time uses the aggregate information they provide as a basis for further disaggregation without recourse to further costly and time consuming data collection and analysis.

Latent Structure Modelling

Latent structure modelling is a method of analysing and measuring unobservable phenomena which cannot be satisfactorily operationally defined. It is a technique derived from psychology and has been used to differentiate between people, objects or collectivities either by classifying, ordering or positioning them along some continuum with respect to underlying characteristics that cannot be explicitly measured.

In the context of trip distribution modelling, it is a methodology which has potential to disaggregate a body of data into latent classes on the basis of the underlying latent variables which exist within that aggregate information, but without the need for further data collection. It thus provides a means of disaggregation, if such classes exist, which is quick, inexpensive and yet statistically reliable. The theory of latent structure analysis is described in detail by Lazarsfield and Henry (1965, 1968) and examples of its use in practice can be found in Goodman (1973, 1974, 1979), Clogg (1980) and many others. The traditional body of trip data — the trip distribution matrix, in conjunction with a trip cost matrix and a deterrence function, could be used to provide the aggregate information from which latent classes might be derived. Clearly, such classes are likely to exist. It is a matter of common sense that people of differing incomes live in different areas, and tend to generate different trips and travel to different places.

If one knew in advance what the categories were that made up the total population, one could attempt to identify into which class each fell. But this would require extensive data on income, socio-economic group, etc. which is largely unavailable — and in any case, one might not know what the underlying categories are. The latent structure model attempts to disaggregate the trip distribution matrix when the data to do so straightforwardly is unavailable.

Within latent structure modelling, in a two way contingency table, let N_{ij} be the number of objects classified into category i on the first dimension and category j on the second dimension (assuming two latent classes). The model for independence is:

$$N_{ij} = A_i B_j \tag{2}$$

If data is a mixture from two different populations, within each of which independence holds, we obtain:

$$N_{ij} = \varrho Ai \; Bj + (1-\varrho) \quad Ci \quad Dj \tag{3}$$

where (under constraints on the other parameters) ϱ is the proportion in the first population. The similarity of (2) to the traditional trip distribution model (1) is clear.

Thus, with the discussion of disaggregate modelling in mind we can propose a latent structure model for trip distribution:

$$T_{ij} = \varrho A_i \; B_j \exp\left(-\lambda c_{ij}\right) + (1-\varrho) \, C_i \, D_j \exp\left(-UC_{ij}\right) \tag{4}$$

This can be interpreted as trip data coming from two populations, within each of which the conventional model holds. The model itself will (if these two classes exist) divide the aggregate matrix into two matrices representing trips associated with the two groups. Initially only two groups are used to verify the model and to ensure simplicity at this early stage. Quite clearly an infinite number of groups might emerge, but attempts to provide for this are unjustified at this time.

Objectives

The objectives of the research can be stated quite simply

(i) To write an efficient computer program to obtain the parameters of model (4), from given matrices of T_{ij} and C_{ij}.

(ii) To apply the program to real data in order to determine whether model (4) is a significant (both in the practical and statistical senses) improvement over model (1).

This report discusses the first stage of the research which aims to satisfy objective (i).

Validation

Before the latent structure model could be applied within a trip distribution context it was necessary to establish the validity of the results it produces and its ability to reflect and reproduce a known pattern of spatial interaction. Consequently it was decided to create a number of artificial trip matrices derived using a specific model formula, trip deterrence function, cost matrix and set of attraction and generation parameters. Attempts would then be made to reproduce these trip distribution matrices using both a conventional model and a latent structure model. The latter would break down the matrix into two component parts.

The trip matrices which were artificially created varied in a number of ways,

— in size. Matrices of between 5×5 and 18×18 cells were modelled.

— In the number of components. Artificial matrices were created using two different values of deterrence function and attraction and generation parameters to produce two differing trip patterns and these were then summed to produce a single trip matrix. The latent structure model was then used to recreate the two matrices, whilst attempting to achieve the benefit of aggregated matrices. Some attempts were also made with 3 component latent structure models and with single component models for comparative purposes.

— In deterrence function. Each matrix and component size was tested using 4 different deterrence functions.

Negative exponential	$e^{-\lambda C_{ij}}$
power	$C_{ij}^{-\lambda}$
negative exponential quadratic	$e^{-(\lambda_1 C_{ij} + \lambda_2 C_{ij}^2)}$
Tanner	$e^{-\lambda C_{ij}} C_{ij}^{\lambda}$

The latent structure models were calibrated to reproduce the original aggregated trip matrix. A minimum x^2 statistic was used to assess goodness of fit. At the same time the sets of attraction and generation parameters used to define the artificial disaggregate matrices (in the case of two matrices/components, two sets of A_i and B_j) were compared with those produced by the latent structure model. It was important that the latent structure approach should be capable of reproducing both the overall matrix in aggregate form, and the parameters which were used in creating the two (or three) component artificial matrix. If this was so, then it is possible that this approach could be used to model a real life situation. If not, then its validity was in doubt.

The method of testing goodness of fit between matrices was the minimum χ^2 statistic. This was calculated after every iteration of the latent structure model until a minimum was found. At this point iteration ceased and the results from the modelling process could be compared with the artificial data. Attraction and generation parameters ought to be the same before and after modelling. The x^2 statistic ought to be very small — reflecting accuracy. For the initial validation procedure, a standard function minimisation procedure, was used (NAG). This was a quasi-Newton algorithm for finding an unconstrained minimum of a function using function valves only. From a starting point supplied by the user, a sequence of points is generated which is intended to converge to a local minimum. These points are generated using estimates of the gradient and curvature of the objective function. An attempt is made to verify that the final point is a minimum (Gill and Murray, 1972).

The validation procedure is outlined in Table 1.

Table 1

The validation process

1. Define trip distribution model, deterrence function, attraction and generation parameters and cost matrix.
2. Define two (or three) values of the deterrence function.
3. Create artificial trip distribution matrices, one for each deterrence function.
4. Aggregate them into a single trip distribution matrix — the artificial two (or three) component matrix.
5. Recreate this artificial matrix using the latent structure approach.
6. Define model to be used — as in the artificial matrix.
7. Set initial estimates of attraction and generation parameters (two (or three) of each for each zone).
8. Using the deterrence function values, estimates of attraction and generation parameters and costs, aim to recreate the aggregate artificial matrix using an iterative function minisation routine with χ^2 as test of fit. Do so by creating two (or three) trip matrices, corresponding to the artificial data. Keep recalculating these matrices and comparing their aggregate sum with the aggregate artificial data until the χ^2 statistic is minimised. Cease iteration.
9. Compare disaggregate attraction and generation parameters. If valid, they should match.
10. Check χ^2 statistic for goodness of fit.
11. Check matrices for cell value accuracy.

Results

Tables 2—5 outline the results from these validation tests. The complete range of matrix sizes and of components was not tested for each model as it was considered unnecessary. The results shown here are ample evidence of the ability of the latent structure approach to recover one, two and three component solutions through an iterative process and to do so accurately. This implies that if the approach is able to model a known multiple or single component structure of trip making, then it is likely to be able to reproduce and indicate where such a structure exists in real data, but where that structure is unclear, or unknown from the aggregate trip data. It would achieve this without recourse to extra data collection or manual disaggregation of trip data that was available.

The results are discussed below:

Table 2

Validation. Negative exponential model

Matrix Size	Components	Differences between Modelled and Original		
		Det. Function Values	χ^2 (accuracy)	Balancing Factors
18 × 18	2	none	negligible (—11)	none
10 × 10	2	none	negligible (—7)	none
9 × 9	2	none	negligible (—7)	none
5 × 5	2	negligible	negligible (—4)	negligible
10 × 10	3	none	negligible (—6)	negligible
18 × 18	1	none	negligible (—13)	none
10 × 10	1	none	negligible (—6)	none
5 × 5	1	none	negligible (—8)	none

Table 3

Validation. Power model

Matrix Size	Components	Difference between Modelled and Original		
		Det. Function Values	χ^2 (accuracy)	Balancing Factors
18 × 18	2	none	negligible (—5)	none
9 × 9	2	none	negligible (—7)	none
5 × 5	2	none	negligible (—9)	none
9 × 9	3	none	negligible (—5)	negligible (poorest model was 1.0 to 0.82, 7.07 to 8.00)
18 × 18	1	none	negligible (—8)	none
10 × 10	1	none	negligible (—5)	none
5 × 5	1	none	negligible (—9)	none

Table 4

Validation. Negative exponential quadratic

Matrix Size	Components	Difference between Modelled and Original		
		Det. Function Values	χ^2 (accuracy)	Balancing Factors
18 × 18	2	none	negligible (−9)	none
10 × 10	2	none	negligible (−8)	none
5 × 5	2	none	negligible (−6)	negligible (poorest fit 3.99 to 4.00 and 1.98 to 2.00)
10 × 10	3	none	negligible (−5)	negligible (1.06 to 1.00, 1.12 to 1.00)
18 × 18	1	none	negligible (−11)	none
10 × 10	1	none	negligible (−7)	none
5 × 5	1	none	negligible (−8)	none

Table 5

Validation. Tanner model

Matrix Size	Components	Difference between Modelled and Original		
		Det. Function Values	χ^2 (accuracy)	Balancing Factors
18 × 18	2	none	negligible (−7)	none
9 × 9	2	none	negligible (−5)	negligible (e.g. 2.92 to 3.00, 0.51 to 0.5)
9 × 9	3	none	small (−2)	small (e.g. 1.09 to 1.00, 2.21 to 2.0)
18 × 18	1	none	negligible (−7)	none
10 × 10	1	none	negligible (−8)	none
5 × 5	1	none	negligible (−8)	none

(i) *Negative Exponential Model*

1, 2 and 3 component, latent structure models were fitted to artificial trip distribution data using a negative exponential deterrence function. The largest number of runs were of two components, with matrix sizes ranging from 18 × 18 to 5 × 5. In each case, except the smallest, the capabilities of the latent structure approach were clear. x^2 values were very small becoming progressively higher and thus representing a worse fit as matrix size decreased. This was expected as derivation of parameters was always going to become more difficult as the number of zonal cells decreased. Only in the case of the smallest matrix (5 × 5) was the model incapable of reproducing the initial values of attraction and generation parameters and deterrence functions. Even so, the values recovered were close (e.g. deterrence function values of 0.04 and 0.10 compared with 0.05 and 0.10).

Three component negative exponential models were run for a 10 × 10 matrix. Despite the extra parameters which had to be estimated (in this case 60 compared with 40 in the two component case), the recovery of the model was very good. The deterrence functions of 0.05, 0.10 and 0.07 were each recovered whilst the parameters were closely matched. A χ^2 value of 0.26844E — 06 was achieved which was particularly good since the derivation of three component solutions inevitably makes recovery of the initial trip matrix more difficult.

Single component models were run for matrix sizes 18 $\times$ 18, 10 $\times$ 10 and 5 $\times$ 5 to reflect the ability of the model to derive solutions where disaggregate information was not required. Attraction and generation parameters and deterrence function values were reproduced exactly, and χ^2 values were very low suggesting a good fit.

(ii) *Power Model*

Single, 2 and 3 component models were again teste dand overall, the recovery of initial values was very good.

Two component solutions were derived for 18 $\times$ 18, 9 $\times$ 9 and 5 $\times$ 5 matrices with deterrence functions of 1.5 and 1.2 in each case. χ^2 values were relatively good — although not as low as for the negative exponential solutions. The recovery of attraction and generation parameters was good for all but the 5 $\times$ 5 solution where inaccuracies crept in again. The larger matrices modelled these parameters almost perfectly with the differences between artificial and modelled parameters attributable to rounding errors.

A three component model was fitted to a 9 $\times$ 9 matrix and a reasonably good χ^2 value was obtained — although less accurate than that for the two component equivalent. Deterrence function values were adequately reproduced but the attraction and generation parameters were slightly less satisfactory implying that larger matrices were required to achieve three component power model solutions. However, despite this, the ability of the model to work towards a three component solution, was clear.

Single component solutions were again derived for comparative purposes and produced accurate representation of deterrence function, and attraction and generation parameters. χ^2 values were very low.

(iii) *Negative Exponential Quadratic Model*

The χ^2 values for the two component negative exponential quadratic model proved to be more accurate than the power model and compared favourably with the negative exponential model. Attraction and generation parameters were well recovered as were the deterrence functions of 0.05, 0.10, 0.08 and 0.11. Once again, the ability of the model to recover original parameter values and to reproduce the total trip matrix declined (marginally) as matrix size decreased. In fact, the negative exponential quadratic model proved itself to be the best model so far in recovering original values using small matrices. Attraction and generation parameters were accurately reproduced with the largest discrepancy being 1.98 compared with 2.00, and 5.98 with 6.00 (negative exponential values were 4.26 with 4.00 and 4.65 with 6.00). The value of the extra parameters in the negative exponential quadratic model was clearly apparent.

The three component solution for a 10 $\times$ 10 matrix proved to be less satisfactory than that using a negative exponential model. Deterrence function values were recovered accurately but the χ^2 statistic was slightly less acceptable; and the attraction and generation parameters showed some noticeable, if only slightly significant, discrepancies — e.g. 4.00 compared with 4.17 and 5.00 with 5.26. Clearly, a larger matrix size would overcome this.

Single component solutions again, were accurately recovered from all points of view.

(iv) *Tanner Models*

The Tanner model was formulated to combine the best of the negative exponential and power models although, inevitably, it has achieved a compromise of the two. Two component solutions were fitted to 18 $\times$ 18 and 9 $\times$ 9 trip distribution matrices. χ^2 values were not as small as for other models although they remained reasonable. Deterrence function values were recovered in both cases whilst in the 18 $\times$ 18 matrix case, the attraction and generation parameters were also well reproduced. The smaller, 9 $\times$ 9 matrix failed to achieve such a good recovery of parameters and values of 3.0 compared with 2.89, 1.0 and 0.98 and 2.0 and 1.96 were typical.

The three component version of the Tanner model, fitted to a 9 $\times$ 9 matrix was least satisfactory of any model fitted so far. The χ^2 value of 0.14911E -02 was comparatively poor whilst the attraction and generation parameters were far from satisfactory. Examples of the poor fit were 0.49 compared with 1.0, 4.77 with 2.0 and 2.0 with 2.30. Clearly the model was working towards a fit but a larger matrix size would have helped considerably. Thus, despite the explicit objectives of the Tanner model to combine the best parts of negative exponential and power models, overall it produced a comparatively poor fit. Its ability to produce a single component solution was also in doubt, although the χ^2 results, and recovery of parameters were adequate.

C. P. U. Time

Whilst carrying out the validation exercises, it was decided to examine the time and resource requirements of the latent structure approach to trip distribution modelling. The iterative nature of this process suggested that it would require substantial quantities of computer time that would increase disproportionally to the size of the problem. This, coupled with the known requirements of conventional trip distribution modelling and the practical need to constrain resources, meant that detail of CPU times was an important indicator of efficiency.

Table 6

Some examples of CPU time

Matrix Size	Function	Components	CPU Time *Min*	CPU Time *Secs*
5 × 5	Power	1	0	6
5 × 5	Neg Exp Quand	1	0	4
6 × 6	Power	1	0	17
6 × 6	Tanner	1	0	11
6 × 6	Neg Exp Quad	1	0	15
9 × 9	Power	1	0	46
9 × 9	Neg Exp Quad	1	0	41
10 × 10	Power	1	1	21
10 × 10	Tanner	1	1	9
10 × 10	Neg Exp Quad	1	1	3
18 × 18	Power	1	14	48
18 × 18	Neg Exp Quad	1	10	46
5 × 5	Neg Exp	2	1	17
5 × 5	Neg Exp Quad	2	0	30
6 × 6	Power	2	2	53
6 × 6	Neg Exp Quad	2	2	06
9 × 9	Neg Exp	2	6	1
9 × 9	Power	2	3	24
10 × 10	Neg Exp Quad	2	7	39
18 × 18	Neg Exp	2	40	24
18 × 18	Neg Exp Quad	2	38	34
9 × 9	Power	3	17	53
10 × 10	Power	3	26	00
10 × 10	Neg Exp Quad	3	27	57
18 × 18	Tanner	3	56	00

Table 6 outlines a selection of CPU times associated with a variety of validation runs. It is clear from this that the requirements of computer time are closely allied to the matrix size and more particularly, to the number of components. Together they determine computer needs. It is important to note that these times are for validation runs only and one would expect that the models would be able to recover artificial values in a quick, concise and efficient way. Clearly, when applied to the vagaries of real data, these requirements are likely to increase substantially in which case, the demands of, say, the three component model and larger matrices, may be prohibitive.

From the earlier table it is clear that both model type and matrix size are significant in determining time requirements. The negative exponential quadratic model is distinctly less efficient in deriving a solution compared with the power model — although we have seen earlier, that it is able to do so rather more accurately than any other. These two features may not be entirely disconnected.

In every case, the value of disaggregating the trip structure into an additional component needs to be carefully assessed. It is expensive in time and resources and the extra information it provides has to be shown to be worthwhile. Subsequent stages of the research will aim to reduce computer requirements so that application to real data trip matrices becomes more viable.

Conclusions

A computer program has been developed that fits a latent structure model for trip distribution to a matrix of trip data and breaks this information down into a specified number of trip matrices, each related to a certain underlying parameter. The number of matrices (or components) tested, is three (1, 2 and 3) and the matrix sizes range from 5 $\times$ 5 zones to 18 $\times$ 18. Four deterrence functions have been used — negative exponential, power, negative exponential quadratic and Tanner.

Examination has been made of the ability of each model and each matrix size for each component number, to recover the original values of attraction and generation parameters and deterrence function values used to create the original artificial trip distribution matrix. The test of goodness of fit, at which point the iterative modelling process ceases, has been the χ^2 statistic. Examination of values recovered by the latent structure models has shown that in all cases, a reasonable fit has been obtained and that in many, the fit has been exact. As matrix size increases, so does goodness of fit. This is also the case as the number of components decreases. The χ^2 statistic showing the relationship of the trip distribution matrices to the original matrices has in general been very good. Deterrence function values have been recovered without exception.

Overall, the latent structure approach has shown itself to be able to take an artificially constructed trip matrix which is known to consist of a set of components (ranging from 1 to 3) and reproduce this matrix accurately whilst deriving the appropriate number of components, the constituent trips and the associated parameters and values. Consequently, it is fair to assume that the process of latent structure trip distribution modelling has been validated. A clear assessment of the capabilities of a range of deterrence functions has emerged, although the purpose of testing these models was not to select one but to discover which were applicable in the latent structure context. Given validation in these terms, it is safe to assume that the approach could be applied to real data and that the results it produces will be meaningful. A preliminary examination of the computer CPU time requirements has shown this to be a significant issue that will require further attention as the demands of real application become more apparent.

Future Research

Following validation, it is possible to develop the latent structure approach in a number of ways:

(i) Its application to real data. A number of relatively small real trip distribution and cost matrices have been assembled. Two and three component models will be fitted to this information with the aim of disaggregation into a separate number of trip matrices as well as an aggregate matrix which will be fitted to the original data (using χ^2 as a test of goodness of fit). An examination will then be made of this χ^2 statistic, the attraction and generation parameters and deterrence function values which are derived and the division of trips into component parts. Clearly, if no such split into components is possible, this might reflect either:

(a) a deficiency in the model; or

(b) the fact that no latent structure exists in real trip data.

It might also suggest that more than two components are needed.

(ii) Current modelling approaches use a general algorithm for function minimisation. Clearly, specific algorithms which are designed to meet the requirements of the latent structure approach, might offer a more precise and efficient modelling method. The effect upon computer resources could be significant.

(iii) Each of the deterrence functions will be fitted to a variety of real data trip matrices. Similarly, a number of components will be derived (1, 2, 3 and possibly 4).

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