NETWORK SCHEDULING LIMITED BY SPECIAL CONSTRAINT AS A FUNCTION OF TIME COST

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Received September 5, 1986

Abstract

The paper deals with the application of mathematical programming techniques to network analysis and network management in general. The authors show the application of linear programming to network scheduling when the problem is to minimize the total crashing cost and the required completion time is given. Finally the application of goal programming is presented in network scheduling in tha case, when all kinds of constraints (time, resource, etc.) are introduced into the problem and the manager has to deal with multiple objectives which may be in conflict. All kinds of network scheduling problems mentioned above are illustrated through a detailed numerical example.

The backbone of the complex production or construction processes with many components (e.g. transport construction investments) is the well-known, time oriented, normative activity network plan (in simpler case only Gantt chart). In order to prepare such a plan whole process of work is to be broken down into activities e.g. on the basis of technical plan documentation as well as of the budget included or partial production processes, resp., that their every important demand, utilization level of resources and the adequate duration could be estimated in the usual way possibly by taking organization standards of the enterprise (eventually workplace) into account. A network representing the whole of production and construction processes can be developed by recognizing the relations and dependencies of the appropriately detailed and identified activities originating from technology, cooperation and other procedural requirements, e.g. legal and financial regulations. (This is called the logical design of work process network.) In a simple case, this flow diagram is only one chain of partial processes between the starting and end points of the whole process.

The guarantee for keeping to the requested time is a certain intensity, amount and utilization level of resources present necessary to carry out the activity in the scheduled time.

By accepting the expertise and possibilities — based on enterprise norms normatives — the necessary time (and the normative, fixed costs belonging to

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it) of the activities in accordance with the composition and utilization level of the resources could be given. Thus the normative network plan contains the types and utilization levels of resources and the time required as basic data to carry out the activities. If the necessary resources can be provided in due time and place according to the plan, the whole work can be performed in minimum length of time according to the normative network schedule.

Scheduling based on the computation of the critical path — starting from the calendar point of time of the scheduled beginning (or expected completion) of the work, taking the working time, shifts and holidays into account provides the date when the activities of a given duration to be carried out for the construction of the building (without interruption) can be and in order to keep the deadline of the objective event must be started and ended. The possible earliest and the allowed latest times of the important events (milestones, but in networks of activity edge events represented by every nodes) also become known.

It is obvious that in the case of organization and management of a big project special attention has to be paid to the activities along the critical path (as they have no reserve time). After reaching certain milestone events and also periodically from time to time (e.g. quarterly) scheduling is expedient to be repeated by updated data that may yield a new critical path as a result of different circumstances.

The task of scheduling based on the computation of critical path could be transformed into the model of linear programming. In this case in requirements (i.e. in the constraints of the optimization task) is given the activities must be carried out until the end of the whole work process and that the precedence, and sequence relations must be kept. The only evaluation factor of this task is the longest (that is the critical) path between the beginning and the end giving simultaneously also the possible earliest completion deadline of the whole process. The network technique, however, does not perform scheduling in this way, but in the network itself, according to the Bellman's principle of optimality in this case by a simple dynamic programming algorithm of multi stage, recurrent optimization.

When, however, the deadline of the end of the program is set to an earlier (or later) time than according to the normative network plan, but the cost of the activity can be provided in the given interval — as a linear function of time, the minimum of the excess costs due to the acceleration of the program will be prefered. The problem what final deadline belongs to a given cost framework can also be examined. In this case, recurrent optimization does not help, the model of linear optimization is to be relied on instead. It is, however, necessary that the relation of time and cost for each activity could be described by linear functions between the possible shortest and the cheapest durations. (The cheapest duration is also called "normal time" that does not necessarily coincide with the duration according to norm.) This task was solved by *J. E. Kelley* with the CPM algorithm published in 1959. As, however, the computation of this task is very time-consuming because of the great number of conditions and variables and as the linear time dependence of the costs of activities is usually artifical, the practical application of the original CPM algorithm was not widespread [1, [2], [3].

The appearance of microcomputers, the ease of interactive operation — at least in case of smaller tasks — resulted in an entirely new situation. For the project manager a many-sided, quick analysis has become available.

The task, when a compromise is to be found between the deadline and the total cost in the knowledge of the specific cost of the acceleration of certain activities, is different. It differs from the former ones as it contains several (namely two) and not one requirements (evaluation factors) of different dimensions different measuring scales belonging to these factors (here time and cost). In this (multi-dimensional) evaluating space, the different possible solutions may be compared with the "smaller-greater" pair of concepts only exceptionally. Thus no optimal solution can be spoken of here, but we have to make a choice. Thus we have to be satisfied with a good compromise and the best solution belonging to it. A compromise can be reached that the most favourable solution should only to the minimal extent differ from the solution that can be realized quickest with the lowest cost in that.

First of all a new relation is to be introduced in the expression of the requirement (denoted by $\approx$); the required performance is to be approximated as closely as possible. This weak equality can be put into the model of linear optimization if it is transformed into real equality by completing it with the variables of short and long deviation, thus the existing constraint system is expanded and the objective function which is to be minimized contains the sum of the absolute and commensurate values of the deviations. But how can this deviations of different dimensions and perhaps of not identical importance be added? Only two of the several possibilities are mentioned.

The deviations can be transformed into "indicators of satisfaction" having no dimension. Let T_n denote the shortest deadline according to scheduling. Thus it is the date of the earliest completion of the whole work (the date of the earliest possible occurrence of the finishing event is marked n). If it is fulfilled (with zero error), our satisfaction in meeting the requirement is 100%. In interactive operations, even in case of multi-criteria evaluation, we can be quickly informed on the level of the fulfillment of the requirements if single requirements are fulfilled optimally. On the basis of this knowledge we can state the deadline that (because of objective reasons or as a compromise) is regarded as unacceptable (let us denote it by T_n°, as our satisfaction is zero). Thus the indicator of satisfaction belonging to keeping the deadline in the interval $S \in [0, 100]$ as a function of deviation s_T is the following:

$$S_T = 100 - p_T s_T ,$$

where $p_T = \dfrac{100}{T_n^{100} - T_n^{\circ}}$ in evaluation can be interpreted as the weight, importance of time deviation with respect to the shortest deadline. Similarly, the indicator of satisfaction of cost is $S_K = 100 - p_K s_K$, where

$$p_K = \frac{100}{K^{100} - K^{\circ}}$$

is in the evaluation the weight of the increase of cost. The objective function is the sum of the indicators of satisfaction that has to be maximized. Identical solution is obtained if the sum of the weighed deviations is minimized after leaving out the constant numbers:

$$p_T s_T + p_K s_K \rightarrow \min .$$

It is apparent that these weight numbers were obtained with due consideration. The value of dissatisfactory fulfilment (T_n°, K°) was determined on the basis of adequate knowledge, thus it can be regarded as a good compromise.

Another compromise is the declaration of priorities among requirements. Taking the previous example, let $p_T \gg p_K$ which means only that in scheduling — in an actual case — the deadline is dominant against cost. Symbol P does not express a weight, but the declaration of priority as a quality index.

In this case all the requirements of a given fulfilment value completed with error variables are put into the condition system of linear optimization and the objective functions providing the deviations are arranged according to their priorities. The simplex algorithm is completed in the following way. The optimal solution according to objective function 1 is determined in the priority order. The simplex table shows the representation of every objective function with respect to the basis, i.e. their modifying factors. They show how the deviation differs from the required fulfilment corresponding the priority giving unit value to the non basic variable. The algorithm is continued with the search of the solution improving the value of the objective function coming next in the priority order if it does not endanger the fulfilment with minimum deviation of the requirement of greater priority. This condition becomes obvious from the modifying factors referring to the next and preceeding objective functions in the simplex table. This methodological possibility known but not used for a long time combined with the new possibilities of computer technique, the interactive computer program of the modified simplex algorithm — called goal programming in special literature — offers the possibility for the system analyzer to approximate the problem quickly from different aspects by changing the priority order. If the dynamic network scheduling of the work processes can be also completed with this possibility of analysis — as it is shown in the following example — and in this way it is fitted better into the operative control system of the process, it will be a great step forward. To do this, the methodological improvement should not forget — certainly among others — about the fact that the linear cost development of activities and reality are usually difficult to make conform. This is mainly why CPM in its original sense was not widespread. This cost development (and the duration change belonging to it) are most of the time periodical and progressively increasing. To handle these cost functions of the activities requires more complicated and much longer procedures than the simplex algorithm based on proportionality.

Numerical example for network scheduling subjected to different conditions and evaluation factors[1]

The task

Nine activities are to be performed to modernize a traffic junction. Table 1 summarizes the sequence relations between the coded (i, j) activities, the duration necessary to normal and crash-time performance (d_{ij}, D_{ij}), costs (K_{ij}, k_{ij}), the incremental crashing cost calculated from them (c_{ij}) that is data expressing extra cost necessary to unit time shortening

$$c_{ij} = \frac{k_{ij} - K_{ij}}{d_{ij} - D_{ij}}$$

Table 1

Symbol of the activity	Code of the activity (i,j)	Symbol of the activity directly following the former activity	The activity				Cost index (10^3 Ft/week) c_{ij}
			normal	crash	normal	crash	
			performance				
			time period (week)		cost (10^3 Ft)		
			d_{ij}	D_{ij}	K_{ij}	k_{ij}	
A	1—2	D, E, F	11	8	15	39	8
B	1—3	G, H	9	6	10	31	7
C	1—4	I	35	28	45	108	9
D	2—3	G, H	13	1	40	76	3
E	2—4	I	22	17	30	60	6
F	2—5	—	40	25	100	130	2
G	3—4	I	16	11	60	80	4
H	3—5	—	22	18	500	70	5
I	4—5	—	10	7	20	50	10

[1] The numerical example was introduced by Professor Berczi, A. at the Production Management Conference of IFIP held in Budapest in 1985. The method — based on this numerical example — is shown in the framework of the application of transport construction.

By scheduling based on the determination of critical path it can be established that the project, under normal conditions, would take 51 weeks to accomplish and would cost 370×10^3 Ft. The activities under crash conditions, i.e. when all activities are performed by their respective maxima, then the reconstruction would take to accomplish only 35 weeks but it would cost 644×10^3 Ft. In the former case the critical path would be activities A—F (respectively, events 1—2—5), in the latter case activities *C—I* (respectively, events 1—4—5) (Figs 1 and 2).

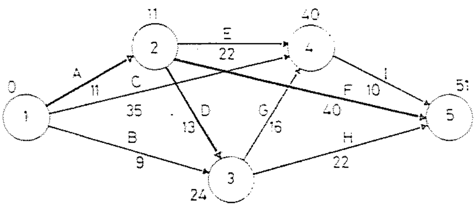

Fig. 1

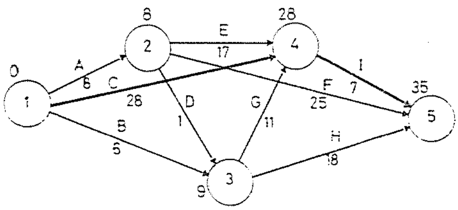

Fig. 2

Every information about the fact how the characteristics of the program under conditions differing from the earlier fixed ones develop, is useful for the manager responsible for the performance of the task. For this purpose, linear programming or in case of taking several requirements into consideration, goal programming can be used efficiently.

The required completion time of the network plan by minimizing the total excess cost

The task described above — by defining suitably chosen decision variables — can be modelled mathematically and solved quickly even by a personal computer.

Let us suppose that the required completion time of the reconstruction of the traffic junction is 40 weeks and the execution of the individual activities is scheduled to minimize the total excess cost compared to the cost of normal execution. This task can be described by the following mathematical model:

$$\min Q = \sum_{i,j} c_{ij} y_{ij}$$

by using the following constraint equations:

$$\left.\begin{array}{l} (x_n - x_1) \leq T_n \\ y_{ij} \leq (d_{ij} - D_{ij}) \text{ and } y_{ij} \geq 0 \\ (x_j - x_i) + y_{ij} \geq d_{ij} \end{array}\right\} \forall_{i,j}$$

where

Q — the total excess cost

c_{ij} — the excess cost needed for unit time-shortening of activity ij

y_{ij} — acceleration time for activity ij (i.e. time-shortening with respect to normal performance)

x_j — earliest occurrence time of the j-th event ($j = 1, \ldots N$; N is the number of the events of the network),

T_n — deadline for the completion of the program,

d_{ij} — normal time of the performance of activity ij,

D_{ij} — minimum time of the performance of activity ij.

On substituting the parameters of the case study into the general model, the following 28 constraint equations can be written for the 14 decision variables of the network plan consisting of nine activities and five events:

$$y_{12} \leq 3;\ y_{13} \leq 3;\ y_{14} \leq 7;\ y_{23} \leq 12;\ y_{24} \leq 5;\ y_{25} \leq 15;\ y_{34} \leq 5;$$

$$y_{35} \leq 4;\ y_{45} \leq 3;$$

$$y_{12},\ y_{13},\ y_{14},\ y_{23},\ y_{24},\ y_{25},\ y_{34},\ y_{35},\ y_{45} \geq 0;$$

$$x_1 = 0,\ x_j \geq 0,\ j = 2, \ldots, 5$$

$$x_2 - x_1 + y_{12} \geq 11;\ x_3 - x_1 + y_{13} \geq 9;\ x_4 - x_1 + y_{14} \geq 35;$$

$$x_3 - x_2 + y_{23} \geq 13;$$

$$x_4 - x_2 + y_{24} \geq 22;\ x_5 - x_2 + y_{25} \geq 40;\ x_4 - x_3 + y_{34} \geq 16;$$

$$x_5 - x_3 + y_{35} \geq 22;\ x_5 - x_4 + y_{45} \geq 10;\ x_5 - x_1 \geq 40.$$

The objective function to be minimized is:

$$\min Q = 8y_{12} + 7y_{13} + 9y_{14} + 3y_{23} + 6y_{24} + 2y_{25} + 4y_{34} + 5y_{35} + 10y_{45}.$$

The optimal solution of the task is:

$$x_1 = 0;\ x_2 = 11;\ x_3 = 17;\ x_4 = 33;\ x_5 = 40$$

$$y_{12} = 0;\ y_{13} = 0;\ y_{14} = 2;\ y_{23} = 7;\ y_{24} = 0;\ y_{25} = 11;\ y_{34} = 0;$$

$$y_{35} = 0;\ y_{45} = 3$$

$$Q = 91.$$

Considering that x_j is the variable of the earliest occurrence time of the events and y_{ij} is the variable of the amount of excess cost of the activities, the network plan for the optimum solution under the given conditions can be drawn on the basis of the results obtained (Fig. 3).

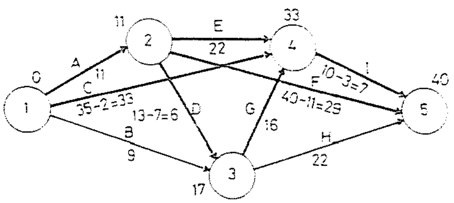

Fig. 3

The slack variables (s_{ij}) belonging to the above optimum solution mean the reserve time of the activities:

$$s_{12} = 0;\ s_{13} = 8;\ s_{14} = 0;\ s_{23} = 0;\ s_{24} = 0;\ s_{25} = 0;\ s_{34} = 0;\ s_{35} = 1;\ s_{45} = 0.$$

The results obtained (and the figure) show that except for activities *B* and *H* (the former one has 8, the latter 1 week reserve-time) all activities are on the critical path. The cost necessary to ensure the completion time of 40 weeks is 91×10^3 Ft, i.e. the total cost of the modernization of the traffic junction is increased from 370×10^3 Ft to 461×10^3 Ft if the completion time taking 51 weeks under normal conditions, is accelerated to 40 weeks.

Incorporating individual activity or event constraints

By means of modelling described above in detail it is possible to fulfil further constraints referring to an arbitrary number of events or activities and to examine the "price" of the fulfilment of the condition. If the condition that the subcontractor responsible for activities *E* and *I* is to be employed at least for 30 weeks is stipulated for the manager of the transport establishment in our case study, and event 3 meaning the partial use of the traffic junction is to be completed by the 12th week, then the *LP* model is to be completed with the following constraint equations;

$$(x_3 - x_1) \leq 12$$

and from the condition $d_{24} - y_{24} + d_{45} - y_{45} \geq 30$ — as $d_{24} = 22$ and $d_{45} = 10$ —

$$\text{so } y_{24} + y_{45} \leq 2.$$

The optimum solution of the LP task completed with two newer restraint equations:

$$x_1 = 0;\ x_2 = 10;\ x_3 = 12;\ x_4 = 32;\ x_5 = 40;$$

$$y_{12} = 1;\ y_{13} = 0;\ y_{14} = 3;\ y_{23} = 11;\ y_{24} = 0;\ y_{25} = 10;\ y_{34} = 0;$$

$$y_{35} = 0;\ y_{45} = 2 \text{ and}$$

$$Q = 108.$$

The slack variables (i.e. the reserve times of the activities) belonging to the optimum solution are:

$$s_{12} = 0;\ s_{13} = 3;\ s_{14} = 0;\ s_{23} = 0;\ s_{24} = 0;\ s_{25} = 0;\ s_{34} = 4;\ s_{35} = 6;\ s_{45} = 0.$$

The network plan belonging to this solution: activity *G* has 4 weeks reserve time, activity *B* has 3 weeks, activity *H* has 6 weeks reserve time, all the other activities are critical (Fig. 4).

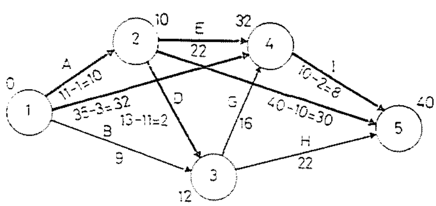

Fig. 4

Comparing this result with the results of the solution obtained by the previous condition system we can state that the excess cost of the fulfilment of the two conditions stipulated subsequently is 17×10^3 Ft.

In the possession of the results obtained by linear programming the persons responsible for the decision may consider how much the constraints "cost". It can be controlled quickly whether a feasible solution exists at all in case of the given constraint system and if it does exist what goal value belongs to it, by a data input corresponding to arbitrary-direction and change of the parameters characterizing the task and by running the program again.

Network scheduling with goal programming

If the scheduling of the modernization of the traffic junction introduced in the case study is to be carried out by considering several evaluation factors and our aim is a best approximation of the required values, instead of the strict constraints the mathematical model of decision making can be described in the following form:

$$\min Q = \sum_{k=1}^{r} (p_{k1}s_{k1} + p_{k2}s_{k2})$$

Under the following constraints

— goal equations are:

$$\sum_{j=1}^{n} a_{kj}x_j - s_{k1} + s_{k2} = a_{0k} \qquad \forall k$$

— conditions for the acceleration of activities are:

$$\left.\begin{aligned} x_j - x_i + y_{ij} &\geq d_{ij} \\ y_{ij} &\leq d_{ij} - D_{ij} \end{aligned}\right\} \quad \forall ij$$

and $x, y \geq 0$

where

$k = 1, \ldots, r$ — index of requirements (goals) taken into account in course of decision;

i and $j = 1, \ldots, n$ — index of the events;

a_{0k} — the quantitative measure of the k-th goal;

x_j — decision variable: the possible earliest completion time of the j-th event;

a_{kj} — coefficient expressing the relation between the k-th goal and j-th event;

d_{ij} — normal (lowest cost) time of the accomplishment of activity (i, j);

D_{ij} — accomplishment time of activity (i, j) under crash conditions;

y_{ij} — accomplishment time decrease of activity (i, j);

$s_{k1} = d_k^+$ — extent of overachievement, deviance from the k-th requested value;

$s_{k2} = d_k^-$ — extent of underachievement, deviance from the k-th requested value;

p_{k1} — evaluation weighted number attached to overachievement of the k-th goal;

p_{k2} — evaluation weighted number attached to underachievement of the k-th goal.

Let us see what solutions are obtained — by changing priorities among the requirements — if the construction of the traffic junction is to be scheduled by the possible "closest" fulfilment of the four requirements.

In course of scheduling, the possible best approximation of the following requested values is to be aimed at:

1. The construction should be completed by the 40th week;
2. Event 3 should be completed by the 12th week;
3. Accomplishment of activities E and I should take 30 weeks;
4. The cost of the accomplishment of the program should be the lowest possible. Substituting the parameters of our case study into the general model, 31 constraint conditions can be postulated.

$$y_{12} + d_1^- = 3;\ y_{13} + d_2^- = 3;\ y_{14} + d_3^- = 7;\ y_{23} + d_4^- = 12;$$
$$y_{24} + d_5^- = 5;\ y_{25} + d_6^- = 15;$$
$$y_{34} + d_7^- = 5;\ y_{35} + d_8^- = 4;\ y_{45} + d_9^- = 3;$$
$$y_{12} - d_{10}^+ = 0;\ y_{13} - d_{11}^+ = 0;\ y_{14} - d_{12}^+ = 0;$$
$$y_{23} - d_{13}^+ = 0;\ y_{24} - d_{14}^+ = 0;\ y_{25} - d_{15}^+ = 0;$$
$$y_{34} - d_{16}^+ = 0;\ y_{35} - d_{17}^+ = 0;\ y_{45} - d_{18}^+ = 0;$$
$$(x_2 - x_1) + y_{12} - d_{19}^+ - 11;\ (x_3 - x_1) + y_{13} - d_{20}^+ = 9;$$
$$(x_4 - x_1) + y_{14} - d_{21}^+ = 35;$$
$$(x_3 - x_2) + y_{23} - d_{22}^+ = 13;\ (x_4 - x_2) + y_{24} - d_{23}^+ = 22;$$
$$(x_5 - x_2) + y_{25} - d_{24}^+ = 40;$$
$$(x_4 - x_3) + y_{34} - d_{25}^+ = 16;\ (x_5 - x_3) + y_{35} - d_{26}^+ = 22;$$
$$(x_5 - x_4) + y_{45} - d_{27}^+ = 10;$$
$$(x_5 - x_1) + d_{28}^- - d_{28}^+ = 40;\ (x_3 - x_1) + d_{29}^- - d_{29}^+ = 12;$$
$$(y_{24} + y_{45}) + d_{30}^- - d_{30}^+ = 2;$$
$$8y_{12} + 7y_{13} + 9y_{14} + 3y_{23} + 6y_{24} + 2y_{25} + 4y_{34} + 5y_{35} + 10y_{45} +$$
$$+ d_{31}^- - d_{31}^+ = 0$$

The goal function to be minimized is:

$$\min Q = p_1(d_{28}^- - d_{28}^+) + p_2(d_{29}^- - d_{29}^+) + p_3(d_{30}^- - d_{30}^+) + p_4(d_{31}^- - d_{31}^+)$$

Table 2 contains the four preference sets ($p_1 \gg p_2 \gg p_3 \gg p_4$) chosen for the solution of the task by goal programming.

Table 2

Requirement (goal)	A	B	C	D
	cases			
1	p_1	p_3	p_2	p_1
2	p_2	p_2	p_1	p_3
3	p_3	p_1	p_3	p_4
4	p_4	p_4	p_4	p_2

Table 3 summarizes the results obtained by taking the four types of priority into account.

Table 3

Requirement (goal)	Dimension	A	B	C	D
		cases			
1	week	40	40	40	40
2	week	17	12	12	17
3	week	29	30	30	29
4	10^3 Ft	461	478	478	461

As Table 3 shows, in spite of the different priorities stated for the acomplishment of the requirements identical results were obtained in cases *A* and *D*, *B* and *C*. The two different results coincide with the schedules in Figs 3 and 4.

It is seen that the solution obtained in cases *A* and *D* is the less expensive; it fulfils the 40 weeks accomplishment but there is an "under-achievement" in the requirement concerning the employment of the subcontractor for a definite time at *E* and *I* activities, and the realization of event 3 within the prescribed deadline. The former one differs by one week (ensuring only 29 week employment instead of 30), the latter by five weeks (the accomplishment of the event is ensured only by the 17th week instead of the 12th). This solution is cheaper by 17×10^3 Ft (108—91) than the earlier one-criterion solution minimizing the excess cost. The solution obtained in cases *B* and *C* is more expensive but here all the other requirements are fulfiled entirely, i.e. the total construction is completed by the 40th week, activity 3 is completed by the 12th week, it takes 30 weeks to carry out activities *E* and *I*.

It has been shown how goal programming can be used for analysis by several criteria and evaluation of scheduling tasks expressed in a network plan. It is clear that excess labour with data describing decision situations that take different conditions into account, has a great advantage, the possibility of examining the effects of considering further goals, requests or conditions by running the program several times.

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