EVALUATION AND PREDICTION OF PERMANENT DEFORMATION IN DELTA WINGS

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Summary

The study evaluates the permanent deformation observed in the delta wings of high-speed aircraft in operation during levelling. The expected degree of permanent deformation is predicted by means of approximating the measuring results with a continuous Markov-chain.

Introduction

There is a certain power load, thermal load and sound load effect varying both in time and extent on up-to-date, high-speed, delta-winged aeroplanes depending on the circumstances of operation (atmospheric conditions and flight tasks) [1]. During operation these loads result in minor and major permanent (structural and plastic) deformations of the aeroplanes. The changed geometrical features call forth the change of the operating characteristics (aerodynamic and flying characteristics) and finally endanger the safe fulfilment of the flight task.

As our experiences in operation show [2], during the levelling of up-to-date, high-speed aeroplanes a significant permanent deformation can be measured—that surpasses the primarily given tolerance values. Certain operating features of the deformed aeroplanes (e.g. maximum speed, flying height) had to be limited in practice, and the geometrical deformations had to be taken into consideration when adjusting the automatopilots.

The permanent deformation of aeroplanes plays an important role not only because of its effect on the operating features, but as a measure of the wear of aeroplanes as well. Therefore it is a very important task to know the permanent deformation of aeroplanes, and to predict how it will change in time.

The present study discusses the data processing of how the geometrical deviations in the aeroplane are checked during its repair. A possible way of predicting the permanent deformation is examined, and a proposal is made for the approximate description of the geometrical deformation in the aerofoils.

Measuring the permanent deformation of aeroplanes

During operation the structure of aeroplanes undergoes macro- and microdeformations. The geometrical deviation resulting in the alteration of the geometry of the whole structural unit, or of the whole aeroplane, is usually called macrodeformation. The most important macrodeformations are the following:

- —the torsional strain of the wing and of the stabilizers,
- —the transverse strain of the wing and of the stabilizers,
- —the deformation of the tie points of the stabilizers,
- —the torsional strain of the fuselage,
- —the transverse strain (deviation) of the fuselage,
- —the deformation of the units of the aeroplane control system,
- —the deformations of the aerodynamic trimmers, plain and split flaps, etc.

The local, small extent geometrical deformation of the structure is usually called microdeformation and does not result in the deformation of the whole structure. For example the local indentation and waviness in the jacket of the wing or in the fuselage can be called microdeformations.

Naturally it is the wing where the effect of microdeformations is the most significant; their role can be especially important in the examination of boundary layer problems and in the study of flight conditions when the angles of attack are large.

The geometrical deformations of aeroplanes in operation are checked by levelling. On the aeroplanes the levelling marks can be found on typical structural units whose geometrical deformations are characteristic of macrodeformations.

The accuracy of measurement of the levelling values in aeroplanes is greatly influenced by the measuring conditions (temperature, natural features, the expertise of those carrying out the levelling, etc.). During the repair of aeroplanes the levelling is carried out under basically identical circumstances. Therefore it is advisable to examine the process of permanent deformation of a particular aeroplanes stock—changing in time—by levelling that is to be carried out anyway in each case during the repair of aeroplanes.

Figure 1 shows a part of the level book of delta wings. During our investigations the permanent deformations of delta wings in single-place, high-speed aeroplanes and in their two-place, traines variants have been evaluated on the basis of the levelling data adopted during average repairs and major overhauls. (Fig. 2). It must be remarked that the measuring results obtained before and after the repair were practically identical. Namely, there were no considerable changes in the geometrical deviations determined in Fig. 1.

The aeroplanes are repaired after a certain period of operation given with a tolerance. Accordingly, the measuring (levelling) values of the permanent deformations in certain aeroplanes of an aircraft type show a certain standard deviation around the mean periods of duty corresponding to the repairs (Fig. 2). It is easier to evaluate the permanent deformation that can be regarded as a standard of the wearing process of the aircraft type if the deviat-

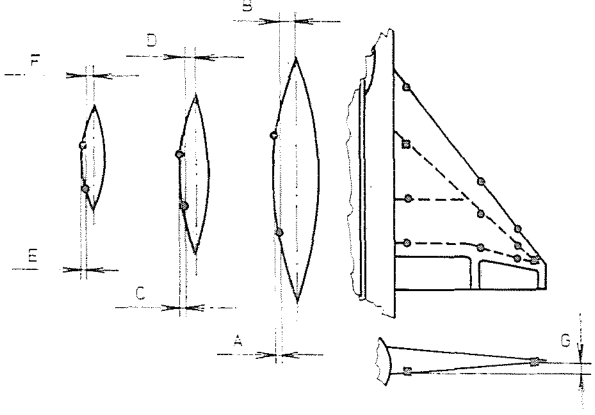

Fig. 1. The level book of the wing

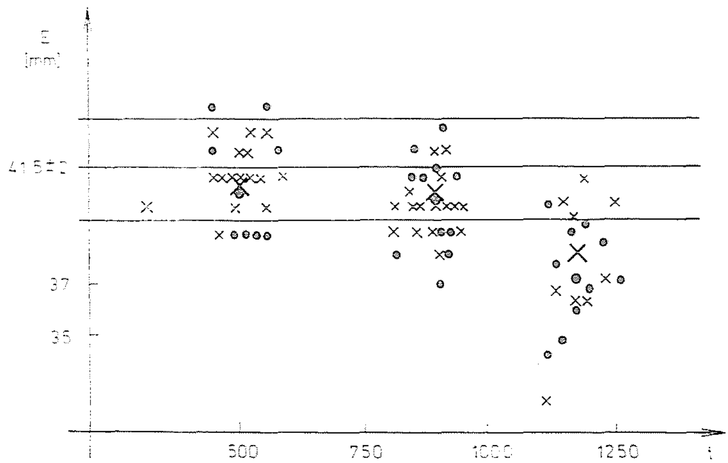

Fig. 2. The standard deviation of measuring results (x — left-hand half-wing, ● — right-hand half-wing, X, . — mean values)

ing measuring results are recalculated to the $\bar{x}_j$ average estimated deformation concerning the t_j average period of duty (flying hour) completed until the particular repair. In this way, it is fundamentally the time function of the average deformation in the type that is obtained.

For the calculations let the serial number of each repair be denoted with $j = 0, 1, 2, \ldots n$ (0 — state of production), the measuring results concerning the individual aeroplanes with $i = 1, 2 \ldots m$. Consequently, the measured values are: $t_{j,i}$; $x_{j,i}$: the concrete $x_{j,i}$ permanent deformation belonging to the $t_{j,i}$ period of duty completed until the j^{th} repair of the i^{th} aeroplane.

The average period of duty completed until the repairs is given as an arithmetical mean value:

$$\bar{t}_j = \frac{\sum_{i=1}^{m} t_{j,i}}{m} . \tag{1}$$

The $\bar{x}_j$ average, estimated deformation can be determined in two steps. First of all the $x_{j,i}$ measured values are recalculated to $\bar{t}_j$ times. Then in the second step the obtained $x_{\bar{t}_j,i}$ values are averaged.

$x_{\bar{t}_j,i}$ can be calculated by means of interpolation or extrapolation carried out on the basis of the survey data adopted during two consecutive repairs (e.g. during the fisrt and the second repair) of individuals of the aircraft type. With a view to this, let us assume that between the two consecutive repairs the permanent deformation has a certain

$$x = a \cdot e^{b.t} \tag{2}$$

exponential character. It is advisable—if possible—to determine the a and b parameters in expression (2) on the basis of the values measured during the repair under survey and the preceding one. From the

$$x_{j,i} = a \cdot e^{bt_{j,i}}, \tag{3a}$$

$$x_{j-1,i} = a \cdot e^{bt_{j-1,i}} \tag{3b}$$

system of equations the following expressions are obtained for the values of a and b:

$$a = x_{j,i} \left(\frac{x_{j,i}}{x_{j-1,i}} \right)^{\frac{-t_{j,i}}{t_{j,i}-t_{j-1,i}}}, \tag{4}$$

$$b = \ln \frac{x_{j,i}}{x_{j-1,i}} \cdot \frac{1}{t_{j,i} - t_{j-1,i}} \tag{5}$$

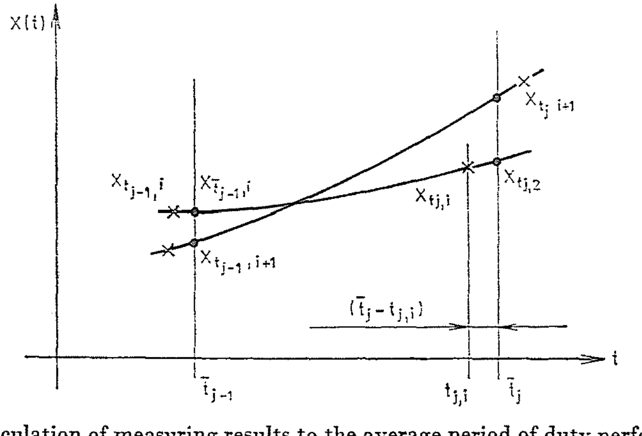

Fig. 3. The recalculation of measuring results to the average period of duty performed until the repairs (explanation in text)

Substituting the values of a, b as well as $\bar{t}_j$ in expression (2), $x_{j,i}$ measuring results can be recalculated to the t_j average period of duty completed until the j^{th} repair by means of the

$$x_{\bar{t}_j,i} = x_{j,i} \left(\frac{x_{j-1,i}}{x_{j,i}} \right)^{\frac{\bar{t}_j - t_{j,i}}{t_{j-1,i} - t_{j,i}}} \tag{6}$$

relation.

As our investigations have shown, the assumption in conformity with the proposed (2) gives satisfactory accuracy, as the approximate function crosses the measuring points; and the $|t_{j,i} - \bar{t}_j|$ values are fairly small (Fig. 3).

With a given $\bar{t}_j$, the expected value of the permanent deformation process is estimated with the arithmetic mean, its standard deviation is estimated with the corrected empirical standard deviation as follows:

$$\bar{x}_j = \frac{\sum_{i=1}^{m} x_{\bar{t}_j,i}}{m} \; ; \tag{7}$$

$$S_j^* = \sqrt{\frac{\sum_{i=1}^{m} (x_{\bar{t}_j-i} - \bar{x}_j)^2}{m}} \; . \tag{8}$$

Predicting the permanent deformation

The permanent deformation of aeroplanes is a process of natural wear, and manufacturing conditions of the aeroplane, and the circumstances of operation (use, maintenance, repair, storage, transportation, stc.). The permanent deformation is basically a determined cumulative process; however, the effects on the structure follow in a way that their chronological order and intensity cannot be predetermined (checked simply). So the permanent deformation of aeroplanes can be described with a stochastic process of continuous state and parameter field whose sampling values are measured when levelling the aeroplane. (These processes are regarded—not quite correctly—as semi-stochastic by certain authors [3]).

The problem of describing the permanent deformation that change during the operation of aeroplanes and aeroplane types, depending on time, has been neglected in professional circles. So there have been no data or recommendations available in the special literature to describe the process of deformation with an approximate function, or to predict its changes in time. During our investigations it was possible to use a set of survey data restricted to only there repairs. In a number of cases it can be supposed in practice that the aeroplanes are always used in identical circumstances and carry out similar flights. (Naturally, a significantly different case—bumpy landing, catastrophe—results in the disregard of survey data concerning the particular aeroplane.) At the same time this involves that the permanent deformation process in aeroplanes has a monotonously-increasing character as a function of the period of duty (flying hours).

In our further investigations we shall avail ourselves of the acceptable supposition that the process describing the permanent deformation in aeroplanes is free from after-effects. During a particular $[t, t + \Delta t]$ period of operation namely the extent of the permanent deformation in aeroplanes can unambiguously be determined knowing the characteristics of the permanent geometrical deformation detected at the beginning of the period examined and of the conditions of operation, consequently it is independent of the circumstances preceding the initial moment of the investigation.

So the permanent deformation process of aeroplanes is supposed to be a monotonous, non-decreasing, continuous stochastic process free from after-effects, whose field of state (expected range) is the $[x_{\min}, x_{\max}]$ interval. The survey data are the discrete sampling values of this random process denoted with $X(t)$. For the approximate description of the permanent deformation process and for the prediction of its changes in time the approximation with the Markov process is suggested in the present study—as one possible method. The advantage of the Markov processes is that they can be relatively easily used in the system of conditions outlined above and they satisfactorily characterize the dynamics of the examined process [4].

On the basis of the elaborated method, from a mathematical aspect, the approximate description of the permanent deformation in aeroplanes and the prediction of its alternation in time is the approximation of an $X(t)$ non-steady state, continuous, random process with a homogeneously continuous process having an $X^*(t)$ steady-state increment and a discrete field of state. An approximate process of this kind is called a Markov chain with a continuous field of parameters [5], or a Markov process with a discrete field of state [6].

The essence of the method is that with $\bar{x}_j$ quantization levels the expected range of the permanent deformation in the aeroplane type is divided into n, not necessarily equidistant Δx_j, in first approximation

$$\Delta x_j = \bar{x}_{j+1} - \bar{x}_j \left(j = 0, 1, \ldots n; \quad x_{\min} = \bar{x}_0 - \frac{\Delta}{4}, x_{\max} = \bar{x}_{n+1} \right) \qquad (9)$$

intervals (quanta) closed only from the left (Fig. 4). The extent of the permanent deformation is said to be corresponding to the S_j state, where the $X(t)$ permanent deformation process can be found in the j^{th} Δx_j quantum. Let us denote the absolute probability of staying in the j^{th} Δx_j quantum with $\boldsymbol{P}_j(t)$. The $m(t)$ expected value and $\sigma(t)$ standard deviation of the $X^*(t)$ approximate process derived with quantization can be expressed according to

$$m(t) = \sum_{j=0}^{n} P_j(t)\ x_j^*, \tag{10}$$

$$\delta(t) = \sqrt{\sum_{j=0}^{n} P_j(t)(x^*)^2 = [m(t)]^2}\,; \tag{11}$$

where x_j^* is the substitution value in about the middle of the Δx_j interval.

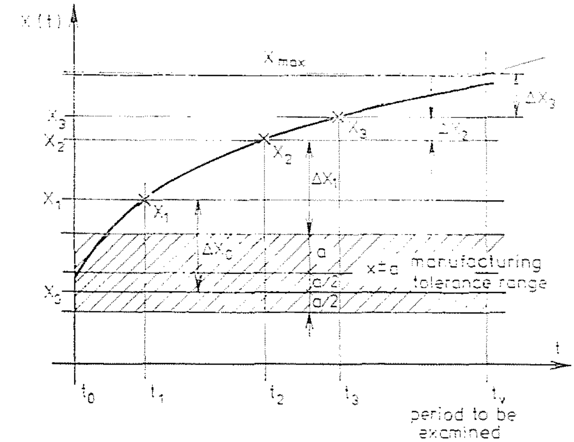

Fig. 4. Making the field of state of permanent deformation discrete (explanation in text)

From the S_j state the $X^*(t)$ approximate process can pass into the S_{j-1} state only—according to our earlier supposition of monotony. At the same time it can be supposed on the basis of the fact that the $X(t)$ process is free from after-effects, and it follows from the Markovian character of the $X^*(t)$ approximate process that the period the process stays in a given S_j state, as the T_j length of the time interval between the two consecutive changes of state, is a random variable with an exponential density function. From this it follows that the expected time (MT_j) during which the deformation process stays in a given S_j state is directly proportional to the Δt time increment. The proportion factor (λ) can be calculated on the basis of the relation

$$MT_j = \int_0^{\infty} e^{-\lambda_j t}\, dt = \frac{1}{\lambda_j} \tag{12}$$

that expresses the expected time during which the process stays in the given j^{th} interval, considering the

$$MT_j \approx \bar{t}_{j+1} - \bar{t}_j \tag{13}$$

estimation* obtained from the measuring values, namely:

$$\lambda_j = \frac{1}{\bar{t}_{j+1} - \bar{t}_j} \,. \tag{14}$$

Accordingly, the λ_j parameter is connected with the average velocity $(\lambda_j \Delta x_j)$ of the passage of the X(t) deformation process through the j^{th} interval. In other words λ_j is the intensity of the passage and crosscut of the X(t) process at the x_j^* quantizing level.

Starting now from the

$$P_j(t = 0) = P_j \tag{15}$$

initial condition, and using the

$$\sum_{j=0}^{n} P_j(t) = 1 \tag{16}$$

relation that is valid at every t time, it can be determined how probable it is to find the X^*(t) deformation process at a $t + \Delta t$ time exactly in the Δx_j quantum. Otherwise, this event can occur in the following cases excluding one another:

— at t time the process was just in the Δx_j quantum, and it will stay there during the Δt period;

— at t time the process was just in the Δx_{j-1} quantum, and during the Δt period it passes to Δx_{j-1};

— at t time the process was just in an Δx_k; $k \leq j - 2$ state, and from there it passes to Δx_j in Δt time.

Choosing a sufficiently small Δt, the probability of the latter event is negligibly small. Consequently it is the sum of the probability of the first two events excluding each other that can answer the question (4):

$$P_j(t + \Delta t) = P_j(t)(1 - \lambda_j \Delta t) + P_{j-1}(t)\lambda_{j-1}\Delta t. \tag{17}$$

From this expressing the

$$\frac{P_j(t + \Delta t) - P_j(t)}{\Delta t} = \lambda_{j-1} P_{j-1}(t) - \lambda_j P_j(t) \tag{18}$$

relation, and realizing that in case of $\Delta t \to 0$

$$\lim_{\Delta t \to 0} \frac{P_j(t + \Delta t) - P_j(t)}{\Delta t} = \frac{dP_j(t)}{dt} \,; \tag{19}$$

we obtain the

$$\frac{dP_j(t)}{dt} = \lambda_{j-1} P_{j-1}(t) - \lambda_j P_j(t) \tag{20}$$

recurrent differential equation.

* As our investigations show, this method of estimation can e.g. imply a maximum of 1,5 . . . . 3% error in case of a normal distribution of the random variable.

This equation can be stated for every interval, and then we obtain the so-called Kolmogorov system of differential equations. (Otherwise, this is "natural", as an $X^*(t)$ continuous Markov chain, or its simplest variant, the Poisson process has been substituted for the $X(t)$ deformation process.)

With this task aimed at, the approximate description of the permanent deformation process and the prediction of its expected development has practically been carried out. However, the practical investigations have shown that with the quantization of the field of state of the $X(t)$ deformation process according to (9), a certain, so-called quantizing error is introduced in the (10, 11) relations the use of an Δx_j value, in the middle of the x_j^* interval, which in itself does not result in a satisfactory accuracy. Instead, it is advisable to give the quantizing level values to be used in the (10, 11) relations in the form

$$x_j^* = \bar{x}_{j-1} - \delta(\bar{x}_{j+1} - \bar{x}_j) \tag{21}$$

e.g. with the introduction of a δ constant parameter. The δ value can be defined with the minimization of the square of the deviation (error) between the $X^*(t)$ approximate description of the permanent deformation process, the expected value of $m(t,\delta)$, and the recalculated, actually measured $\bar{x}_j$ results. Introducing the $t_j = t_i$; $\bar{x}_j = \bar{x}_i$ marking:

$$\frac{\partial}{\partial \delta}\left\{\sum_{i=1}^{n}[m(\bar{t}_i, \delta) - \bar{x}_i]^2\right\} = 0; \tag{22}$$

where at a given t_i time

$$m(\bar{t}_i, \delta) = \sum_{j=0}^{n} P_j(\bar{t}_i)[\bar{x}_{j-1} - \delta(\bar{x}_{j+1} - \bar{x}_j)]. \tag{23}$$

Performing the indicated operations, namely squaring and differentiating, transposing the resulting equation and solving it for δ the following equation is obtained:

$$\delta = \frac{\sum_{i=1}^{n}\left[\sum_{j=0}^{n} P_j(\bar{t}_i)\, x_{j+1} - x_i\right]^2}{\sum_{i=1}^{n}\sum_{j=0}^{n} P_j(\bar{t}_i)\,(x_{j+1} - x_j)}\,. \tag{24}$$

The (20) system of equations can relatively easily be solved with a Laplace transformation [7, 8] in case of $n = 3$ that corresponds to the number of survey data available in the present work:

$$P_0(t) = P_{00}e^{-\lambda_0 t}\,; \tag{25a}$$

$$P_1(t) = \frac{P_{00}\lambda_0}{\lambda_1 - \lambda_2}\,(e^{-\lambda_0 t} - e^{\lambda_1 t}) + P_{10}\,e^{-\lambda_1 t}; \tag{25b}$$

$$P_2(t) = \frac{P_{00}\lambda_0\lambda_1}{(\lambda_1 - \lambda - \lambda_0 - \lambda_1)}[(\lambda_2 - \lambda_1)e^{-\lambda_0 t}(\lambda_2 - \lambda_0)e^{-\lambda_1 t} + (\lambda_1 - \lambda_0)e^{-\lambda_2 t}] +$$

$$+ \frac{\lambda_1 P_{10}}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) + P_{20}e^{-\lambda_2 t}; \quad (25c)$$

$$P_3(t) = 1 - \sum_{j=0}^{2} P_j(t) . \quad (25d)$$

During the practical calculations it has caused problems that only the levelling values of production and operation of the aeroplanes, their tolerance ranges and the levelling values measured during their first three repairs were available. It was not possible to measure the expected range of the deformation process $[x_{min}, x_{max}]$ and the (15) initial conditions. During the calculations it was assumed that the distribution of the manufacturing measurements of the aeroplane is concentrated in the middle of the tolerance range of production. So the x_{min} has been taken up—as Fig. 4 shows—at 1/4 of the tolerance range of production. The x_{max} has been given either with the extreme value of the torelance range of production or by means of extrapolating on the basis of the average deformation values measured during the last two repairs. The (15) initial conditions have been taken up as $P_{00}(t = 0) = 1$: $P_{j0}(t = 0) = 0$ $(j = 0)$.

The concrete results of the calculation are shown in Fig. 5.

It should also be noted that at t time given because of quantization, the density function of the probability index of the X(t) permanent deformation process will differ from the normal (Gaussian) density function having σ(t) standard deviation calculated on the basis of (11). To describe a density function of this kind, differing little from the normal one, the Gramm—Charlie series is generally used [8]. The description of the deformation process can naturally be extended by using the Gram—Charlie series. This is especially important and necessary when we want to determine the percentage of the examined aeroplanes where a deformation beyond certain limits—from the aspect of the safety of flight and other aspects—appears after a certain number of operating hours, as a certain number of aeroplanes have to be ordered in advance, and later replaced in order to prevent the technical level of the stock of aeroplanes from deteriorating.

Characterization of the permanent deformation of aerofoils

Because of the permanent geometrical deformations of the airframe, the characteristic features of the aeroplane significantly change from the aspects of aerodynamics and aviation engineering. The same holds true, to a greater

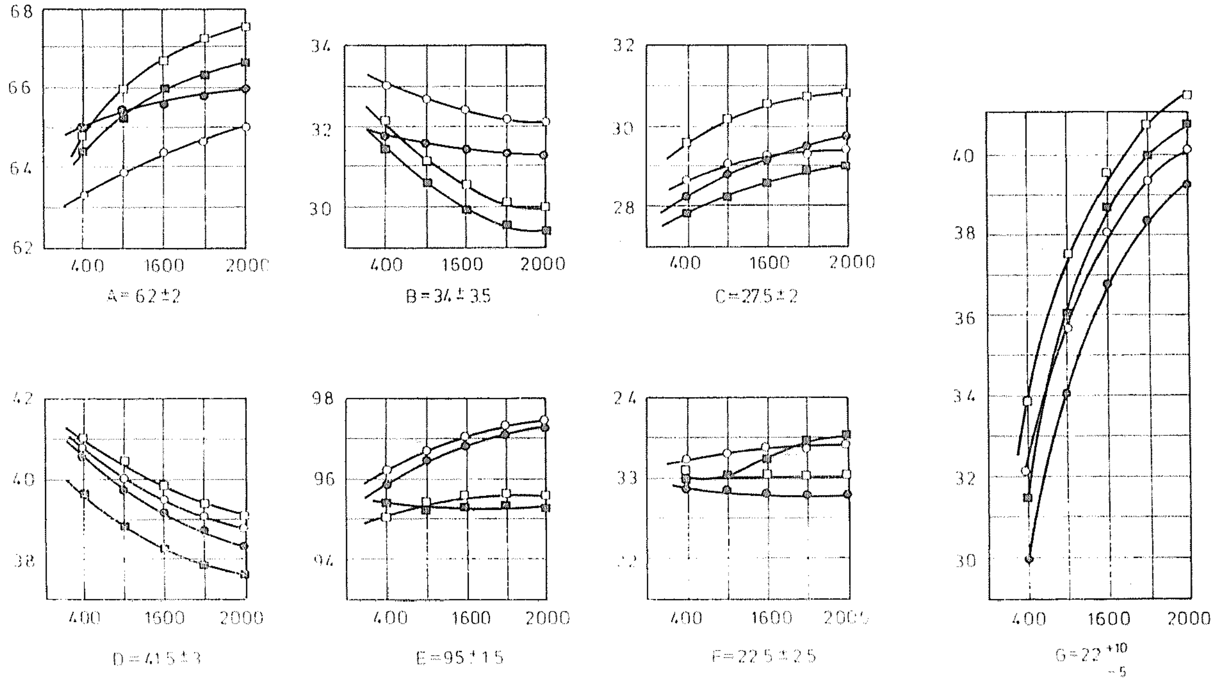

Fig. 5. **The change of the levelling values with tolerance in the wing during operation Symbols:**
○ — right-hand half-wing of a one-seater
□ — right-hand half-wing of a two-seater
⊙ — left-hand half-wing of a one-seater
▪ — left-hand half-wing of a two-seater
size in mm flying hours

extent, in respect of the deformation of air wings. To calculate this change, the deformation of air wings has to be indicated in some "manageable" form. As it is for the deformation of certain aerofoils only that survey data are available, knowing these deformations it is possible to calculate the distribution of circulation and lift force around the air wing, we confine ourselves to the approximate description of the permanent deformation in aerofoils. In order to facilitate the further calculations, the following procedure is considered to be the most usable.

Let us assume that the lower and upper outlines (surfaces) of the aerofoils are deformed to the same extent in a plane perpendicular to the median plane of the aerofoils having thin, mostly slightly cambered profiles (approximately to the plane of the chords of aerofoils). At the same time the deformation of the contour points of the aerofoil profiles in the direction of the chord of the profiles (symmetry axis) is practically negligible. With these conditions the measuring results can be regarded as the deformation of the centre line or chord of the aerofoils and this can be approximated with a parabola that inevitably crosses the front point of the profile (Fig. 6):

$$(x - x_{f0})^2 = 2p(y - f_0). \tag{26}$$

x coincides with the chord of the profile, and y is perpendicular to it, and in the XOY co-ordinate system crossing the front point of the profile they are the co-ordinates of the points of the approximate parabola. x_{f0}, f_0 co-ordinates the cusp of the approximate parabola, and the p parameter can be determined on the basis of Fig. 6 using $x_0 = 0$, $y_0 = 0$; x_1, y_1 and x_2, y_2 deformation values calculated from the measuring results as in Fig. 1:

$$x_{f_0} = \frac{y_1 x_2^2 - y_2 x_1^2}{2(y_1 x_2 - y_2 x_1)}; \tag{27a}$$

$$P = \frac{x_1^2 - 2x_1 x_{f_0}}{2y_1}; \tag{27b}$$

$$f_0 = \frac{-x_{f_0}^2}{2p}. \tag{27c}$$

Practice proves that the deformation of profiles to be measured along the chord length is of a negligibly small degreee. As Fig. 6 shows, it is possible to cut out terminal k of the profile on the approximate parabola from the $x_0 = 0$, $y_0 = 0$ point with the chord length taken up as constant. The straight line joining the terminal and the centre of the co-ordinate system can be defined as the position of the chord of the profile after the permanent deformation. That is, the permanent deformation of the profile can be considered as a process consisting of a α_d turn and the change of a Δf profile camber as compared to the deformed position of the chord. As a consequence, the permanent deformation

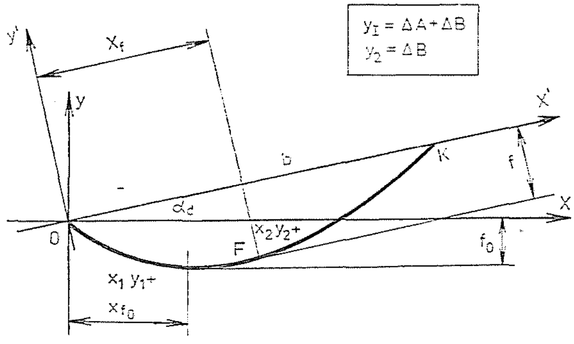

Fig. 6. Approximate description of the deformation of profile with a parabola

of the aerofoils consists of a (α_d) torsional strain and a (Δf) transverse strain. The α_d torsional strain value can be calculated with the $x = b$ (b is the chord length of the profile) substitution on the basis of the arc Sin y/x relation expressed from (26):

$$\alpha_d = \text{arc Sin}\left[\frac{f_0}{b} + \frac{(b - x_{f_0})^2}{2pb}\right]. \tag{28}$$

As Figure 6 shows, a new co-ordinate system can be taken up according to the deformed state of the profile. The new $X'\ OY'$ co-ordinate system can be produced by turning off the original one with an α_d angle. The most distant F point of the approximate parabola is at an f distance from the new X' axis. It is this f value that causes the maximum camber-change of the deformed profile. It is easy to see, and can be proved with calculations as well, that the x_f value giving the place of point K in the $X'OY'$ co-ordinate system is the exact equal of the half of the chord length. ($xl = b/2$). At the same time value f can approximately be defined on the basis of the

$$f \approx y\,|_{x \sim x_f} + x(\approx x_f)\ \text{Sin}\ \alpha_d \tag{29}$$

relation, as the α_d is a fairly small value. To determine the $\Delta\bar{f} = f/b$ relative camber-change of the profile related to the chord length of the profile, characterizing the transverse strain the following relation was obtained after simple mathematical steps:

$$\Delta\bar{f} = \frac{f_0}{b} + \frac{(0{,}5b - x_{f_0})^2}{2pb} + 0{,}5\ \text{Sin}\ \alpha_d. \tag{30}$$

In the XOY co-ordinate system referring to the unstrained, initial state the Δy deformation of the chord of the profile, and at the same time of its outline can be given in the following simple form as well, by means of α_d and $\Delta\bar{f}$:

$$\Delta y = b\Delta\bar{f} + \frac{(x - 0{,}5b)^2}{2p} + x\ \text{Sin}\ \alpha_d. \tag{31}$$

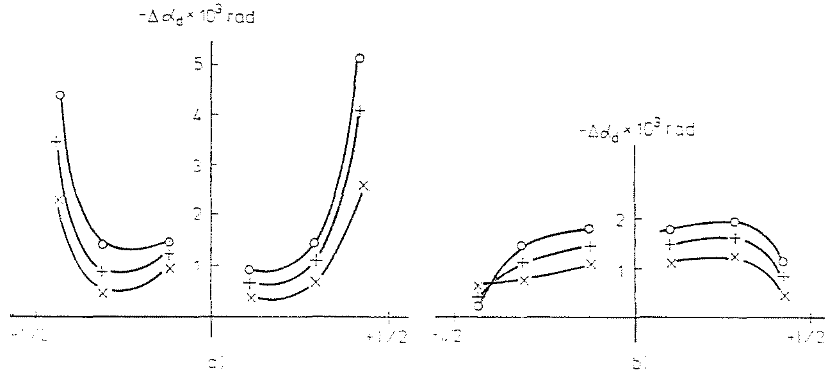

Fig. 7. Torsional strain of aerofoils $\Delta\alpha_d$ during operation a.) in one-seaters, b.) in two-seaters (after x — 400, + — 1200, ○ — 200 flying hours, l — the wing span)

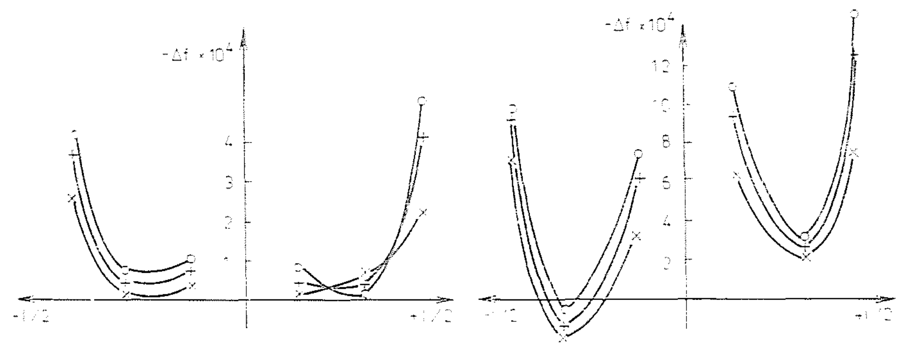

Fig. 8. The change of the camber of aerofoils (Δf) during operation a.) in one-seaters, b.) in two-seaters (after x — 400, ¼ — 1200, ○ — 200 flying hours, l — the wing span)

In Figs 7, 8 the relative camber of the aerofoils ($\Delta\bar{f}$) and their torsional (α_d) strains can be seen as a function of the flying time (t) and wing span (l).

It is remarkable that to check this way of the approximate description of deformation, concrete measurements have been carried out, too. First of all the permanent deformation of the aerofoils was defined from the levelling values. Then, using these values, the deformations of the aerofoils were described with different approximate functions. Later on the calculation results were compared to the deformation values measured at 20 points on the outline of the particular aerofoils. From among the examined approximate functions it was the parabolic approach outlined above that yielded the best, practically accurate results.

Evaluating the measurement and calculation results

During our investigations it was established that the survey data adopted when levelling an aeroplane can be used as primary information to determine the permanent deformation of the airframe. When studying an aeroplane type, the levelling results are characteristic of the average deformation of the type and they deviate considerably. (Fig. 2). Approximating the stochastic process representing the permanent deformation of the wings with a discrete Markov process it is possible to describe the deformation process approximately and predict its expected formation in time. From the measurement results processed like this, it can be seen that the permanent deformation of delta wings consists of

—the upward inclination of the wing toes, (Fig. 5)
—the torsional deflection of the aerofoils in the direction of the decrease of the attack angle (Fig. 7),
—the aerofoils becoming negatively cambered (Fig. 8).

From Fig. 5 it can be seen that as early as during the initial period of operation a relatively large permanent deformation can be observed. This can unambiguously be accounted for with the so-called alignment of the airframe. (During the initial period of operation, after some flying hours the airframe undergoes a permanent structural deformation that results from a slight displacement of the structural elements.) From Fig. 5 it can again be well seen that the right-hand and left-hand half-wings, as well as the wings of the single-seaters and two-seaters are deformed in a considerably different way. These divergencies result from the differences in aerial operation, and the characteristic features of aerial operation (mostly left turn realization).

From Figs 7 and 8 it is obvious that the permanent deformation of delta wings is not a monotonic variable along the wing span. It is possible because of the great forces arising in the tie points of the main landing gears that exert an effect on exactly the spot in question. At the same time it means that the extent of the permanent deformation in the wing also depends on the circumstances of operation (bumpy landing, the frequency of landings).

In Figs 7 and 8 the difference between the deformation of the right-hand and left-hand half-wings is even more conspicuous. Such considerable difference can no longer be accounted for with the characteristic features of the operation alone. The reason for this can be found in the pilot's (man's) physiological features. For, as experience during the operation shows, pilots prefer left turns, and most of them land by heeling the aeroplane over to the left. This fact is supported with a considerable amount of survey data in [9]. Moreover, as [9] says, during landing and at the very moment of landing the aeroplane not only heels over mostly to the left, but in the majority of cases it has a left-side angular velocity.

The measuring and calculating results show that the extent of the permanent deformation in delta wings is almost identical with the elastic deformation values of the wings [10].

It has to be pointed out above all that in this case in the approximate description of the permanent deformation we set out from the supposition that "the aeroplane is always used to perform almost identical, similar flights". However, this supposition does not prove correct in every case. If it is well-known that during the operation of the aeroplanes there is a considerable change in the circumstances of operation, it is not enough to approximate the stochastic process describing the deformations with the process of birth alone. The approximate description with the processes of birth and decay results in complex formulas requiring ever-increasing calculation time as the equivalent of attainable accuracy.

Conclusions

It is relatively easy to provide an approximate description of the permanent deformation in aeroplanes, and to characterize its expected formation on time, by means of approximating the levelling data with a Markov chain of a continuous field of parameters.

The permanent deformation in the delta wings of up-to-date, high-speed aeroplanes is relatively great (the turn of the aerofoils in a direction where the incidence angle decreases—0,005° . . . 0,1°, the change of their relative camber—0,06 . . . 0,12 in a negative direction), and it changes considerably as the flying time increases.

The extent and form of permanent deformation in delta wings greatly depends on the duty of the aeroplane to be carried out (the deformation of two-seaters is about twice as great as that of one-seaters), on the flying time, on the circumstances of operation, on the technique of piloting the aeroplane (especially on the landing mode of operation), and on man's physiological characteristics.

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