THE SAFETY OF RAILWAY VEHICLES NEGOTIATING A CURVE

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Abstract

Among the fundamental technical conditions of railway transport, the safety of the railway vehicle moving along the track is of outstanding importance. This basic condition requires that the railway vehicle should be retained within the gauge of the track under all operating conditions of the track, loading and speed so that it should not be derailed by any normal service effects.

This basic requirement is valid to a greater extent for railway vehicles negotiating a curve because the curve may rightly be considered as a distinguished section of the railway track owing to the particular forces applied on the railway vehicle moving along the curve and those acting against its safety of motion.

The aim of this discussion is to determine these particular forces as well as to examine their effect on the safety of railway vehicles negotiating a curve.

Scientific preliminaries

As early as the initial stage of railway transport, the following requirement was raised: for the safety of motion of railway vehicles, a thorough investigation should be carried out concerning the geometrics, speed and force conditions of the railway vehicle, the running gear, particularly its leading wheel pair.

Since according to our experiences, the curve is a critical section of the railway track, the first problem to be investigated scientifically concerning the contact between the curve and vehicle running-gear was the geometrical investigation of the position occupied by the railway vehicle in the curve.

After initial experiments, the circular- and parabolic methods — especially the latter one — were established and widely introduced for describing the curve under consideration. Within the curve represented by the parabolic method, the longitudinal axis of the railway vehicle is perpendicular to the radius of the curve: the lateral dimensions and the displacements of the railway vehicle are represented proportionally to the real ones. So, by means of this procedure, it could be discovered whether a railway vehicle of a certain wheel arrangement or a bogie vehicle can move along the curve under consideration without being wedged up between the rails.

But the fundamental problem in the investigation of negotiating a curve by a railway vehicle can be formulated as follows: even if the railway vehicle can traverse the curve under consideration without any stresses from geometrical point of view, the question remains as to what a position it will assume in the curve and what the factors will be determining this position. No answer can be obtained to this question from a pure geometrical investigation because the favourable geometrical conditions provide only the necessary but not the satisfactory conditions of traversing the curve.

Further investigations were enabled by the examination of the balanced force-system acting on the railway vehicle moving along the curve in an identical position (conformly).

Accordingly, in the sense of the theory of force-conditions of the railway vehicle moving along the curve, as developed by numerous researchers in the past, the railway vehicle has generally not a pure rolling motion along the curve. Due to this fact, through the action of frictional forces arising on the contact areas of the railhead and the tread areas of the railway vehicle tyres moving along the curve, the conical tyre-surface of the leading wheel of the railway vehicle is pressed against the inside surface of the outside rail of the curve with its guiding-point A (Fig. 1).

In a straight section there is no lateral direction friction: lateral force $K = O$. But as the railway vehicle enters the transition section of the curve from the straight section of the track, tread-frictions occur, and by their action, lateral resulting force K arises. The leading wheel of the railway vehicle is raised by resulting guiding-force K arising during its moving along the curve and acting on guiding-point A of the railway vehicle. While this raising effect is less than perpendicular force Q acting on the leading wheel and loading partly the tread-surface of the rail, partly guiding-point A, the leading wheel pair is retained within the gauge.

If resultant guiding-force K is exactly equalized by gravity-force Q loading the leading wheel, the *tyre-tread of the leading-wheel — though it is still in contact with the running-surface of the rail —, the load is nevertheless not transmitted through it but only through guiding-point A on the flange.* As a result of this, there occurs a basically differing loading-state designated by sign Δ.

This force-system, though critical, but still in equilibrium will go through a change if resulting lateral force K exceeds the value limited by critical state of equilibrium Δ because under these conditions the leading wheel is starting to leave the track of the curve.

For the sake of achieving the safety of motion of the railway vehicle negotiating a curve, the development of critical state Δ should by all means be prevented. But for the fulfilment of this basic requirement, the exact knowledge of the factors bringing about critical state Δ of guiding-point A, as well as that of their effects is needed.

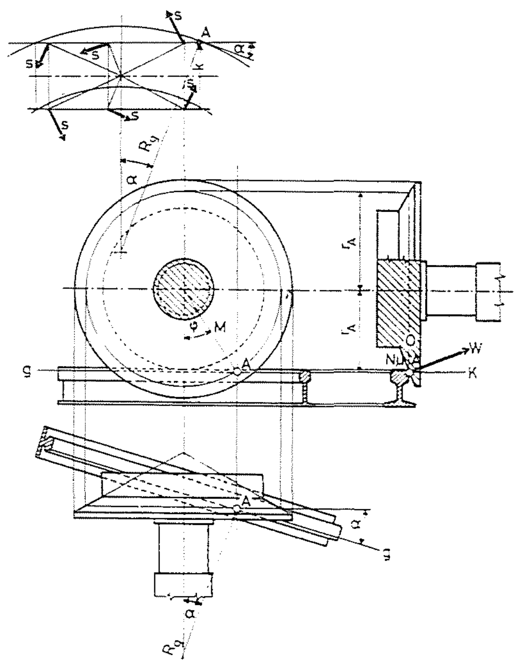

Fig. 1

Though it was established by respective investigations that force-vectors determining critical state Δ of guiding-point A include lateral force K and perpendicular wheel-force Q, the knowledge of K was insufficient for the dynamical characterization of guiding-point A. At the same time, the magnitude and the feature of the other factors remained unknown.

This problem was tried to be solved by a method considered as approximative. With this method applied, though the force-equilibrium similar to critical state Δ could be determined in case speed $V = 0$ and guiding-angle $\alpha \neq 0$, but no information could be provided about either the quality or quantity of the force effects producing equilibrium in critical state Δ in the case of motion along the curve at a speed V and with guiding-angle $\alpha \neq 0$. Neither could the force conditions of critical state Δ be determined in an experimental way.

As a summarization, it can be stated that due to the accepted theory of the railway vehicle moving along the track, statements have been made both of the geometrical relationships of the railway vehicle moving along the track, and the effect of the wheel-tread frictions made on the rising of lateral force K of the leading wheel, and finally, the basic importance of critical state of equilibrium Δ of guiding-point A has been stressed in ensuring the safety of the railway vehicle negotiating a curve.

In spite of its partial results, the running-gear theory applied widely was not able to develop a procedure solving the two fundamental problems despite repeated efforts. These two basic problems are:

1. What are the factors resulting in critical state Δ of the leading wheel?
2. What is the process-mechanism of the factors leading to the development of critical state Δ?

With these basic problems unsolved, the approximation theory of the railway vehicle negotiating a curve could be neither exact nor complete, and as a consequence, it could not direct practice expediently and effectively either. The main problem of the safety of railway transport has been left unsolved up to now.

Method of investigation

The aim of investigation is unchanged:

1. Determination of the factors resulting in critical state of equilibrium Δ of the leading wheel of railway vehicles negotiating a curve.

2. Determination of the processes used with these factors by which critical state Δ is developed.

The two main requirements of the new procedure are: completeness and exactness. The requirement of completeness demands that all the decisive factors should co-operate in the solution of a problem. While the requirement of exactness demands that the factors determining the problem should be taken into consideration not with their assumed or approximate values but with the exact ones. Critical state Δ of the railway vehicle does not allow any approximate or rough procedure because otherwise the unsafety of operation would not be diminished but increased.

The introduction of new concepts, denominations and designations became necessary because without them the exact description of the examined processes would not have been possible.

This task had to be started from "a".

The position of guiding-point A

Since the railway vehicle is deprived from the conditions of pure rolling when negotiating the curve, under the action of the frictional forces arisen on the contact areas of the wheel-tyre and the rail-head, the conical surface of the leading wheel of the railway vehicle negotiating a curve is pressed against the inner surface of the outside rail of the track in the curve at guiding-point A (Fig. 1).

In Fig. 2 the following are shown: if the leading wheel axis of the railway vehicle does not fit the direction of radius R_g of the guiding-circle g of the curve, then, as a consequence, guiding angle α is resulting at guiding-point A.

The straight section of the railway track is connected to its curved section by the transition curve of varying curvature. Before the transition curve would join the curve of constant radius, on the initial section of the former there already occur initial guiding-point A_0, guiding angle α_0 and meridian-plane M_0 owing to the developing curvature of the initial section of the transition curve. This guiding-point A_0 starts from its position in the meridian-plane, moves along the transition curve and after entering the curved section of the railway track appears as guiding-point A, located at the point of the constituent of guiding-cone m in meridian-plane M where guiding-circle g comes into contact with guiding hyperbola h (Fig. 2).

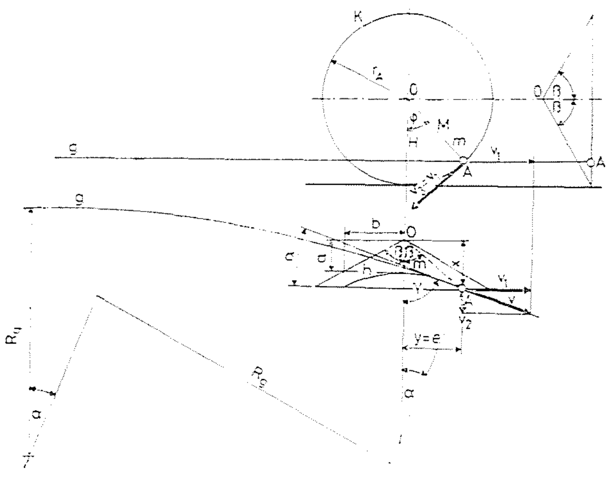

Fig. 2

Moving along the curve of constant radius R_g, guiding-point A is situated in meridian-plane M at an "anticipation"-distance e from meridian-plane H of the leading wheel guiding-cone (Fig. 2).

"Anticipation"-distance e is calculated from the equation of hyperbola h (Fig. 2):

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

after differentiation:

$$y' = \frac{b^2}{a^2} \cdot \frac{x}{y}$$

hence reckoning with

$$\frac{y}{x} = \frac{b^2}{a^2 y'} = \frac{\text{tg}^2 \beta}{y'} : \quad \frac{1}{\text{tg}\,\beta}$$

the following result is obtained:

$$\frac{y}{x \cdot \text{tg}\,\beta} = \frac{y}{r_A} = \frac{e}{r_A} = \sin\varphi = \frac{\text{tg}\,\beta}{y'} = \frac{\text{tg}\,\beta}{\text{tg}\,\gamma} = \text{tg}\,\beta\,\text{tg}\,\alpha$$

Thus the functions of interest of "anticipation" angle are:

$$\sin\varphi = tg\,\beta\,tg\,\alpha \tag{1}$$

$$\cos\varphi = \sqrt{1 - \text{tg}^2\beta\,\text{tg}^2\alpha} \tag{2}$$

$$e = r_A \sin\varphi = r_A\,tg\,\beta\,tg\,\alpha \tag{3}$$

The speed-conditions of guiding-point A

Lateral force K arising owing to tread-friction S of the railway vehicle wheels negotiating the curve presses the guiding-cone of the leading wheel against the inner surface of the outside rail of the curve at its guiding point A.

The railway vehicle moves along the curve at a speed V. The speed of guiding point A is V, so at guiding point A frictional force S arises, the direction of which is given by resulting speed V_ε. Now, the speed conditions of guiding point A should be determined.

Figure 3 shows the resolution of the speeds at guiding point A into its components, as well as the composition of the latter with resultant sliding speed V_ε.

Speed-component $\cos\alpha\,(1-\cos\varphi)$ is perpendicular to meridian-plan M. On the guiding-cone projection at the top right of Fig. 3, resultant V_a of speed-components $\cos\alpha\cdot\sin\varphi$ and $\sin\alpha$ lies within cone constituent OA, i.e. in meridian-plane M because formula 1 taken into consideration:

$$\text{tg}\,\vartheta = \frac{\cos\alpha\,\sin\varphi}{\sin\alpha} = \frac{1}{\text{tg}\,\alpha}\,\text{tg}\,\alpha\,\text{tg}\,\beta = \text{tg}\,\beta$$

The aim of further examinations is to find out whether resultant speed V_ε of guiding point A lies within tangent-plane E fitting cone-constituent OA.

To solve this problem three points lying within tangent-plane E should first be determined (Fig. 4). One of these is apex-point O of the guiding cone, the other one is guiding point A and the third is point B.

The coordinates of guiding-cone apex-point O are:

$$x_1 = 0, y_1 = 0, z_1 = 0$$

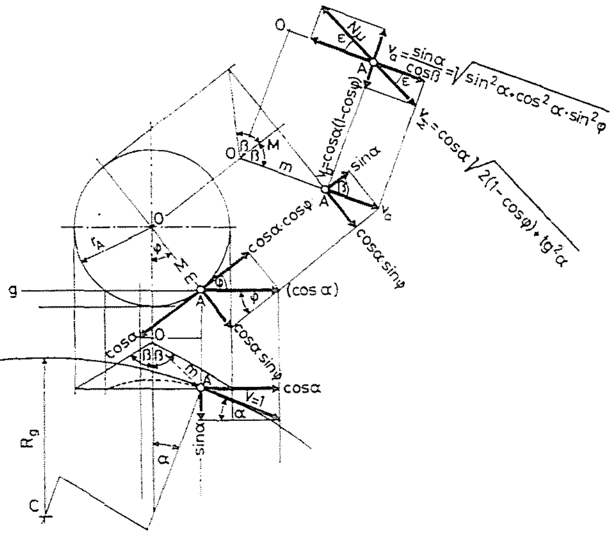

Fig. 3

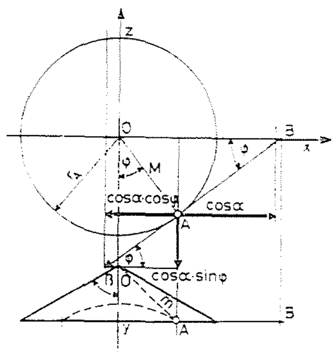

Fig. 4

The coordinates of guiding point A are:

$$x_2 = r \cdot \sin \varphi, \; y_2 = \frac{r}{\operatorname{tg} \beta}, \quad z_2 = r \cdot \cos \varphi$$

The coordinates of point B are:

$$x_3 = \frac{r}{\sin \varphi}, \; y_3 = \frac{r}{\operatorname{tg} \beta}, \; z_3 = 0$$

The coordinates of the end-point of resultant speed V_z are:

$$x = r \cdot \sin \varphi + \cos \alpha \,(1 - \cos \varphi)$$

$$y = \frac{r}{\operatorname{tg} \beta} + \sin \alpha$$

$$z = -r \cdot \cos \varphi - \cos \alpha \sin \varphi$$

Speed V_ε of guiding point A lies within tangent plane E fitting cone-constituent OA in the case if coordinates x, y, z of resultant speed V_ε satisfy the equation of the plane determined by points O, A and B, i.e. if coordinates x, y, z of resultant speed V_ε satisfy the condition of the vanishing of the following determinant:

$$D = \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$ 0, the proper values substituted, the following condition results

$$\begin{vmatrix} r \sin \varphi + \cos \alpha (1 - \cos \varphi) & \frac{r}{\operatorname{tg} \beta} + r \cdot \sin \alpha & -(r \cos \varphi + \cos \alpha \sin \varphi) \\ r \cdot \sin \varphi & \frac{r}{\operatorname{tg} \beta} & -r \cos \varphi \\ \frac{r}{\sin \varphi} & \frac{r}{\operatorname{tg} \beta} & 0 \end{vmatrix} = 0$$

With the calculations performed, it is obvious that condition $D = O$ is really satisfied, so resultant speed V_ε lies within tangent-plane E fitting cone-constituent OA (Fig. 4).

According Fig. 3:

the speed-component lying within guiding-cone constituent OA in meridian-plane M is:

$$v_a = \sqrt{\sin^2 \alpha + \cos^2 \alpha \sin^2 \varphi} = \frac{\sin \alpha}{\cos \beta} \tag{4}$$

the speed-component perpendicular to guiding-cone constituent OA and meridian-plane M is:

$$v_b = \cos \alpha (1 - \cos \varphi) \tag{5}$$

the resultant speed is:

$$v_\varepsilon = \sqrt{v_a^2 + v_b^2} = \sqrt{\cos^2 \alpha (1 - \cos \varphi)^2 + \sin^2 \alpha + \cos^2 \alpha \sin^2 \varphi}$$

$$v_\varepsilon = \cos \alpha \sqrt{(1 - \cos \varphi)^2 + tg^2 \alpha + \sin^2 \varphi} =$$

$$= \cos \alpha \sqrt{1 - 2 \cos \varphi + \cos^2 \varphi + tg^2 \alpha + 1 - \cos^2 \varphi}$$

$$v_\varepsilon = \cos \alpha \sqrt{2(1 - \cos \varphi) + \operatorname{tg}^2 \alpha} \tag{7}$$

The trigonometrical functions of angle ε are:

$$\sin\varepsilon = \frac{v_b}{v_\varepsilon} = \frac{\cos\alpha(1-\cos\varphi)}{\cos\alpha\sqrt{2(1-\cos\varphi)+\mathrm{tg}^2\alpha}} = \frac{1-\cos\varphi}{\sqrt{2(1-\cos\varphi)+\mathrm{tg}^2\alpha}}$$

$$\cos\varepsilon = \frac{v_a}{v_\varepsilon} = \frac{\sqrt{\sin^2\alpha+\cos^2\alpha\sin^2\varphi}}{\cos\alpha\sqrt{2(1-\cos\varphi)+\mathrm{tg}^2\alpha}} =$$

$$= \frac{\cos\alpha\sqrt{\sin^2\varphi+\mathrm{tg}^2\alpha}}{\cos\alpha\sqrt{2(1-\cos\varphi)+\mathrm{tg}^2\alpha}} = \sqrt{\frac{\sin^2\varphi+tg^2\alpha}{2(1-\cos\varphi)+\mathrm{tg}^2\alpha}} \tag{9}$$

$$\mathrm{tg}\,\varepsilon = \frac{v_b}{v_a} = \frac{\sin\varepsilon}{\cos\varepsilon} = \frac{1-\cos\varphi}{\sqrt{\sin^2\varphi+\mathrm{tg}^2\alpha}} \tag{10}$$

The force-conditions of guiding point A

Figure 4 illustrates the force-system applying on guiding point A of the leading wheel of the railway vehicle in the critical state of equilibrium Δ of the leading wheel, when the leading wheel tyre comes into contact with the running surface of the rail without loading it because weight Q of the leading wheel is acting only upon guiding point A.

The entire force-system can be divided into two groups:

1. Force $D_\Delta = Q_\Delta \cdot \sin\varphi_\Delta - N_\Delta \cdot \sin\varepsilon_\Delta$ perpendicular to meridian-plane M acts only on the axle-box, the vector of torque: $N_\Delta \cdot \sin\varepsilon_\Delta r_\Delta$ lies within the axis of the driven wheel-set.

2. Balanced force-system $Q_\Delta \cos\varphi_\Delta + K_\Delta + N_\Delta + S_\Delta = 0$ lies within meridian-plane M because: wheel-force component $Q_\Delta \cdot \cos\varphi_\Delta$ lies within meridian-plane M,

lateral force K_Δ is parallel to the wheel-axis and lies within meridian-plane M, railhead reaction-force N_Δ is perpendicular to plane E fitting upon meridian-constituent m, and $S_\Delta = N_\Delta \cdot \mu\cos\varepsilon_\Delta$

Critical factor of safety: b_Δ

According to the equilibrium-diagram in Fig. 4:

$$\frac{K_\Delta}{Q_\Delta \cdot \cos\varphi_\Delta} = \mathrm{tg}[\beta - \varrho_\Delta],\ \text{hence:}$$

$$b_\Delta = \frac{Q_\Delta}{K_\Delta} = \left[\cos\varphi_\Delta \mathrm{tg}\,[\beta-\varrho_\Delta]\right]^{-1}\ \text{where according to Fig. 4:}$$

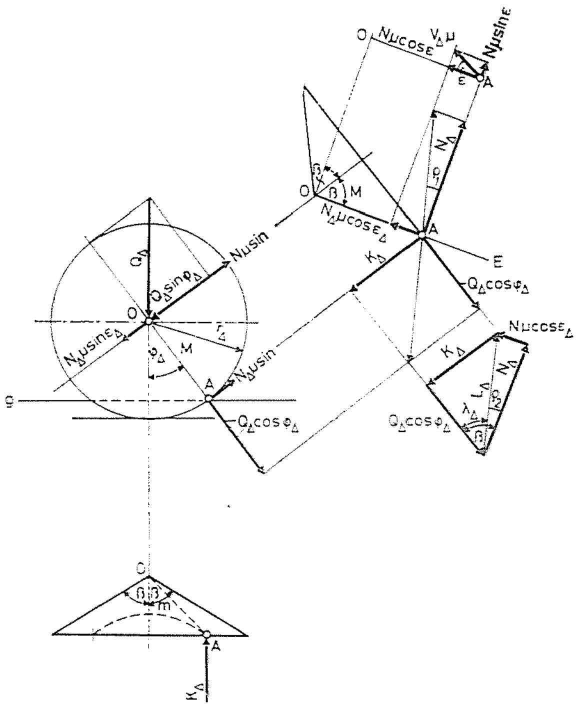

Fig. 5

$$\varrho_\Delta = \text{arc. tg}(\mu \cdot \cos \varepsilon_\Delta), \text{ whereby:}$$

$$b_\Delta = \frac{Q_\Delta}{K_\Delta} = [\cos \varphi_\Delta \text{tg}[\beta - \text{arc. tg } (\mu \cdot \cos\varepsilon_\Delta)]]^{-1} \quad (11)$$

α_Δ = guiding-angle

β = guiding-cone apex angle

μ = friction-coefficient

$$\cos \varphi_\Delta = \sqrt{1 - \text{tg}^2 \alpha_\Delta \cdot \text{tg}^2 \beta}$$

$$\cos \varepsilon_\Delta = \sqrt{\frac{\sin^2\varphi_\Delta + \text{tg}^2 \alpha}{2(1-\cos\varphi_\Delta)\,\text{tg}^2\alpha_\Delta}}$$

If $\beta = 60°$ and $\mu = 0.27$, then:

α_Δ	$\delta_\Delta = \frac{Q_\Delta}{K_\Delta}$
0	
1	1.001
2	1.002
3	1.004
4	1.007
5	1.012
10	1.048

Effective factor of safety

If gravity-force Q_e and guiding-force K_e is applied on the leading wheel of the railway vehicle, the effective safety factor of the leading wheel is:

$$b_e = \frac{Q_e}{K_e}$$

If $Q_{\acute{e}} > Q_e$ or $K_{\acute{e}} < K_e$, the safety of the railway vehicle increases with the simultaneous increase in safety factor: $b_{\acute{e}} > b_e$.

If $Q_{\acute{e}} < Q_e$ or $K_{\acute{e}} > K_e$, the safety of the railway vehicle decreases with the simultaneous decrease in safety factor: $b_{\acute{e}} < b_e$.

From the point of view of the results achieved in this paper, the following can be emphasized:

Critical safety function: $\left[b_\Delta = \frac{Q_\Delta}{K_\Delta}\right]$ is complete and exact.

The resolution of critical safety function: $\left[b_\Delta = \frac{Q_\Delta}{K_\Delta}\right] = F(\alpha, \beta, \mu)$ into the partial functions enables their effect to be examined separately by means of the variation of the factors determining function: b_Δ, thus e.g. that of guiding angle: α or guiding cone: β. Since there are a great number of functional-relationships to be derived in this way through the variation in the values of the factors, for the sake of detecting these functional relationships and their technical applicability, it seems inevitable to adopt new, effective procedures on a wide scope.

Examples for the application of the safety factors

The determination of the critical and effective safety factors of the railway vehicle negotiating a curve

If given:

Radius of curve R_g,

the wheel arrangement of the running-gear of the vehicle, vertical force on the leading wheel: Q_1 and if guiding-angle: α_Δ and lateral force $K = K_\Delta$ belonging to it are determined by calculations or construction, then with the help of Q_Δ, from formula (11) the

critical safety factor: $\left[b_\Delta = \dfrac{Q_\Delta}{K_\Delta}\right]$ can be determined.

If $Q_1 > Q_\Delta$, then the effective safety factor of the vehicle:

$$b_e = \frac{Q_1}{Q_\Delta} > \frac{Q_\Delta}{K_\Delta}$$

ensures avoiding derailment;

If $Q_1 < Q_\Delta$, the effective safety factor of the vehicle is:

$$b_e = \frac{Q_1}{Q_\Delta} < \frac{Q_\Delta}{K_\Delta}$$

consequently, the leading wheel derails.

Locomotive design

One of the basic tasks of the locomotive-design is the determination and possible increase of safety of the railway locomotive negotiating a curve. Let be given:

Radius of curve R_g,

Wheel-arrangement of the running-gear of the locomotive,

vertical leading wheel force Q_1.

If guiding angle: α and lateral force $K = K_\Delta$ belonging to it are determined by calculations or graphical methods, and Q_Δ has been determined from formula (11) and $Q_1 > Q_\Delta$ is valid, then:

$$Q_0 = Q_1 - Q_\Delta$$

is the excessive force equalizing the centrifugal force arising in the curve and the lateral force originating from the wind pressure.

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