DYNAMIC PROCESSES CAUSED BY TRACK UNEVENNESSES IN BRAKED RAILWAY VEHICLES

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Abstract

Unevennesses always present in railway tracks give rise to vibrations in the vehicles moving along the rails. In vehicles equipped with direct friction-brakes, vertical exciting effects act upon the sprung vehicle parts through the brake suspension system with the intervention of friction-forces acting upon the wheel-sets when the brake gear is in action. In this paper, the formation of a dynamic model is described which is suitable for the examination of exciting effects transmitted through the brake suspension system, and the description of the system is given as required for digital simulation taking into consideration a two-axle vehicle equipped with block-brakes. The results obtained can be applied directly to the case of bogie vehicles.

1. Introduction

Unevennesses always present in railway tracks give rise to vibrations in the vehicles moving along the rails. As a consequence of vibrations, the structural elements and the load of the vehicle are exposed to dynamic overloads. The exciting effect acts directly upon the wheel-sets contacted with the rails, from where it is transmitted onto the vehicle superstructure through the intermediate structural parts. In a basic case, the transmission of the vertical forces is ensured by the elastic-dissipative force-connection of the spring-suspension system. But in case of vehicles equipped with friction-brakes, when the brake-gear is in action, vertical force components are transmitted onto the sprung vehicle parts through the brake suspension system — with the intervention of the frictional force acting upon the wheel-sets —, which act quasi parallel with the structural parts of the spring-suspension system. The parallel force transmission mentioned above implies the fact that, on the one hand, themotion of the sprung vehicle parts is intensively damped by the frictional fo ces arising due to the brake-gear in action parallel with the elastic dissipative force-connection of the sprung suspension system but, on the other hand, exciting, undamped forces are transmitted from the primarily excited wheel-sets onto the sprung vehicle parts through the brake suspension system. This latter force transmission determines, among others, the loading conditions of the brake suspension system, and thus knowledge of the variation dynamics of forces is of paramount importance for the dimensioning of the system. It also follows from the above that, in case the brake-gear is in action, the vertical wheel-tread forces arise in a way differring from the case of the unbraked vehicle-running under the same track-unevenness conditions. And as a consequence, it can be also deduced from the foregoing that the creep-dependent track-directional wheel-tread froce — which is, in fact, the braking force acting upon the vehicle —, since it is the sum of the products of the vertical wheel-tread forces and the creep-dependent force-connection factors, can be determined only by means of joint examination of the track excitation and the processes associated with braking. In this way, the development of brake performances — and within it, e.g. that of the stopping-distance — can be calculated with due exact-

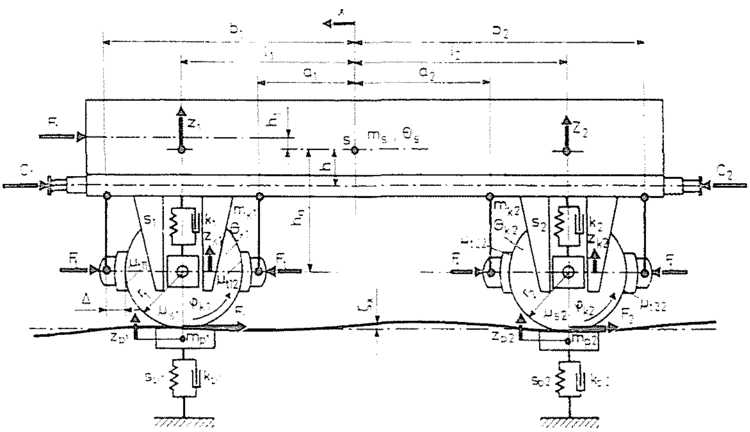

Fig. 1

ness only in knowledge of the actual track excitation. In this paper, the conditions of the two-axle railway vehicles are examined in the case of block-tread braking and uneven tracks. For the dynamic processes to be analized, the simplified planar dynamic model containing several non-linearities is formed, and the description of the system required for digital simulation is given. The depth of the system decomposition used in our treatment is in accordance with the level applied in the technical literature on similar subjects [1], [2].

2. Dynamic model

The planar dynamic model formed for the purpose of analizing dynamic processes is shown in Fig. 1. The superstructure of the two-axle railway vehicle (underframe, body-work and load) is simulated by a rigid body having mass m_s and moment of inertia Θ_s regarding the axis in the gravity centre vertically to the plane in the Figure. The two wheel-sets of the vehicle are simulated by two rotating discs having masses m_{k1} and m_{k2}, respectively, and moments of inertia Θ_{k1} and Θ_{k2}, respectively, calculated for the axis of rotation. The inertia of the permanent way parts supporting the two wheel-sets are represented by masses m_{p1} and m_{p2}, respectively. The relationships between the masses mentioned above are of a different character. In service conditions with the brakes inoperative, the spring suspension system realizing the vertical force transmission between the vehicle superstructure and the wheel-sets were mapped with the use of simplifications — by means of linear springs of stiffness s_1 and s_2, respectively, and linear shock-absorbers of damping factor k_1 and k_2, respectively. As a first step, no clearance was considered between the axle-box guides and axle-box cases. In this way, the horizontal forces of the axle-box guide were treated as internal forces arising in a displacement-free froce connection. Under service-conditions of the braked vehicle, force-connection develops also through the brake suspension system parallel with the spring suspension system, between the vehicle superstructure and the wheel-sets. The transmitted force is determined by the frictional force arising on the brake-blocks and the geometry of the suspension system. The vertical supporting reactions developing in the connection of the wheel-sets and the rail-head were treated as internal forces. The representative track-masses located under the wheel-sets, at the start, were considered to be connected to the stationary reference-plane with linear spring of stiffness s_{p1} and s_{p2}, respectively, and linear shock-absorbers of damping factors k_{p1} and k_{p2}, respectively. As a vertical external force, the force of gravity acts upon each mass of the system forming the model. As a state-dependent force of horizontal direction acting upon each mass of the system simulating the vehicle superstructure, the air-drag force and the longitudinal forces transmitted through the buffer- and draw-gear should be taken into consideration. As horizontal external forces, the state-dependent creep-forces arising on the wheel-treads act upon the masses of the wheel-sets (the resultant of the horizontal brake-block forces acting upon one wheel-set as resulting from the brake-block on the right- and left-hand sides is equal to zero). The representative track masses can be displaced only in a vertical direction consequently, they are considered as braced horizontally.

The free co-ordinates of the dynamic model are the following:

- x — the horizontal displacement of the centre of gravity in the vehicle superstructure
- z_1 — the vertical displacement of the horizontal median of the vehicle superstructure at the point above the axle of the front wheel-set relative to the standstill condition of the vehicle,
- z_2 — the vertical displacement of the horizontal median of the vehicle superstructure at the point above the axle of the rear wheel-set relative to the standstill condition of the vehicle,

z_{k1} the vertical displacement of the centre of gravity of the front wheel-set relative to the standstill condition of the vehicle,

z_{k2} the vertical displacement of the centre of gravity of the rear wheel-set relative to the standstill condition of the vehicle,

φ_{k1} the angular displacement of the front wheel-set as interpreted in the vertical plane,

φ_{k2} the angular displacement of the rear wheel-set as interpreted in the vertical plane.

On the basis of actual evaluation of the set of free co-ordinates and their first derivatives, the (motion)state-dependent forces arising in the force-connections between the masses forming the model can be determined. From the point of the examined dynamic model, the force-connection between the tyre and the brake-block, as well as the force-connection between the wheel and the rail are of basic importance. The effect of the elementary tangential friction forces arising in the friction-connection between the tyre and the brake-block as exerted upon the periphery of the wheel is taken into consideration with a concentrated force-action derived from the horizontal brake-block forces and the sliding friction coefficient in the form of a product. The sliding-friction coefficient was given as the function of the mean brake-block pressure buildt up and of the sliding speed developing in the friction-connection in the form of a non-linear two-variable functional relationship based upon measurements. The peripheral force rising in the wheel-rail connection was derived as the product of the vertical axle-force and the force-connection coefficient. The force-connection coefficient was given as the two-variable non-linear function of the creepage/slipping speed interpreted as the difference between the peripheral speed and the track-directional travelling speed of the wheel, as well as of the track-directional travelling-speed of the wheel. The geometrical characteristics including also track unevennesses required for the setting-up of the motion equations of the dynamic model are shown in Fig. 1.

3. Motion equations of the dynamic model

The motion equations of the dynamic model with seven degrees of freedom discussed in point 2. were determined by means of the synthetical method with the force-actions and torques taken into consideration, as represented in Fig. 2. If

$$\mathbf{X} = [x, z_1, z_2, z_{k1}, z_{k2}, \varphi_{k1}, \varphi_{k2}] \qquad (1)$$

symbolizes the vector of the generalized co-ordinates, then the motion equations of the dynamic model can be included — after proper rearrangement — into the non-linear implicit differential equation of second order

$$\mathbf{M}(\mathbf{X}, \dot{\mathbf{X}})\ddot{\mathbf{X}} = \mathbf{F}(\mathbf{X}, \dot{\mathbf{X}}, t) \qquad (2)$$

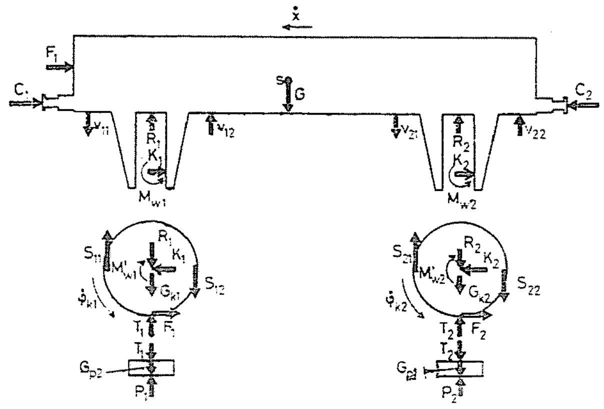

Fig. 2

related to vector-valued function $\mathbf{X}(t)$, which differential equation will be converted into a stochastic differential equation [4], [5] in case of the stochastic variation of track unevenness ζ_x. Matrix $\mathbf{M}$ on the left-hand side of Eq. (2) contains the constants formed from the geometrical and inertia characteristics of the dynamic model, the $\dot{\mathbf{X}}$-dependent wheel-rail force-connection coefficients (μ_{s1}, μ_{s2}), and the $\mathbf{X}$-dependent first derivatives $\left(\zeta'_x, \zeta'_{x-(l_1+l_2)}\right)$ of the vertical track-unevenness function. The detalied description of matrix $\mathbf{M}$ is shown in Fig. 3. It should be noted that — when writing the elements of $\mathbf{M}$ — it was considered that the origo of the co-ordinate system of track-unevenness function ζ_x — is situated under the front wheel-set at the initial time-point of

$$\mathbf{M}(\mathbf{X}, \dot{\mathbf{X}}) = \begin{bmatrix} m_s + m_{k1} + m_{k2} + \mu_{s1} m_{p1} \zeta'_x + \mu_{s2} m_{p2} \zeta'_{x-(l_1+l_2)} & (m_{k1}+m_{k2}) \frac{h_a}{l_1+l_2} & (m_{k1}+m_{k2}) \frac{h_a}{l_1+l_2} & -\mu_{s1} m_{p1} & -\mu_{s2} m_{p2} & 0 & 0 \\ 0 & m_s \frac{l_2}{l_1+l_2} & m_s \frac{l_1}{l_1+l_2} & 0 & 0 & 0 & 0 \\ (m_{k1}+m_{k2}) h_a - \mu_{s1} m_{p1} \zeta'_x h_a - \mu_{s2} m_{p2} \zeta'_{x-(l_1+l_2)} h_a & \frac{\Theta_s + (m_{k1}+m_{k2}) h_a^2}{l_1+l_2} & \frac{\Theta_s + (m_{k1}+m_{k2}) h_a^2}{l_1+l_2} & \mu_{s1} m_{p1} h_a & \mu_{s2} m_{p2} h_a & 0 & 0 \\ -m_{p1} \zeta'_x & 0 & 0 & m_{k1} + m_{p1} & 0 & 0 & 0 \\ -m_{p2} \zeta'_{x-(l_1+l_2)} & 0 & 0 & 0 & m_{k2} + m_{k2} & 0 & 0 \\ -r_1 \mu_{s1} m_{p1} \zeta'_x & 0 & 0 & r_1 \mu_{s1} m_{p1} & 0 & \Theta_{k1} & 0 \\ -r_2 \mu_{s2} m_{p2} \zeta'_{x-(l_1+l_2)} & 0 & 0 & 0 & r_2 \mu_{s2} m_{p2} & 0 & \Theta_{k2} \end{bmatrix}$$

Fig. 3

the examination. So, relationships $z_{k1} = z_{p1} + \zeta_x$ and $z_{k2} = z_{p2} + \zeta_{x-(l_1+l_2)}$ are in force between the vertical unevennesses and the displacements of the wheel-set masses and track-masses, respectively. The co-ordinates of vector function **F** on the right-hand side of Eq. (2) are formed from the expressions of the different time-and (motion)state-dependent force-actions and torques. The detailed inscription of vector **F** is shown in Fig. 4, where the symbols are

$$\mathbf{F} = \begin{bmatrix} -F_1 - C_1 - F_1^* - F_2^* + C_2 \\ -V_{11} + R_1 + V_{12} - G_s - V_{21} + R_2 + V_{22} \\ -F_1 h_1 + C_1 h_{\ddot{u}} + V_{11} b_1 - R_1 l_1 - V_{12} a_1 + F_1^* h_a + \\ + F_2^* h_a - V_{21} a_2 + R_2 l_2 + V_{22} b_2 - C_2 h_{\ddot{u}} + M_{w1} + M_{w2} \\ V_{11} - R_1 - G_{k1} - V_{12} - G_{p1} + P_1 + m_{p1} \zeta_x'' \dot{x}^2 \\ V_{21} - R_2 - G_{k2} - V_{22} - G_{p2} + P_2 + m_{p2} \zeta_{x-(l_1+l_2)}'' \dot{x}^2 \\ -S_{11} r_1 - S_{12} r_1 + F_1^* r_1 - M'_{w1} \\ -S_{21} r_2 - S_{22} r_2 + F_2^* r_2 - M'_{w2} \end{bmatrix}$$

Fig. 4

the same as those applied in Figs 1 and 2. Note that M_{w1} and M_{w2} indicate bearing-friction resistance-torques, while M'_{w1} and M'_{w2} indicate the sum of bearing-friction and rolling-friction resistance-torques.

The expressions of force-actions F_1^* and F_2^* are the following:

$$F_1^* = -\mu_{s1} G_{p1} + \mu_{s1} P_1 + \mu_{s1} m_{p1} \cdot \zeta_x'' \dot{x}^2,$$

$$F_2^* = -\mu_{s2} G_{p2} + \mu_{s2} P_2 + \mu_{s2} m_{p2} \cdot \zeta_{x-(l_1+l_2)}'' \dot{x}^2. \tag{3}$$

The forces transmitted through the brake suspension elements are derived from the friction-forces arising on the brake-blocks with the help of relationships:

$$V_{ij} = \frac{r_i}{r_i + \Delta} S_{ij}; \quad i, j = 1, 2. \tag{4}$$

4. Time- and state-dependence of the forces occurring in the equation of motion

The forces occurring in Eq. (2) should be described as the functions of time and (motion) state. In the first place, virtual friction-forces S_{ij} transmitted through the friction-connection of the brake-blocks will be dealt with. According to relationship $S_{ij} = \mu_{t_{ij}} F_t$, the virtual friction-force is derived as the product of friction coefficient $\mu_{t_{ij}}$ and brake-block force F_t. Brake-block force F_t

represents an external, time-dependent force-action, while friction coefficient $\mu_{t_{ij}}$, for a start, depends on brake-block pressure $p_t = F_t/A$, and sliding-speed v_{ij} occurring on the sliding surface of the brake-block. (Here A symbolizes the sliding surface of the brake-blocks on one side of the wheel-set.) The sliding-speeds evolving on the sliding surface of the brake-blocks are obtained in consideration of the following relationships:

$$\begin{aligned} v_{11} &= r_1\dot{\varphi}_{k1} + \dot{z}_{11} - \dot{z}_{k1}, \\ v_{12} &= r_1\dot{\varphi}_{k1} - \dot{z}_{12} + \dot{z}_{k1}, \\ v_{21} &= r_2\dot{\varphi}_{k2} + \dot{z}_{21} - \dot{z}_{k2}, \\ v_{22} &= r_2\dot{\varphi}_{k2} - \dot{z}_{22} + \dot{z}_{k2} \end{aligned} \tag{5}$$

where

$$\begin{aligned} \dot{z}_{11} &= \dot{z}_1 + \frac{b_1 - l_1}{l_1 + l_2}(\dot{z}_1 - \dot{z}_2), \\ \dot{z}_{12} &= \dot{z}_1 - \frac{l_1 - a_1}{l_1 + l_2}(\dot{z}_1 - \dot{z}_2), \\ \dot{z}_{21} &= \dot{z}_2 + \frac{l_2 - a_2}{l_1 + l_1}(\dot{z}_1 - \dot{z}_2), \\ \dot{z}_{22} &= \dot{z}_2 - \frac{b_2 - l_2}{l + l_{21}}(\dot{z}_1 - \dot{z}_2). \end{aligned} \tag{6}$$

So virtual friction-forces S_{ij} will be provided by the expression:

$$S_{ij} = \mu_t(p_t, |v_{ij}|)(\text{sign } v_{ij}) \cdot F_t \tag{7}$$

in case $i, j = 1, 2$. The two-variable friction-coefficient function μ_t is represented above the positive plane-quadrant in Fig. 5.

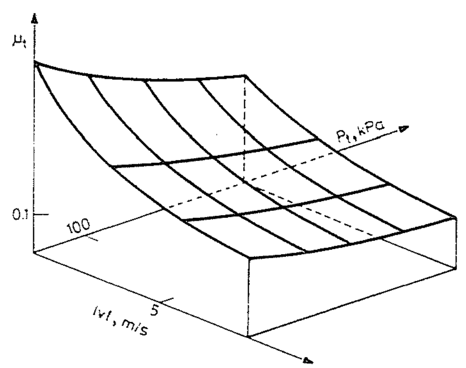

Fig. 5

Track-directional force F_i transmitted through the wheel-rail connection is yielded by the expression:

$$F_i = -\mu_{si}G_{pi} + \mu_{si}P_i + \mu_{si}m_{pi}\zeta''_x\dot{x}^2 + \mu_{si}m_{pi}\zeta'_x\ddot{x} - \mu_{si}m_{pi}\ddot{z}_{ki} =$$
$$= F_i^* + \mu_{si}m_{pi}\zeta'_x\ddot{x} - \mu_{si}m_{pi}\ddot{z}_{ki}; \qquad i = 1, 2. \tag{8}$$

The decisive role of force-connection coefficient μ_{si} is obvious. Force-connection coefficient μ_{si} — as mentioned above — is the function of the wheel-tread creepage/sliding speed $\dot{x} - r_i\dot{\varphi}_{ki}$ and travelling-speed $\dot{x}$. The values of μ_{si} are yielded by the expression:

$$\mu_{si} = \mu_s(\dot{x}, \ \dot{x} - r_i\dot{\varphi}_{ki}) \operatorname{sign}(\dot{x} - r_i\dot{\varphi}_{ki}) \qquad i = 1, 2 \tag{9}$$

from the force-connection coefficient function μ_s shown above the positive plane-quadrant in Fig. 6.

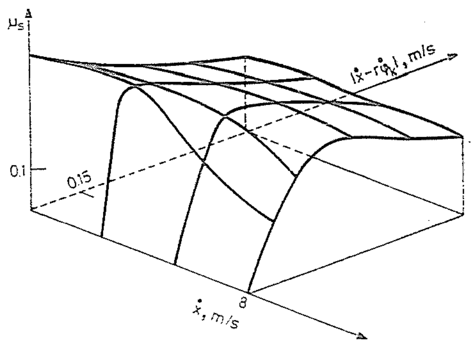

Fig. 6

The motion-state dependence of force-actions R_1 and R_2 transmitted from the spring-suspension system onto the wheel-sets and the vehicle-superstructure, respectively, were obtained in the form:

$$R_i = R_{i0} + s_i(z_{ki} - z_1) + k_i(\dot{z}_{ki} - \dot{z}_1); \qquad i = 1, 2 \tag{10}$$

where the springing is considered to be linear and the damping is also taken into consideration.

The air-resistance force acting upon the vehicle-superstructure was calculated as the function of speed according to relationship

$$F_1 = a_1\dot{x}^2 \operatorname{sign} \dot{x}. \tag{11}$$

Forces C_1 and C_2 were equally considered as zero in our examinations, and their occurrence in vector-valued function $\mathbf{F}$ on the right-hand side of Eq. (2) intends to ensure the subsequent considerability of the longitudinal dynamic interactions.

The state of motion-dependence of force-actions P_1 and P_2, respectively, transmitted to the representative track-masses placed under the wheel-sets is obtained in the following form:

$$P_1 = P_{10} - s_{p1}(z_{k1} - \zeta_x) - k_{p1}(\dot{z}_{k1} - \zeta'_x\dot{x}) \tag{12}$$
$$P_2 = P_{20} - s_{p2}(z_{k2} - \zeta_{x-(l_1+l_2)}) - k_{p2}(\dot{z}_{p2} - \zeta'_{x-(l_1+l_2)}\dot{x})$$

with the elasticity initially also considered as linear, and with the dissipation taken into consideration.

Torques M_{w1} and M_{w2} of the friction forces arising in the bearing-supports of the wheel-sets as transmitted to the vehicle-superstructure are obtained in the following form:

$$M_{wi} = (a + b \mid r_i\dot{\varphi}_{ki} \mid + c(r_i\dot{\varphi}_{ki})^2) \operatorname{sign}\dot{\varphi}_{ki} \qquad i = 1\ 2 \tag{13}$$

depending on the peripheral speeds of the wheel-sets, while the values of the bearing-friction resistance torque acting upon the wheel-sets as increased with the rolling-friction torques were obtained by the expression:

$$M'_{wi} = (a' + b' \mid r_i\dot{\varphi}_{ki} \mid + c'(r_i\dot{\varphi}_{ki})^2) \operatorname{sign} \dot{\varphi}_{ki} \qquad i = 1, 2 \tag{14}$$

given also as the function of the peripheral speed.

The constant force of gravity acting in the centre of gravity, too, was operated on each mass of the model (G_s, G_{k1}, G_{k2}, G_{p1}, G_{p2}). Between the mentioned forces of gravity and the constants occurring in formulas (10) and (12), respectively, relationships $R_{i0} = G_s\alpha_i$ and $P_{i0} = G_s\alpha_i + G_{ki} + G_{pi}$; $i = 1, 2$ are in force, where

$$\alpha_1 = \frac{l_2}{l_1 + l_2}, \quad \alpha_2 = \frac{l_1}{l_1 + l_2}.$$

The state-dependence of forces F_1^* and F_2^* occurring in vector $\mathbf{F}$ on the right-hand side of the motion equaions were already given through the pair of formulae (3), while the state-dependence of force actions V_{ij}; $i, j = 1, 2$ is also determined by formulae (4) and (7).

In connection with track-unevenness ζ_x, it should be noted that if it is considered to be a deterministic one, then the first and second derivative functions can also be considered as known, consequently, the elements of vector $\mathbf{F}$ and those of matrix $\mathbf{M}$ can be calculated. If track-unevennesses ζ_x are considered to be a stationary stochastic process with zero expectation having a spectral density function $S(\Omega)$ (given as the function of angular frequency $[\Omega] = \text{rad}/m$), then the numerical values characteristic of excitation as required for the simulation are obtained on the basis of the formula of realization function of the process in the following form:

$$\zeta_x = \psi_0 + \sum_{k=1}^{N} 2\sqrt{S(\Omega_k)\Delta\Omega}\ \cos(\Omega_k x + \psi_k) \tag{15}$$

developed on the basis of the discrete sequence of angular frequency Ω_k [3], [6]. In formula (15), ψ_0 symbolizes a normally distributed random number with zero expectation and with a variance $S(\Omega_0)\Delta\Omega$, while ψ_k stands for uniformly distributed random numbers in $[-\pi, \pi]$. Derivative functions from (15) are obtained by the expressions:

$$\zeta'_x = -\sum_{k=1}^{N} 2\Omega_k \sqrt{S(\Omega_k)\Delta\Omega}\, \sin(\Omega_k x + \psi_k) \tag{16}$$

and

$$\zeta''_x = -\sum_{k=1}^{N} 2\Omega_k^2 \sqrt{S(\Omega_k)\Delta\Omega}\, \cos(\Omega_k x + \psi_k). \tag{17}$$

5. Numerical solution of the motion equation

If both sides of motion Eq. (2) are multiplied by the inverse of state-dependent matrix $\mathbf{M}(\mathbf{X}, \dot{\mathbf{X}})$ on the left-hand side of the equation, then the expression of acceleration vector $\ddot{\mathbf{X}}$ is obtained in an $\mathbf{X}$- $\dot{\mathbf{X}}$- and t-dependent explicit form like this:

$$\ddot{\mathbf{X}} = \mathbf{M}^{-1}(\mathbf{X}, \dot{\mathbf{X}})\; \mathbf{F}(\mathbf{X}, \dot{\mathbf{X}}, t) = \mathbf{\Phi}(\mathbf{X}, \dot{\mathbf{X}}, t). \tag{18}$$

To solve the explicit equation obtained in this way in a numerical form, it is expedient to change over to the differential equation of the first order:

$$\dot{\mathbf{Y}} = \begin{bmatrix} \dot{\mathbf{Y}}_1 \\ \dot{\mathbf{Y}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}(\mathbf{Y}_2, \mathbf{Y}_1, t) \\ \mathbf{Y}_1 \end{bmatrix} = \mathbf{\Psi}(\mathbf{Y}, t) \tag{19}$$

related to the 14-dimensional state-vector $\mathbf{Y} = [\mathbf{Y}_1, \mathbf{Y}_2]^*$, with the application of substitutions $\mathbf{Y}_1 = \dot{\mathbf{X}}$ and $\mathbf{Y}_2 = \mathbf{X}$. With the system of initial speeds $\mathbf{Y}_1(t_0) = \dot{\mathbf{X}}(t_0) = \dot{\mathbf{X}}_0$, and the system of initial displacements $\mathbf{Y}_2(t_0) = \mathbf{X}(t_0) = \mathbf{X}_0$ assigned to initial time-point t_0 as connected to Eq. (19), the initial value problem set in the way mentioned above can be solved numerically. To solve this problem, the Runge-Kutta method of fourth order can be used well, the essence of which is that starting from the system of initial values, the approximate values of the solution-function can be generated on the sequence of time-points $\{t_i\}_{i=1}^N$ of spacing Δt, by means of definition:

$$\mathbf{Y}(t_{i+1}) = \mathbf{Y}(t_i) + \frac{1}{6}(\mathbf{k}_{i1} + 2(\mathbf{k}_{i2} + \mathbf{k}_{i3}) + \mathbf{k}_{i4}) \tag{20}$$

where coefficients $\mathbf{k}_{ij}$; $j = 1, 2, 3, 4$ are determined as follows:

$$\begin{aligned} \mathbf{k}_{i1} &= \Psi(\mathbf{Y}(t_i), t_i)\Delta t, \\ \mathbf{k}_{i2} &= \Psi\left(\mathbf{Y}(t_i) + \frac{\mathbf{k}_{i1}}{2}, t_i + \frac{\Delta t}{2}\right)\Delta t, \\ \mathbf{k}_{i3} &= \Psi\left(\mathbf{Y}(t_i) + \frac{\mathbf{k}_{i2}}{2}, t_i + \frac{\Delta t}{2}\right)\Delta t, \\ \mathbf{k}_{i4} &= \Psi(\mathbf{Y}(t_i) + \mathbf{k}_{i3}, t_i + \Delta t)\Delta t. \end{aligned} \tag{21}$$

With the successive application of relationship (20) as relying on (21), the approximate values of the state-vector-function $\mathbf{Y}(t)$ are obtained on the sequence of time-points of spacing Δt chosen as the basis of calculations.

It should be noted that owing to the relatively high calculation-demand character of the Runge-Kutta method, in case of pıeliminary dynamic analyses of lower calculation demand character, the use of a numerical approximation method of solution can be justified which renders Eq. (2) explicit so that by grouping the state-dependent members of matrix $\mathbf{M}$ on the right-hand side of the equation, they are determined through multiplying them by the value of acceleration as related to the previous time-point.

6. Concluding remarks

The application of the dynamic model dealt with in the previous chapter and the calculation process associated with it leads to a numerical determination of state-vector $\mathbf{Y}(t)$ of a braked railway vehicle travelling along uneven track, as has taken place on a given sequence of time-points $\{t_i\}_{i=1}^{N}$.

The input process of the dynamic system represented by a braked vehicle travelling along uneven track will be the time-function of brake-block force F_t. The state-space method, according to its aspects, transforms time-function F_t into state-vector $\mathbf{Y}(t)$, as the first step of the dynamic system, then — in the knowledge of $\mathbf{Y}(t)$ and F_t — into the requested system-response vector $\mathbf{V}(t)$. In Fig. 7, the block-diagram of the mentioned transforms are shown, and the character of development is outlined for the time-function of brake-block force F_t, and for the response-function system $\mathbf{V}(t) = [V_{11}(t), V_{12}(t), V_{21}(t), V_{22}(t)]^*$ containing forces arising in the brake-suspension elements, in the case of stop-braking.

Functions $V_{ij}(t)$ follow the character of variation of function F_t with smaller or greater deviations in the first half period of the braking process. But in case of a significant reduction in the vehicle-speed — and first of all, at the moments immediately before stoping —, the speed of the tyre-brake-block sliding is basically determined, according to relationships (5) and (6), by speeds $\dot{z}_i$ and $\dot{z}_{ki}$; $i = 1, 2$, depending on the vibrational state of the vehicle. So with (4) and (7) taken into consideration, a sign-reversal force-variation can develop also in functions $V_{ij}(t)$, indicating a significant dynamic overload of the brake-suspension system.

To describe and analyse the dynamic processes in the braked railway vehicles as brought about by the track-unevennesses, the examination-model

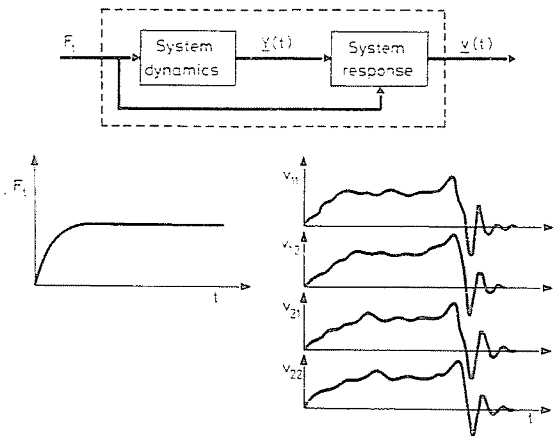

Fig. 7

and calculation-method elaborated in this paper are advisable to be developed in the following directions:

1. Development of a more detailed dynamic model of the spring-suspension system-connection with the elements of non-linear elasticity and dissipation, as well as the effect of a loose axle-box guidance taken into account;
2. Recknoning with the mass, elasticity, damping of the brake suspension system and the effects of backlashes at the links in the model;
3. Reckoning with the dependence of the friction-force arising between the brake-block and the tyre upon the temperature conditions of the friction interface in the calculation process;
4. Reckoning with the stochastic process-couple describing the random fluctuations of the coefficient of friction between the brake-block and the tyre, as well as those of the wheel-rail force-connection coefficient in the calculation process;
5. Reckoning with the non-linearity of the track-compliance and dissipation, as well as the local elasticity present at the wheel and rail interface in the model and with the calculation process.

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