

IDENTIFICATION OF THE HYDRODYNAMIC DRIVE-SYSTEM CHARACTERISTIC CURVES

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Summary

The torque- and power-transmission properties of hydrodynamic drive-systems are judged on basis of the steady-state characteristics, in the overwhelming majority of practical applications. But with the greater part of the hydrodynamic drive-systems, there often occur nonsteady operational states, too, in the course of their normal operation, and these very operational states are the decisive ones with respect to dynamic excess loads. In this paper, the general relationships of the identification of the characteristics of hydrodynamic drive-systems are formulated and a numerical procedure is described for the approximation of the nonsteady torques by means of the method of least squares.

The Investigation Model

The scheme of the hydrodynamic drive-system is shown in Fig. 1. This system is made up of prime mover E , hydrodynamic turbo-transmission H and unit T applying load (braking). On the input-shaft of the hydrodynamic turbo-transmission, speed n_1 and torque M_1 , while on the output-shaft speed n_2 and torque M_2 have been developed, respectively. The direction of the energy flow is indicated by arrows. From a system-theoretical point of view, the hydrodynamic turbo-transmission can be identified as the transfer member shown in Fig. 2. Speeds n_1 and n_2 are considered as the two input characteristics, while torques M_1 and M_2 represent the two output characteristics.

Speed-ratio:

$$i \stackrel{\text{def}}{=} \frac{n_2}{n_1}; \quad (n_1 \neq 0) \tag{1}$$

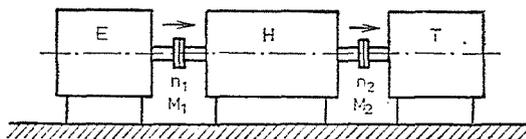


Fig. 1

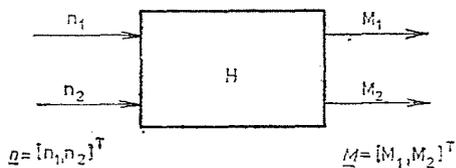


Fig. 2

is introduced as the quotient of speeds n_1 and n_2 of the hydrodynamic turbo-transmission. In the steady-state operation of the hydrodynamic turbo-transmission of a given layout, the torques applied on the input and output shafts can be described with a good approximation by means of the accepted expressions [1]:

$$M_1 = K_1(i) n_1^2 \quad (2)$$

and

$$M_2 = K_2(i) n_2^2. \quad (3)$$

The torque-coefficient functions $K_1(i)$ and $K_2(i)$ can be determined from the transmission measurements. But functions $K_1(i)$ and $K_2(i)$ are not independent from each other. If the generally accepted torque-ratio (multiplication):

$$k \stackrel{\text{def}}{=} \frac{M_2}{M_1} \quad (4)$$

is introduced, then by substituting (2) and (3) the following expression is received:

$$k = \frac{M_2}{M_1} = \frac{K_2(i) n_2^2}{K_1(i) n_1^2} = \frac{K_2(i) i^2}{K_1(i)}, \quad (5)$$

i.e. k is also a function of i ($k = k(i)$). From the foregoing it follows:

$$K_2(i) = \frac{K_1(i) k(i)}{i^2}. \quad (6)$$

Since in practical cases, functions $K_1(i)$ and $K_2(i)$ are continuous and take bounded-values at $i = 0$, therefore

$$\lim_{i \rightarrow 0} K_2(i) = \infty \quad (7)$$

As a preparatory task for the identification of the characteristic curve, it should be considered that the smooth variation of function $K_1(i)$ enables it to be approximated by the polynomial form of the l^{th} power of variable i.e.:

$$K_1(i) \approx a_0 + a_1 i + \dots + a_l i^l. \quad (8)$$

Similarly, if the torque-ratio (multiplication) function $k(i)$ is approximated in the form of

$$k(i) \approx b_0 + b_1 i + \dots + b_m i^m \quad (9)$$

on the basis of expression (6), with $r = l + m$ being in force as for $K_2(i)$, the following relationship is obtained:

$$K_2(i) \approx \frac{c_0}{i^2} + \frac{c_1}{i} + c_2 + c_3 i + \dots + c_r i^{r-2}. \quad (10)$$

So the following relationships are in force with a good approximation:

$$M_1 = K_1(i)n_1^2 \approx \left(\sum_{j=0}^l a_j i^j \right) n_1^2 = \left(\sum_{j=0}^l a_j \left(\frac{n_2}{n_1} \right)^j \right) n_1^2, \quad (11)$$

$$M_2 = K_2(i)n_2^2 \approx \left(\sum_{j=0}^r c_j i^{j-2} \right) n_2^2 = \left(\sum_{j=0}^r c_j \left(\frac{n_2}{n_1} \right)^{j-2} \right) n_2^2. \quad (12)$$

In this way, the mathematical model required to identify the steady-state turbo-transmission characteristic curves is provided by the expression received in (11) and (12)*.

There is a possibility to determine the unknown coefficients a_0, a_1, \dots, a_l and c_0, c_1, \dots, c_r contained in relationships (11) and (12), with the use of the real quaternions n_1, n_2, M_1 and M_2 received by measurements. With vectors:

$$\mathbf{n} = [n_1, n_2]^T \text{ and } \mathbf{M} = [M_1, M_2]^T$$

being introduced as the input and output characteristics of the transfer member to be identified (Fig. 2.), and with the parameter-vectors: $\mathbf{a} = [a_0, \dots, a_l]^T$ and $\mathbf{c} = [c_0, \dots, c_r]^T$ being used, respectively, the minimum problem**

$$\bar{\Phi}(\mathbf{a}, \mathbf{c}) = \sum_{(s)} (\mathbf{M}_s - \mathbb{H}(\mathbf{n}_s, \mathbf{a}, \mathbf{c}))^2 = \min ! \quad (13)$$

can be formulated, the solution of which results in receiving the optimum parameter-vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{c}}$.

Vector-function \mathbb{H} in (13) can be considered as a concise form of relationships (11) and (12), since it gives the mathematical model of the identification mentioned above.

The objective function (13) can be basically considered as an optimum criterion formulated by means of the method of least squares. This criterion provides the required optimum parameters by the solution of the linear algebraic set of equations determined through the vanishing of the partial derivatives of function $\bar{\Phi}(\mathbf{a}, \mathbf{c})$.

The identification of the nonsteady-state characteristic curves of the hydrodynamic turbo-transmission can be carried out by expanding the statements made in the foregoing, as described further on.

The general problem of the characteristic curve identification

An investigation model containing the nonsteady-state operating conditions, too, is obtained if the expressions reflecting the time-dependencies of speeds n_1 and n_2 are added to the expressions (11) and (12) taken from the

* As for the detailed explanation of the theoretical basis for the system identification, see [4] and [5].

** In (13) subscript s identifies the couples of torques and speeds measured at the same instant.

foregoing chapter. This involves that the derivatives $\dot{n}_1, \ddot{n}_1, \dots$ and $\dot{n}_2, \ddot{n}_2, \dots$ with respect to time are to be included in both expressions but with unknown coefficients.

According to the above said, it can be written:

$$M_1 \approx H_1(n_1, n_2, \mathbf{a}) + H_1^*(n_1, \dot{n}_1, \ddot{n}_1, \dots, n_2, \dot{n}_2, \ddot{n}_2, \dots, \mathbf{A}_1, \mathbf{C}_1) \quad (14)$$

and

$$M_2 \approx H_2(n_1, n_2, \mathbf{e}) + H_2^*(n_1, \dot{n}_1, \ddot{n}_1, \dots, n_2, \dot{n}_2, \ddot{n}_2, \dots, \mathbf{A}_2, \mathbf{C}_2) \quad (15)$$

where H_1 and H_2 are the two coordinate-functions of the vector-function \mathbb{H} given in (13), while H_1^* and H_2^* are the two coordinate-functions characterizing the nonsteady-state behaviour.**

Functions H_1^* and H_2^* in (14) and (15) are reckoned through Taylor-series when setting the identification problem. The first few members of Taylor's expansion are concerned with the identification problem. Thus e.g. with the linear members retained:

$$H_1^* \approx A_{11}\dot{n}_1 + A_{12}\ddot{n}_1 + \dots + C_{11}\dot{n}_2 + C_{12}\ddot{n}_2 + \dots \quad (16)$$

$$H_2^* \approx A_{21}\dot{n}_1 + A_{22}\ddot{n}_1 + \dots + C_{21}\dot{n}_2 + C_{22}\ddot{n}_2 + \dots \quad (17)$$

With the constant coefficients in expressions (16) and (17) included in parameter-vector $\mathbf{A}_1, \mathbf{C}_1, \mathbf{A}_2$ and \mathbf{C}_2 , the linearized identification problem can be set in the following way using the mathematical model:

$$\mathbb{M} \approx \mathbb{H}_m(n_1, \dot{n}_1, \ddot{n}_1, \dots; n_2, \dot{n}_2, \ddot{n}_2, \dots; \mathbf{a}, \mathbf{e}, \mathbf{A}_1, \mathbf{C}_1, \mathbf{A}_2, \mathbf{C}_2). \quad (18)$$

According to Fig. 3, let us assume that time-function $n(t)$ as an input process arrives at the input of transfer system outlined here. The input process

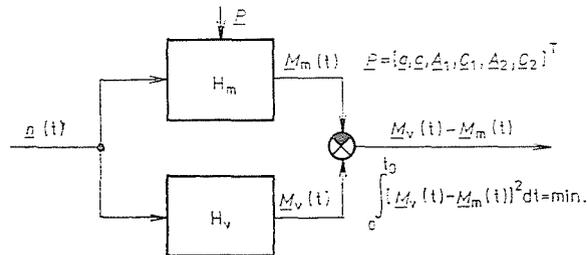


Fig. 3

arrives at the input of the mathematical model with a transfer property determined by formula (18) and at the input of the member with a transfer property H_v , as representing the real drive-system, through the input branching

** In expressions (14) and (15), respectively, functions H_1 and H_2 can depend on n_1 and n_2 , respectively, only in a way that their steady-state values be equal to zero.

in an unchanged magnitude. The above mentioned problems can be imagined in a way that the realization of the torque response process is carried on simultaneously in the mathematical model with a transfer property H_m and in the real system with a transfer property H_v , i.e. the response process $\bar{M}_m(t)$ realized in the mathematical model is yielded by calculations and the response process $M_r(t)$ of t-e real system is yielded by measurements.

The difference between the two response processes should be minimized by means of an optimal selection of the parameters in vector function \bar{H}_m . So the objective function can be written as follows:*

$$\bar{\Phi}(a, c, A_1, C_1, A_2, C_2) = \int_0^{t_0} (M_r(t) - \bar{M}_m(t))^2 dt = \min ! \quad (19)$$

So the optimum criterion is set again on the principle of the method of least squares but now it involves the minimum value according to the norm of function-space L_2 because of the time-functions used here, i.e. with the parameters optimized, the function $\bar{M}_m(t)$ lies closest to the real torque response function $M_v(t)$ in the mean square sense.

The extremization problem formulated in (19) can be solved through the numerical solution of the algebraic set of equations determined by the vanishing of the partial derivatives of function $\bar{\Phi}$.

The solution of the identification problem can be carried out expediently in two steps. As the first step, the parameter-vectors a and b of vector-function \bar{H} in equation (13) can be determined in the knowledge of measurement results of the steady-state operating conditions. As a second step, with the help of the measurement results received under nonsteady-state operating conditions, the components of the yet unknown parameter-vectors A_i and C_i of vector-function \bar{H}_m in (18) satisfying the minimum criterion in (19) can be calculated.

Identification of the steady-state characteristic curves

In the knowledge of the steady-state characteristic curves of the hydrodynamic drive-system, coefficients \hat{a}_i and \hat{c}_i of the torque functions satisfying the minimum criterion in (13) can be determined. For the measuring investigations of the steady-state characteristic curves, torque values M_1 and M_2 are generally determined at discrete speed-ratios of i calculated from steady-state speeds n_1 and n_2 related to the given operating states. For carrying out proper calculations, the minimum criterion in (13) is used in the form of two scalar equations, since only torque coordinate M_1 is influenced by parameter-vector a , and only torque coordinate M_2 by parameter-vector c . Accordingly, the identification of the two torque-functions can be carried out separately.

* In (19) t_0 stands for the upper limit of the time interval used as a basis of identification.

Identification of the primary-side steady-state torque-function

According to the above, the first equation obtainable from criterion (13), with the measurement results assumed of the discrete operating states N_1 in number, and with substituting relationship (11), can be written in the following form:

$$\Phi_1(a) = \sum_{s=1}^{N_1} \left(M_{1s} - \sum_{j=0}^l a_j n_{2s}^j n_{1s}^{2-j} \right)^2 = \min! \quad (20)$$

Further on, let the partial derivatives of function Φ_1 be formed with respect to a_j ; $j = 0, 1, 2, \dots, l$ which should be of zero value because of the minimum criterion. In a general case, the set of equations

$$\frac{\partial \Phi_1(a)}{\partial a_j} = - \sum_{s=1}^{N_1} 2 \left(M_{1s} - \sum_{k=0}^l a_k n_{2s}^k n_{1s}^{2-k} \right) n_{2s}^j n_{1s}^{2-j} = 0, \\ j = 0, 1, 2, \dots, l \quad (21)$$

is obtained, which is a linear, inhomogeneous set of equations with respect to parameters a_j . With the introduction of matrix notations, the following relationship can be written:

$$\mathbf{B}_1 \mathbf{a} = \mathbf{d}_1 \quad (22)$$

where the form of matrix \mathbf{A}_1 of a size $l \times l$ will be:

$$\mathbf{B}_1 = \begin{bmatrix} \sum n_{1s}^4 & \sum n_{1s}^3 n_{2s} & \dots & \sum n_{1s}^{4-l} n_{2s}^l \\ \sum n_{1s}^3 n_{2s} & \sum n_{1s}^2 n_{2s}^2 & \dots & \sum n_{1s}^{3-l} n_{2s}^{l+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum n_{1s}^{4-l} n_{2s}^l & \sum n_{1s}^{3-l} n_{2s}^{l+1} & \dots & \sum n_{1s}^{4-2l} n_{2s}^{2l} \end{bmatrix} \quad (23)$$

while the l -dimension vector on the right side can be written as follows:

$$\mathbf{d}_1 = [\sum n_{1s}^2 M_{1s}, \sum n_{1s} n_{2s} M_{1s}, \dots, \sum n_{1s}^{2-l} n_{2s}^l M_{1s}]^T \quad (24)$$

The summations in expressions (23) and (24) are related to index s . The optimum values \hat{a}_j of the constants in the expression of torque function M_1 are provided by the solution of a set of equations (22).

Identification of the secondary-side steady-state torque-function

The determination of the parameters of secondary-side torque-function M_2 is accomplished in a way quite similar to that of the primary-side. Let the measured secondary torque be known at the measuring points N_2 in number. In this way, the following minimum problem is determined:

$$\Phi_2(c) = \sum_{s=1}^{N_2} \left(M_{2s} - \sum_{j=0}^r c_j n_{1s}^{2-j} n_{2s}^j \right)^2 = \min! \quad (25)$$

by the second equation obtainable from minimum-criterion (13) considering (12). With the partial derivatives of function $\Phi_2(\mathbf{c})$ formed with respect to variables c_j ; $j = 0, 1, 2, \dots, r$ which should be of zero value because of the minimum criterion, again a linear set of equations is received which can be written in the following form using matrix notations:

$$\mathbf{B}_2 \mathbf{c} = \mathbf{d}_2 \quad (26)$$

Here

$$\mathbf{B}_2 = \begin{bmatrix} \Sigma n_{1s}^4 & \Sigma n_{1s}^3 n_{2s} & \dots & \Sigma n_{1s}^{4-r} n_{2s}^r \\ \Sigma n_{1s}^3 n_{2s} & \Sigma n_{1s}^2 n_{2s}^2 & \dots & \Sigma n_{1s}^{3-r} n_{2s}^{r+1} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma n_{1s}^{4-r} n_{2s}^r & \Sigma n_{1s}^{3-r} n_{2s}^{r+1} & \dots & \Sigma n_{1s}^{4-2r} n_{2s}^{2r} \end{bmatrix} \quad (27)$$

and

$$\mathbf{d}_2 = [\Sigma n_{1s}^2 M_{2s}, \Sigma n_{1s} n_{2s} M_{2s}, \dots, \Sigma n_{1s}^{2r} n_{2s}^r M_{2s}]^T. \quad (28)$$

The summations in relationships (27) and (28) are related to index s .

The optimum values \hat{c}_j of the parameters in the expressions of torque function M_2 are provided by the solution of the linear set of equations (26).

If functions M_1 and M_2 are to be approximated by the equal number of members ($l = r$), as well as, if the measured primary and secondary-side torques are related to the same operating states with the same index s , then $\mathbf{B}_1 = \mathbf{B}_2$. It should be noted, however, that more simple calculations are involved if during the measurements, the steadiness of primary-side speed n_1 can be ensured by the regulation of the driving prime mover because, in this case, n_1 becomes independent from index s identifying the operating states and its powers can be factored out of the sum-expressions determining the matrix elements.

Identification of the nonsteady-state characteristic curves

For the identification process of the nonsteady-state primary and secondary torques in expressions (14) and (15), respectively, the optimum parameter-vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{c}}$ valid for the steady-state operating conditions determined in the foregoing are assumed to be known, as also the steady-state torque-components H_1 and H_2 . The nonsteady-state torque deviations can be calculated as the difference between real torque-time functions $M_1(t)$ and $M_2(t)$ measured in a sufficiently long* time-interval $[0, t_0]$ and the steady-state torques calculable from the actual values of time-functions $n_1(t)$ and $n_2(t)$ according to (11) and (12) with the help of parameter-vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{c}}$ as follows:

$$\Delta M_1(t) = M_1(t) - H_1(n_1, n_2, \hat{\mathbf{a}}) \quad (29)$$

$$\Delta M_2(t) = M_2(t) - H_2(n_1, n_2, \hat{\mathbf{c}}).$$

In the following, identification represents the approximation to these torque "deviations" with the help of functions H_1^* and H_2^* according formulae (16) and (17). In this way, the optimum-criterion in (19) can also be written by means of the following two scalar conditions:

$$\begin{aligned} \Phi_{\text{lin}}(\mathbf{A}_1, \mathbf{C}_1) &= \int_0^{t_0} (\Delta M_1 - (A_{11}\dot{n}_1 + A_{12}\ddot{n}_1 + \dots + C_{11}\dot{n}_2 + C_{12}\ddot{n}_2 + \dots))^2 dt = \\ &= \min! \end{aligned} \quad (30)$$

and

$$\begin{aligned} \Phi_{\text{2in}}(\mathbf{A}_2, \mathbf{C}_2) &= \int_0^{t_0} (\Delta M_2 - (A_{21}\dot{n}_1 + A_{22}\ddot{n}_1 + \dots + C_{21}\dot{n}_2 + C_{22}\ddot{n}_2 + \dots))^2 dt = \\ &= \min! \end{aligned} \quad (31)$$

In a way similar to the identification of the steady-state torques, the identification of the nonsteady-state torques at the primary and secondary side can also be carried out separately.

Identification of the nonsteady-state primary torques

The identification of the nonsteady-state primary torque involves the determination of parameter vectors \mathbf{A}_1 and \mathbf{C}_1 satisfying minimum criterion (30). In order to achieve this, the necessary partial derivatives with respect to the components of parameter vectors \mathbf{A}_1 and \mathbf{C}_1 should be formed in turn, which must be equal to zero according to the minimum criterion. Thus e.g.

$$\begin{aligned} \frac{\partial \Phi_{\text{lin}}(\mathbf{A}_1, \mathbf{C}_1)}{\partial A_{11}} &= - \int_0^{t_0} 2(\Delta M_1 - (A_{11}\dot{n}_1 + A_{12}\ddot{n}_1 + \dots + C_{11}\dot{n}_2 + C_{12}\ddot{n}_2 + \dots)) \times \\ &\quad \vdots \quad \times \dot{n}_1 dt = 0 \end{aligned} \quad (32)$$

$$\begin{aligned} \frac{\partial \Phi_{\text{lin}}(\mathbf{A}_1, \mathbf{C}_1)}{\partial C_{11}} &= - \int_0^{t_0} 2(\Delta M_1 - (A_{11}\dot{n}_1 + A_{12}\ddot{n}_1 + \dots + C_{11}\dot{n}_2 + C_{12}\ddot{n}_2 + \dots)) \times \\ &\quad \vdots \quad \times \dot{n}_2 dt = 0 \quad \vdots \end{aligned}$$

With the integral resolved into members and the constants factored out, a linear set of equations results expressed in the usual matrix-form as follows:

$$\mathbf{B}_{\text{in}} \alpha_1 = \Delta u_1 \quad (33)$$

* The length of the measurement interval is to be determined so that the normally expectable operating conditions should be represented by the operating states and operating state-variations realized within the time-interval in the drive-system.

where

$$\mathbf{B}_{in} = \begin{bmatrix} \int \dot{n}_1^2 dt & \int \dot{n}_1 \ddot{n}_1 dt \dots \int \dot{n}_1 \dot{n}_2 dt & \int \dot{n}_1 \ddot{n}_2 dt \dots \\ \vdots & \vdots & \vdots \\ \int \dot{n}_1 \dot{n}_2 dt & \int \ddot{n}_1 \dot{n}_2 dt \dots \int \dot{n}_2 \dot{n}_2 dt & \int \dot{n}_1 \ddot{n}_1 dt \dots \\ \vdots & \vdots & \vdots \end{bmatrix} \quad (34)$$

and

$$\alpha_1 = [A_1, C_1]^T, \quad (35)$$

$$\Delta\psi_1 = [\int \Delta M_1 \dot{n}_1 dt, \dots, \int \Delta M_1 \dot{n}_2 dt, \dots]^T. \quad (36)$$

The integral-signs indicate an integration in the whole domain $[0, t_0]$. The optimum parameter values are contained by solution-vector $\hat{\alpha}_1$ received as the solution of the set of equations.

Identification of the nonsteady-state secondary torque

The identification of the nonsteady-state secondary torque involves the determination of vectors A_2 and C_2 representing the solution of the minimum problem (31). Basically, this takes place in a way quite similar to that used in describing the identification of the primary torque. Inasmuch, derivatives of the same order are considered in both cases of identification, the coefficient-matrix of the resulting set of equations will be common with them, so the linear set of equations

$$\mathbf{B}_{in} \alpha_2 = \Delta\mu_2 \quad (37)$$

is obtained where

$$\alpha_2 = [A_2, C_2]^T, \quad (38)$$

$$\Delta\mu_2 = [\int \Delta M_2 \dot{n}_1 dt, \dots, \int \Delta M_2 \dot{n}_2 dt, \dots]. \quad (39)$$

The optimum values of the parameters in the nonsteady-state torque function are provided by the solution-vector $\hat{\alpha}_2$ of equation-system (37).

Concluding remarks

The application of the identification process described here and the numerical method involved enable the results of the experimental investigations of the hydrodynamic drive-systems to be evaluated by means of computer-technique within the frame of a uniform mathematical model for both steady-state and time-dependent operating conditions.

With the elaboration of the identification method, the concept of the speed-ratio generally used in the theory of hydrodynamic drive-systems for the description of the steady-state operation was taken as a starting point.

But with the description of the nonsteady-state operation, the speed-ratio loses its emphasized significance, and owing to the difficulty in treatment, it is more expedient to consider the torque characteristics as functions depending on the input and output speeds.

The resulting relationships were derived in the frame of a polynom model but the essence of the train of thoughts applied here will be retained in the case of the application of mathematical models based on functions of an other type as well. With the Taylor-series expansion applied, in the end, an algebraic set of equations is obtained to determine the optimum parameters.

Concerning the further development of the identification method discussed here, it can be stated that a more complete description of the nonsteady-state processes is received if the inner flow processes of the hydrodynamic drive-gear are taken into consideration when constructing the mathematical model. In connection with this, reference [3] and the investigations under way at the Department of Vehicle Engineering at the Budapest Technical University, are referred to. The results of the latter investigations show that, in certain cases, (when the rows of blades within the hydrodynamic element are relatively far from each other), the hydrodynamic drive-system should be modelled as a system having a memory, and this should be taken into consideration with the identification process.

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