TRANSFORMATION PROCEDURES TO ACCELERATE FINITE ELEMENT ANALYSES

L. VARGA

Department of Mechanics, Faculty of Transport Engineering,
Technical University, H-1521, Budapest

Received October 3, 1986

Presented by Prof. Dr. P. Michelberger

Abstract

The method presented is advantageous for structures comprising recurrent part elements by applying the five basic transformations and their combinations, thus eliminating much of tedious data input. Substructuring avoids the need to produce consecutive stiffness matrices and time-consuming reduction. Recurrent structural parts can be composed to panels — without high-capacity mass storage — suiting the rapid composition of various models in an explicit form (reduced to appropriate points).

Introduction

Two problems facing us in computerized finite element analysis are increased memory and mass storage capacity demand as well as long running time. The so-called substructure method lends itself to the analysis of complex structures or of simpler structures by minor compters. Without entering into details of this method, it essentially consists in decomposing the structure into part units (substructures), then producing their stiffness matrices, to be reduced subsequently to connecting points (common points of substructures) [1]. Thereby it is sufficient to solve the reduced equation system and equation systems of each different substructure.

In order to increase the efficiency of this method, let us have a look at the time-consuming steps of the substructure finite element method, such as:

1. to define the geometry;
2. to produce the substructure stiffness matrix;
3. to reduce the substructure stiffness matrix to connecting points;
4. to solve the equation system of the structure;
5. to solve substructure equation systems.

The substructure method is advantageous as it involves much fewer operations hence has a shorter running time demand compared to solving the complete equation system. Another advantage is the possibility to examine the effect of modifications within a substructure independent of the other substructures.

Let us present now a method that aids in reducing running time demand for steps 1, 2, 3 for structures comprising several parts of identical geometries.

Transformation algorithm of a substructure stiffness matrix

The structure in Fig. 1 can be decomposed into five substructures, the first four having an identical geometry. Stiffness matrix of substructure 1 may be supposed as strictly related to those of substructures 2, 3 and 4. This relation is similar to producing the stiffness matrix of an element in the local (element-bound) coordinate system, to be transformed into the global coordinate system, i.e. a transformation by rotation [2], [3]. Actually, the problem is to produce

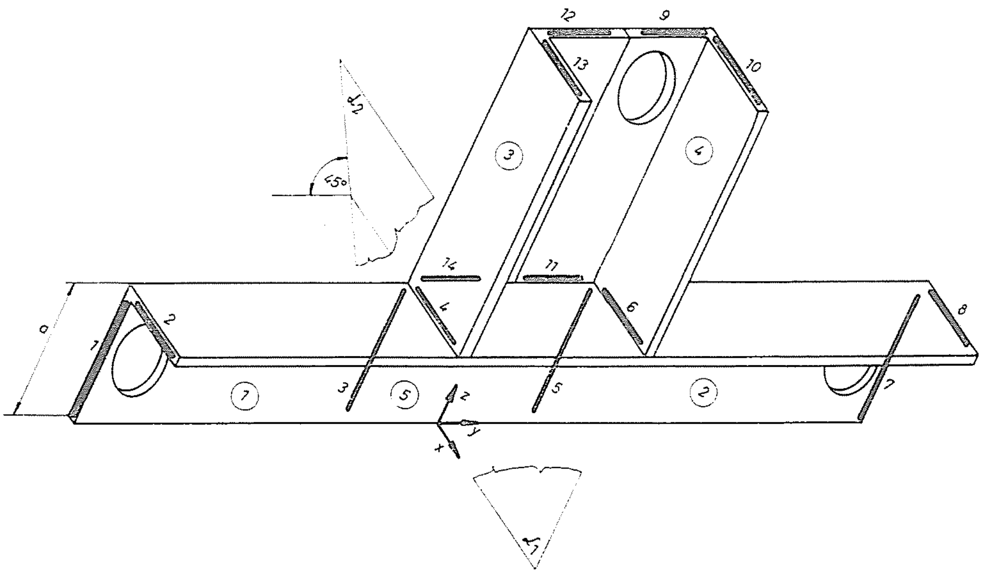


Fig. 1

the stiffness matrix of an arbitrary substructure in a position different from that of a substructure known in the global system by simple means, using its stiffness matrix, without repeatedly composing it. Substructures 1 and 2 are seen not to be registerable by rotation, so it is advisable to interpret transformation in the general meaning of the word. It will be shown how to rapidly determine substructure stiffness matrices in case of arbitrary transformation true to form and dimensions, together with transformation matrices for essential transformations.

Let us have an arbitrary stiffness matrix (①) in the global coordinate system 0 (Fig. 2), and let its local coordinate system be denoted by 1. Let us find a transformation to determine the unknown stiffness matrix (②) in the global system. The unknown stiffness matrix may be assigned a local coordinate system 2. The transformation to shift coordinate system 1 to 2 (e.g. rotation,

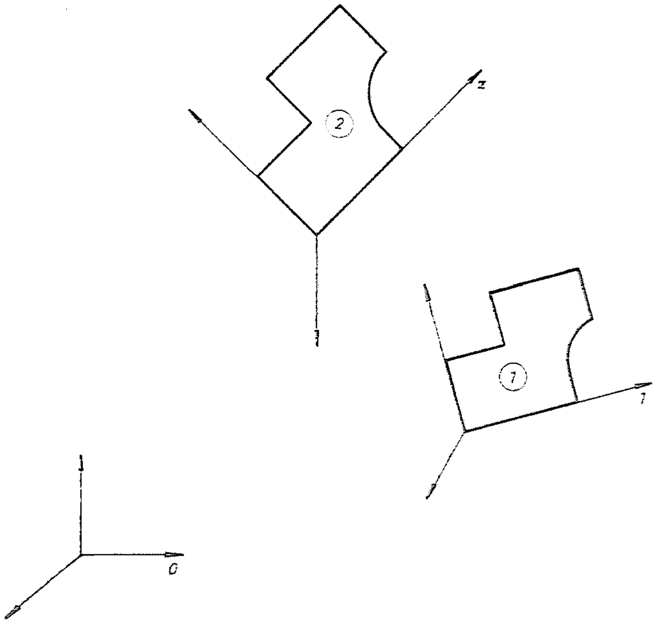


Fig. 2

reflection, etc.) is known. (Shifting has to be understood in a broader sense, namely e.g. a reflection to a given point cannot be replaced by shifting, nevertheless, transformation is true to form and dimensions.)

The transformed to an arbitrary point can be written in the form:

$$R_2 = \mathbf{A}_{12} R_1 + B_{12}$$

where:

$R_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ coordinate vector before transformation;

$R_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ coordinate vector after transformation;

$\mathbf{A}_{12} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ rotational part of tranformation from 1 to 2;

$B_{12} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ translational part of transformation from 1 to 2.

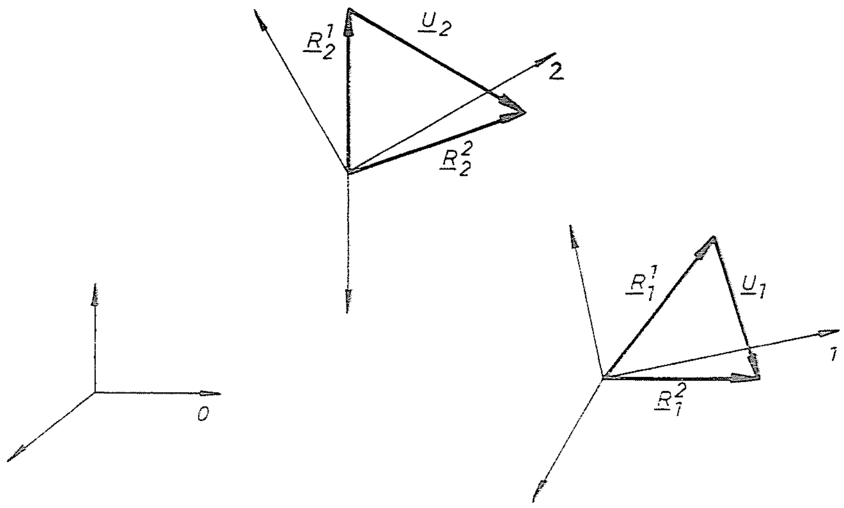


Fig. 3

Displacement vectors in the original and the transformed systems are, according to Fig. 3:

$$U_1 = R_1^2 - R_1^1$$
$$U_2 = R_2^2 - R_2^1.$$

Subscripts refer to the coordinate system, while superscripts identify the vector in the given coordinate system.

Substituting the relation between displacement vectors:

$$U_2 = (\mathbf{A}_{12}R_1^2 + B_{12}) - (\mathbf{A}_{12}R_1^1 + B_{12}) = \mathbf{A}_{12}(R_1^2 - R_1^1).$$

Hence, the two displacement vectors are related as:

$$U_2 = \mathbf{A}_{12}U_1.$$

Assuming in a given node 3 + 3 degrees of freedom to be interpreted (displacement vector U, rotation vector V), and the substructure to be transformed to have n nodes, a $6n \times 6n$ transformation matrix can be produced, such as (indicating only non-zero elements):

$$\mathbf{T}_{12} = \left[\begin{array}{cc:cc:c:cc}
\mathbf{A}_{12} & & & & & & \\
 & \mathbf{A}_{12} & & & & & \\
\hdashline
 & & \mathbf{A}_{12} & & & & \\
 & & & \mathbf{A}_{12} & & & \\
\hdashline
 & & & & \ddots & & \\
\hdashline
 & & & & & \mathbf{A}_{12} & \\
 & & & & & & \mathbf{A}_{12}
\end{array}\right]$$
$$\quad 1 \qquad 2 \qquad \cdots \qquad n$$

Stiffness equations in their respective local coordinate systems are:

$$\mathbf{K}_1^1 f_1 = F_1$$

$$\mathbf{K}_2^2 f_2 = F_2$$

where:

$\mathbf{K}_1^1 = \mathbf{K}_2^2 = \mathbf{K}$ stiffness matrix of the substructure in its local coordinate system;

$f_1 = \begin{bmatrix} U_1^1 \\ V_1^1 \\ \cdot \\ \cdot \\ \cdot \\ U_1^n \\ V_1^n \end{bmatrix}$, $f_2 = \begin{bmatrix} U_2^1 \\ V_2^1 \\ \cdot \\ \cdot \\ \cdot \\ U_2^n \\ V_2^n \end{bmatrix}$ displacement vectors written in local systems (composed of displacement and rotation vectors);

$F_1 = \begin{bmatrix} N_1^1 \\ M_1^1 \\ \cdot \\ \cdot \\ \cdot \\ N_1^n \\ M_1^n \end{bmatrix}$, $F_2 = \begin{bmatrix} N_2^1 \\ M_2^1 \\ \cdot \\ \cdot \\ \cdot \\ N_2^n \\ M_2^n \end{bmatrix}$ load vectors written in local systems (composed of force and moment vectors).

Transforming displacement and load vectors to the global coordinate system:

$$f_1 = \mathbf{T}_{10} f_0 \quad \text{and} \quad F_1 = \mathbf{T}_{10} F_0$$

$$f_2 = \mathbf{T}_{20} f_0 \quad \text{and} \quad F_2 = \mathbf{T}_{20} F_0$$

where

f_0, F_0 displacement and load vectors, resp., written in the global system;

$\mathbf{T}_{10}, \mathbf{T}_{20}$ matrices for transforming from the local systems to their global counterparts.

Resubstituting into the stiffness equations:

$$\mathbf{KT}_{10} f_0 = \mathbf{T}_{10} F_0$$

$$\mathbf{KT}_{20} f_0 = \mathbf{T}_{20} F_0.$$

Arranging yield stiffness matrices ($\mathbf{K}_0^1, \mathbf{K}_0^2$) in the global coordinate system:

$$(\mathbf{T}_{10}^* \mathbf{K} \mathbf{T}_{10}) f_0 = F_0$$

$$\mathbf{K}_0^1 f_0 = F_0$$

$$(\mathbf{T}_{20}^* \mathbf{K} \mathbf{T}_{20}) f_0 = F_0$$

$$\mathbf{K}_0^2 f_0 = F_0.$$

Making use of the orthogonality of the transformation matrix has led to:

$$\mathbf{T}^{-1} = \mathbf{T}^*.$$

Stiffness matrix of substructure 1 being, however, assumed to be known in the global system, the local stiffness matrix becomes:

$$\mathbf{K} = \mathbf{T}_{10} \mathbf{K}_0^1 \mathbf{T}_{10}^*.$$

In conformity with the above relationships, local displacements are related as:

$$f_2 = (\mathbf{T}_{20} \mathbf{T}_{10}^*) f_1.$$

By definition, this is identical to a transformation between systems 1 and 2:

$$\mathbf{T}_{12} = \mathbf{T}_{20} \mathbf{T}_{10}^*.$$

Arranged:

$$\mathbf{T}_{20} = \mathbf{T}_{12} \mathbf{T}_{10}$$

$$\mathbf{T}_{20}^* = \mathbf{T}_{10}^* \mathbf{T}_{12}^*.$$

Substituted into the relationship for the stiffness matrix written in the global system of substructure 2:

$$\mathbf{K}_0^2 = (\mathbf{T}_{10}^* \mathbf{T}_{12}^* \mathbf{T}_{10}) \; \mathbf{K}_0^1 (\mathbf{T}_{10}^* \mathbf{T}_{12} \mathbf{T}_{10}).$$

This relationship is suitable to determine substructure stiffness matrix $(\mathbf{K}_0^2)$ derived by arbitrary transformation from a known substructure matrix $(\mathbf{K}_0^1)$ by direct transformation (matrix multiplication):

$$\tilde{\mathbf{T}} = \mathbf{T}_{10}^* \mathbf{T}_{12} \mathbf{T}_{10}$$

$$\tilde{\mathbf{T}}^* = \mathbf{T}_{10}^* \mathbf{T}_{12}^* \mathbf{T}_{10}$$

$$\boxed{\mathbf{K}_0^2 = \tilde{\mathbf{T}}^* \; \mathbf{K}_0^1 \; \tilde{\mathbf{T}}}\,.$$

Provided substructure 1 has been written in the global system, and also the geometry transformation $(\mathbf{T}_{12})$ had been referred to the global system $(0 \equiv 1)$, the relationship is further simplified to:

$$\mathbf{T}_{10} = \mathbf{T}_{10}^* = \mathbf{E} \quad \text{(unit matrix)}.$$

Hence:

$$\tilde{\mathbf{T}} = \mathbf{E}^*\mathbf{T}_{12}\mathbf{E} = \mathbf{T}_{12}$$

$$\tilde{\mathbf{T}}^* = \mathbf{E}^*\mathbf{T}_{12}^*\mathbf{E} = \mathbf{T}_{12}^*.$$

This transformation may be obtained after the reduction of the substructure stiffness matrix, and has the following advantages:

1. stiffness matrix of identical (inter-transformable) substructures has to be produced but once;
2. identical substructures (with identical connecting points) have to be reduced but once;
3. recurrent substructure stiffness matrices and reduced substructure stiffness matrices have to be stored but once.

In composing the geometrical model, recurrent part units (substructures) can be fitted by means of various transformations. The five fundamental transformations true to dimension and form (shifting, rotation, reflection to a point, reflection to a straight line, reflection to a plane) permit a simple realization of practically any geometrical variation. Transformations can also be linked to a chain. Let us apply transformations $\mathbf{T}_1, \mathbf{T}_2, \ldots, \mathbf{T}_m$ in series. Now, the resultant transformation matrix

$$\mathbf{T} = \mathbf{T}_1\mathbf{T}_2 \ldots \mathbf{T}_m.$$

Just as matrix multiplication, also repeated transformations are not exchangeable. Certain transformations are not independent of each other, that is, a given geometrical correlation can be described in terms of several different transformations [4].

Transformation matrices of geometrical transformations

Without detailed calculations, transformation matrices of fundamental transformations will be presented in a three-dimensional, Euclidean space:

a) *Shifting* (by a given vector):

$$\mathbf{A}_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{B}_{12} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

b) *Rotation* (by a given angle around a straight line with a given direction vector passing through a given point):

$$\mathbf{A}_{12} = \mathbf{A}_{12}^1 \mathbf{A}_{12}^2 \mathbf{A}_{12}^3 \mathbf{A}_{12}^4 \mathbf{A}_{12}^5 \mathbf{A}_{12}^6 \mathbf{A}_{12}^7$$

$$\mathbf{A}_{12}^1 = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \end{bmatrix}$$

$$\mathbf{A}_{12}^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{C}{V} & -\frac{B}{V} & 0 \\ 0 & \frac{B}{V} & \frac{C}{V} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_{12}^3 = \begin{bmatrix} \frac{V}{L} & 0 & -\frac{A}{L} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{A}{L} & 0 & \frac{V}{L} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_{12}^4 = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 & 0 \\ \sin\varphi & \cos\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_{12}^5 = \begin{bmatrix} \frac{V}{L} & 0 & \frac{A}{L} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{A}{L} & 0 & \frac{V}{L} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_{12}^6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{C}{V} & \frac{B}{V} & 0 \\ 0 & -\frac{B}{V} & \frac{C}{V} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_{12}^7 = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \end{bmatrix}$$

where:

$$V = \sqrt{B^2 + C^2}$$

$$L = \sqrt{A^2 + B^2 + C^2}.$$

If $B = C = 0$, that is, $V = 0$, then:

$$\mathbf{A}_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix}$$

In either case:

$$B_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

c) *Reflection to a point* (of given coordinates):

$$\mathbf{A}_{12} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$B_{12} = \begin{bmatrix} 2x_0 \\ 2y_0 \\ 2z_0 \end{bmatrix}$$

d) *Reflection to a straight line* (with a given direction vector, passing through a given point):

$$\mathbf{A}_{12} = \begin{bmatrix} 2\frac{A^2}{L} - 1 & 2\frac{AB}{L} & 2\frac{AC}{L} \\ 2\frac{AB}{L} & 2\frac{B^2}{L} - 1 & 2\frac{BC}{L} \\ 2\frac{AC}{L} & 2\frac{BC}{L} & 2\frac{C^2}{L} - 1 \end{bmatrix}$$

$$\mathbf{B}_{12} = \begin{bmatrix} 2x_0 - 2x_0 & \frac{A^2}{L} - 2y_0 & \frac{AB}{L} - 2z_0 & \frac{AC}{L} \\ 2y_0 - 2x_0 & \frac{AB}{L} - 2y_0 & \frac{B^2}{L} - 2z_0 & \frac{BC}{L} \\ 2z_0 - 2x_0 & \frac{AC}{L} - 2y_0 & \frac{BC}{L} - 2z_0 & \frac{C^2}{L} \end{bmatrix}$$

where:

$$L = \sqrt{A^2 + B^2 + C^2}.$$

e) ***Reflection to a plane*** (of a given normal, passing through a given point):

$$\mathbf{A}_{12}=\begin{bmatrix} 1-2\frac{A^2}{L} & -2\frac{AB}{L} & -2\frac{AC}{L} \\ -2\frac{AB}{L} & 1-2\frac{B^2}{L} & -2\frac{BC}{L} \\ -2\frac{AC}{L} & -2\frac{BC}{L} & 1-2\frac{C^2}{L} \end{bmatrix}$$

$$\mathbf{B}_{12}=\begin{bmatrix} 2x_0\frac{A^2}{L} & +2y_0\frac{AB}{L} & +2z_0\frac{AC}{L} \\ 2x_0\frac{AB}{L} & +2y_0\frac{B^2}{L} & +2z_0\frac{BC}{L} \\ 2x_0\frac{AC}{L} & +2y_0\frac{BC}{L} & +2z_0\frac{C^2}{L} \end{bmatrix}$$

where:

$$L=\sqrt{A^2+B^2+C^2}\,.$$

Let us follow the steps of the method on hand of an example corresponding to Fig. 1. The entire structure is seen to be geometrically constructible from two substructures.

A given substructure comprises inner nodes and connecting points (b) and (c), respectively. Accordingly the stiffness, equation system of the substructure becomes:

$$\begin{bmatrix} \mathbf{K}_{bb} & \mathbf{K}_{bc} \\ \mathbf{K}_{cb} & \mathbf{K}_{cc} \end{bmatrix} \begin{bmatrix} U_b \\ U_c \end{bmatrix} = \begin{bmatrix} F_b \\ F_c \end{bmatrix}$$

$$(\mathbf{K}_{cc}-\mathbf{K}_{cb}\mathbf{K}_{bb}^{-1}\mathbf{K}_{bc})\; U_c = F_c-(\mathbf{K}_{cb}\mathbf{K}_{bb}^{-1})F_b$$

Assuming inner nodes to have no load, and introducing the concept of reduced substructure stiffness matrix:

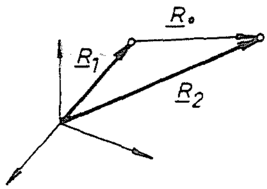
$$\tilde{\mathbf{K}}\,U_c = F$$

where:

$\tilde{\mathbf{K}}$ is the stiffness matrix of the substructure reduced to connection points.

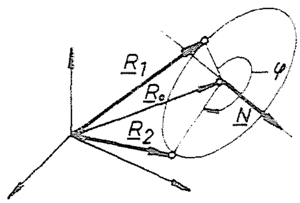
Example on the application

Reduced stiffness matrices of substructures 2, 3 and 4 can be produced by proper transformations from substructure 1:



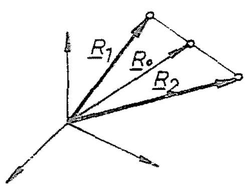
$$\underline{R}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

4/a.



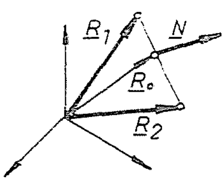
$$\underline{R}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \qquad \underline{N} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

4/b.



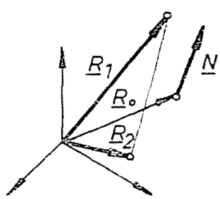
$$\underline{R}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

4/c.



$$\underline{R}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \qquad \underline{N} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

4/d.



$$\underline{R}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \qquad \underline{N} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

4/e.

Fig. 4

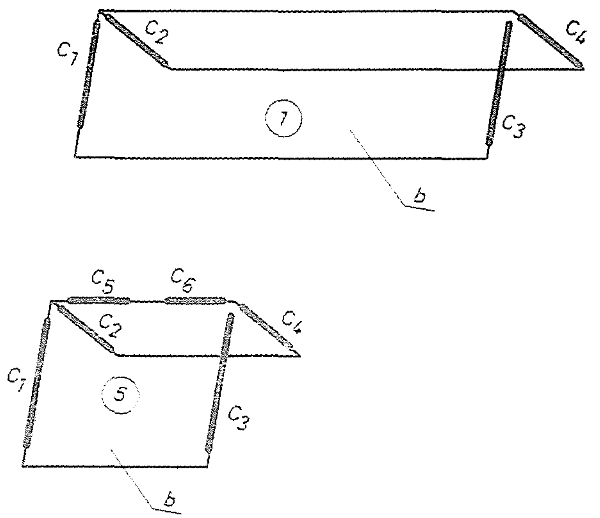


Fig. 5

I. Transformation from 1 to 2

Reflection to plane α_1:

$$R_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad N = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Transformation matrix for the reflection to the given plane:

$$\mathbf{A}_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

II. Transformation from 1 to 3

Reflection to plane α_2:

$$R_0 = \begin{bmatrix} 0 \\ -a \\ a \end{bmatrix} \quad N = \begin{bmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}.$$

Transformation matrix for the reflection to the given plane:

$$A_{13} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}.$$

III. Transformation from 1 to 4

Reflection to plane α_1, then to plane α_2:

Transformation matrix for the given transformation results as a product of the former ones:

$$\mathbf{A}_{14} = \mathbf{A}_{12}\mathbf{A}_{13} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}.$$

In possession of the reduced stiffness matrices of substructures 1 and 5, all the other matrices are simple to establish, in conformity with the above:

$$\tilde{\mathbf{K}}^1$$
$$\tilde{\mathbf{K}}^2 = \mathbf{T}_{12}^* \tilde{\mathbf{K}}^1 \mathbf{T}_{12}$$
$$\tilde{\mathbf{K}}^3 = \mathbf{T}_{13}^* \tilde{\mathbf{K}}^1 \mathbf{T}_{13}$$
$$\tilde{\mathbf{K}}^4 = \mathbf{T}_{14}^* \tilde{\mathbf{K}}^1 \mathbf{T}_{15}$$
$$\tilde{\mathbf{K}}^5 .$$

In order to fit reduced substructure stiffness matrices, stiffness equations will be partitioned in conformity with Fig. 5:

$$\begin{bmatrix} \mathbf{K}_{11}^1 & \mathbf{K}_{12}^1 & \mathbf{K}_{13}^1 & \mathbf{K}_{14}^1 \\ \mathbf{K}_{21}^1 & \mathbf{K}_{22}^1 & \mathbf{K}_{23}^1 & \mathbf{K}_{24}^1 \\ \mathbf{K}_{31}^1 & \mathbf{K}_{32}^1 & \mathbf{K}_{33}^1 & \mathbf{K}_{34}^1 \\ \mathbf{K}_{41}^1 & \mathbf{K}_{42}^1 & \mathbf{K}_{43}^1 & \mathbf{K}_{44}^1 \end{bmatrix} \begin{bmatrix} U_1^1 \\ U_2^1 \\ U_3^1 \\ U_4^1 \end{bmatrix} = \begin{bmatrix} F_1^1 \\ F_2^1 \\ F_3^1 \\ F_4^1 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{K}_{11}^5 & \mathbf{K}_{12}^5 & \mathbf{K}_{13}^5 & \mathbf{K}_{14}^5 & \mathbf{K}_{15}^5 & \mathbf{K}_{16}^5 \\ \mathbf{K}_{21}^5 & \mathbf{K}_{22}^5 & \mathbf{K}_{23}^5 & \mathbf{K}_{24}^5 & \mathbf{K}_{25}^5 & \mathbf{K}_{26}^5 \\ \mathbf{K}_{31}^5 & \mathbf{K}_{32}^5 & \mathbf{K}_{33}^5 & \mathbf{K}_{34}^5 & \mathbf{K}_{35}^5 & \mathbf{K}_{36}^5 \\ \mathbf{K}_{41}^5 & \mathbf{K}_{42}^5 & \mathbf{K}_{43}^5 & \mathbf{K}_{44}^5 & \mathbf{K}_{45}^5 & \mathbf{K}_{46}^5 \\ \mathbf{K}_{51}^5 & \mathbf{K}_{52}^5 & \mathbf{K}_{53}^5 & \mathbf{K}_{54}^5 & \mathbf{K}_{55}^5 & \mathbf{K}_{56}^5 \\ \mathbf{K}_{61}^5 & \mathbf{K}_{62}^5 & \mathbf{K}_{63}^5 & \mathbf{K}_{64}^5 & \mathbf{K}_{65}^5 & \mathbf{K}_{66}^5 \end{bmatrix} \begin{bmatrix} U_1^5 \\ U_2^5 \\ U_3^5 \\ U_4^5 \\ U_5^5 \\ U_6^5 \end{bmatrix} = \begin{bmatrix} F_1^5 \\ F_2^5 \\ F_3^5 \\ F_4^5 \\ F_5^5 \\ F_6^5 \end{bmatrix}$$

$$
\begin{bmatrix}
K^1_{11} & K^1_{12} & K^1_{13} & K^1_{14} & & & & & & & & & & \\
K^1_{21} & K^1_{22} & K^1_{23} & K^1_{24} & & & & & & & & & & \\
K^1_{31} & K^1_{32} & K^1_{33}+K^5_{11} & K^1_{34}+K^5_{12} & K^5_{13} & K^5_{14} & & & & & K^5_{16} & & & K^5_{15} \\
K^1_{41} & K^1_{42} & K^1_{43}+K^5_{21} & K^1_{44}+K^4_{44}+K^5_{22} & K^5_{23} & K^5_{24} & & & & & K^5_{26} & K^4_{41} & K^4_{42} & K^4_{43}+K^5_{25} \\
 & & K^5_{31} & K^5_{32} & K^2_{33}+K^5_{33} & K^2_{34}+K^5_{34} & K^2_{31} & K^2_{32} & & & K^5_{36} & & & K^5_{35} \\
 & & K^5_{41} & K^5_{42} & K^2_{43}+K^5_{43} & K^2_{44}+K^3_{44}+K^5_{44} & K^2_{41} & K^2_{42} & K^3_{41} & K^3_{42} & K^3_{43}+K^5_{46} & & & K^5_{45} \\
 & & & & K^2_{13} & K^2_{14} & K^2_{11} & K^2_{12} & & & & & & \\
 & & & & K^2_{23} & K^2_{24} & K^2_{21} & K^2_{22} & & & & & & \\
 & & & & & K^3_{14} & & & K^3_{11} & K^3_{12} & K^3_{13} & & & \\
 & & & & & K^3_{24} & & & K^3_{21} & K^3_{22} & K^3_{23} & & & \\
 & & K^5_{61} & K^5_{62} & K^5_{63} & K^3_{34}+K^5_{64} & & & K^3_{31} & K^3_{32} & K^3_{33}+K^5_{66} & & & K^5_{65} \\
 & & & K^4_{14} & & & & & & & & K^4_{11} & K^4_{12} & K^4_{13} \\
 & & & K^4_{24} & & & & & & & & K^4_{21} & K^4_{22} & K^4_{23} \\
 & & K^5_{51} & K^4_{34}+K^5_{52} & K^5_{53} & K^5_{54} & & & & & K^5_{56} & K^4_{31} & K^4_{32} & K^4_{33}+K^5_{55}
\end{bmatrix}
\cdot
\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \\ U_9 \\ U_{10} \\ U_{11} \\ U_{12} \\ U_{13} \\ U_{14} \end{bmatrix}
=
\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \\ F_{10} \\ F_{11} \\ F_{12} \\ F_{13} \\ F_{14} \end{bmatrix}
$$

Fig. 6

Reduced stiffness matrices of substructures 2, 3 and 4 are partitioned in a similar way. Grouping connecting points in conformity with Fig. 5, the stiffness matrix of the complete structure reduced to its connecting points will have the following built-up (for the sake of clearness, denoting $\mathbf{K}^i_{k1}$ by K^i_{k1}):

The reduced equation system can be solved, followed by calculating the nodal displacements of the substructure. In the problem concerned, stiffness matrices of two substructures (1 and 5) and their reduced are seen to be sufficient. The remaining substructures may be determined by a simple matrix multiplication—transformation. (In reductions and fittings, the fact that substructures are joined according to Fig. 5 has been made use of.)

References

1. Substructuring and Equation System Solutions in Finite Element Analysis. Computers and Structures, Vol. 7 (1977) pp. 197—206
2. PRZEMIELECKI, J. S.: Theory of Matrix Structural Analysis. McGraw-Hill, 1968
3. MICHELBERGER, P.—HORVÁTH, S.—VARGA, L.: Mechanics, V. (Annex). (In Hungarian) Tankönyvkiadó, 1985
4. HARRINGTON, S.: Computer Graphics Programming Approach. McGraw-Hill, 1985

László VARGA H-1521 Budapest