THE STRESS AND STRAIN OF THE RUBBER BELTING OF BELT CONVEYORS I

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Received October 30, 1984
Presented by Prof. Dr. P. Michelberger

Summary

Among the material handling machines suitable for the transportation of both bulk goods and parcels the belt conveyor is one of the most common ones. One of its special part, the belt, is often made of cord embedded in rubber. The article analyzes the connection of structure and elastic behaviour of rubber belts on the basis of the interaction of matrix and reinforcement. Mechanical loads appearing under the influence of load are listed, the continuous material models of belts are introduced. All the examinations are based on the connection of rubber and reinforcement taking the mechanical guiding role of cord into account. The elastic behaviour of conveyor belts is examined on the ideal cord rubber material models. These models according to the deformation of reinforcing frame are in the group of elastic continuity reinforced by tensile ($\varepsilon_k \lessgtr 0$) and intensile cords ($\varepsilon_k = 0$). Nowadays the belts reinforced by steel cord are spreading widely in the field of material handling. In belts produced from a steel rubber material-pair, the Young modulus ratio of the components is $7 \cdot 10^4$. This figure on the one hand supports specific elastic behaviour of a complex system and on the other calls attention to a high modulus ratio on the boundary surface of the two materials.

Introduction

Among the material handling machines suitable for bulk material, piece goods and persons the belt conveyor with rubber belting is one of the most wide-spread ones. One of its special parts, the belt is produced from rubber strengthened with cord.

In the following the relation of the structure of rubber beltings and their elastic behaviour is analyzed on the basis of the interaction between the matrix and the strengthening. The mechanical loads developing as a result of loads are listed and the continuum material models of belts are introduced. All the tests are based on the relation of rubber and strengthening by taking the mechanical control role of the cord material into consideration.

Structure of the belt

The belts of conveyors are usually made from reinforcing insert embedded in vulcanized rubber. The most common reinforcing material is cord produced from warp and weft cord threads. According to the principle of uniform strength, the cord materials are embedded in a way that the axis of warp threads should be parallel to the longitudinal axis of the belt, x_1 and that of the weft threads to the lateral axis, x_2 (Fig. 1).

In case of long belts and great load special reinforcing: parallel steel wire inserts are used (Fig. 2). As in these belts the steel wires are parallel to the longitudinal axis of the belt, x_1 the strength of the compound material is practically equal to the strength of the rubber in direction x_2 perpendicular to the wires because of the lack of transversal threads.

According to the strength test of the belts, the strength of the rubber can be usually neglected besides the strength of the reinforcing material, a consequence mainly of the rate of the elastic modulus of the two materials.

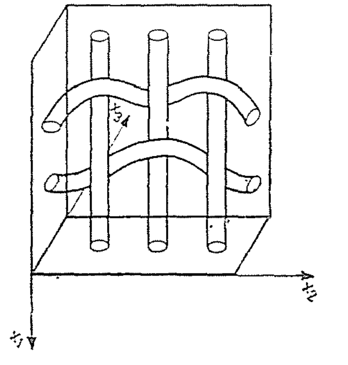

Fig. 1. Structure of the belt reinforced with carcass

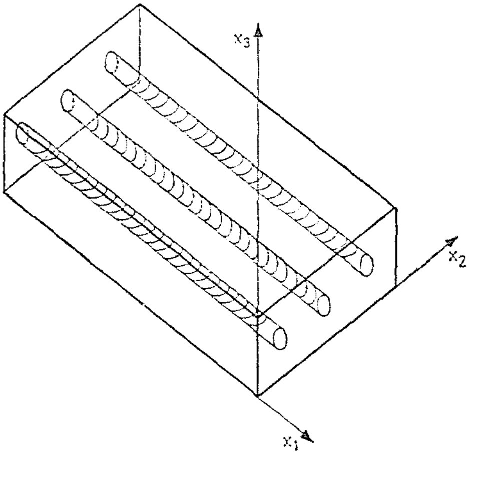

Fig. 2. Arrangement of steel reinforcement in the belt

Let us examine the relation of elastic moduli in case of rubber-steel belts. Let the elastic modulus of the steel be

$$E_a = 2.1 \cdot 10^5 \text{ MPa}$$

that of the bedding rubber

$$E_g = 3 \text{ MPa}$$

Thus the ratio of the elastic moduli of the two materials

$$\frac{E_a}{E_g} = 7 \cdot 10^4 \tag{1}$$

The value of the torsional shear modulus in case of a steel-rubber material pair

$$G_a = 8 \cdot 10^4 \text{ MPa}$$

$$G_g = 1 \text{ MPa}$$

The ratio of the moduli of torsional shear of the two materials taking the above mentioned facts into account is

$$\frac{G_a}{G_g} = 8 \cdot 10^4 \tag{2}$$

Figures 1 and 2 — that refer to rubber-steel belts — result on the one hand, in a specific elastic attitude of the complex system, and on the other call attention to a modulus-step on the boundary surface of the two materials working together. In the course of mechanical tests of the compound material, the local effect of steel cords embedded in rubber is to be taken into account because of the high modulus ratios (1) (2).

Testing the development of structural belt materials we can state that the spread of steel cord plays an increasingly important role in the field of reinforcing material. Keeping in mind the fact that we use import rubber and steel cord to produce belts, the detailed mechanical test of compound belt materials is justified.

Mechanical load of belts

The conveyor belt is affected by static load in a static condition and by dynamic load during operation. The static load is the tensile load coming from the tension of the belt. The weight of the material to be transported causes load.

Dynamic load comes from the movement of the belt itself on the one hand and from the material falling on the belt on the other. These two dynamic loads have different effects because of their characteristics. The first one loads the belt mainly in a longitudinal direction and causes a swing-like load,

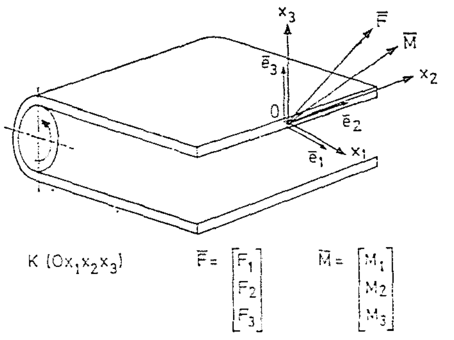

Fig. 3. Vectors F, M representing the mechanical load of plain belt

the second one loads both longitudinally and transversally and its effect appears in the form of impulses.

Analyzing the effects of dynamic loads we can state that loads coming from the movement of the belt are less dangerous because they — with good approximation — appear in the form of even, constant power impulses.

The dynamic effect of the material on the belt can be tested on the basis of the interaction between the concentrated or distribution force and the elastic continuum.

Weight force originating in the mass of the belt causes load that exerts its influence as an evenly distributing force along the total length. Passing through pulleys and drums causes significant mechanical load to the belt.

Under the influence of the load on the belt, mechanical loads develop in the complex body that are carried by the matrix and reinforcement jointly and are working together.

The resultant vectors F and M describe mechanical load in the centre, 0, of an arbitrary cross-section of the plain belt. (Fig. 3.) In order to analyse them in detail the mechanical effect of the resultant vectors F and M is tested by dividing then into components in the local coordinate system K $(0x_1x_2x_3)$ belonging to the cross-section.

Let unit vectors

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

mark the axes of the rectangular, straight acis coordinate system K in the above order. According to analytic notation the mark of unit vectors is e_i $(i = 1, 2, 3)$.

Let the vector of resultant force F be

$$F = F_1 e_1 + F_2 e_2 + F_3 e_3,$$

or in a short form

$$F = F_i e_i \qquad (i = 1, 2, 3)$$

and the expression of resultant moment vector M

$$M = M_1 e_1 + M_2 e_2 + M_3 e_3$$

or in a short form

$$M = M_i e_i \qquad (i = 1, 2, 3)$$

Among the above components F_1 causes tension or pressure, F_2 and F_3 cause shear in the belt. Moment M_1 means torsion, M_2 and M_3 mean bending load or the cross-section of the belt. It is clear from the list that the material of the belt is loaded in a complex way.

Summarizing the above mentioned facts: the control or dimensioning of conveyor belts can be carried out on the basis of the following load matrices by taking the mechanical role of the bedding and reinforcing material into account.

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \qquad M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}$$

Real mechanical attitude of the belt and the continuum models of its materials

The rubber layer covering the belts is usually thicker on the transportation side than on the lower side to compensate abrasive effect thus the complex material belongs to the quasi orthotropic bodies on the basis of the elastic characteristics. We can see that the rubber and cord frame follow each other alternately so this system belongs to laminated bodies on the basis of the material structure.

The mechanical functions of matrix and reinforcement in the conveyor belt can be distinguished and divided easily.

The rubber plays first of all the role of matrix in the complex body. This matrix connects the cord material layers and promotes the interworking of inserts during strain. The matrix prevents the instability of cord threads when pressing reinforcement thus it protects the frame from failure.

The mechanical characteristics of elastic plates reinforced with cord are dealt with in the following.

The simplest inhomogeneous orthotropic plate is produced from cord material layers embedded in rubber at an angle $\pm\alpha$, alternately. The insert number of the complex body is 2 m where m means the unilateral inserts. The elastic system is characterized by cord geometry $|\pm\alpha| =$ const. α is the interpretation range of embedding angle $\alpha \in [0, 90°]$. Another case of the theory is the orthotropic insert pair where one layer cord thread was embedded in angle $\alpha_1 = 0°$, the other one in $\alpha_2 = 90°$. From the viewpoint of conveyor belts the test of the latter orthogonal material model deserves special attention.

The total strain of the material system reinforced with cord is characterized partly by the elongation rate of the threads $\lambda^k \gtreqless 1$, partly by the scissors movement of the threads $\alpha \to \beta$ (Fig. 4). With this we also emphasize that the mechanical behaviour of the complex body is directed by the reinforcing material. According to the tests of R. S. Rivlin the junction points of the cord threads embedded in the rubber — if there are such — behave as a relatively suitable knuckle during strain [1]. These knuckles in the belt are in the junction points of the warp and weft threads.

In connection with the model of the belt in the course of deformation, we always suppose that the strain of the reinforcing frame is always followed perfectly by the matrix. This phenemenon is expressed by W. Hofferberth's compatibility equation [2].

In real rubber-cord systems the volume of the rubber is several times as much as that of the reinforcing material thus the mechanical role of the matrix is to be taken into consideration during the energetics test of the complex body. Contrary to this fact we can say that during the strength test of the rubber-cord system the effect of rubber can be generally neglected besides that of cord. This fact, in connection with the rubber cord system, led to the elaboration of W. Hofferberth's reticulate theory [3].

Testing the rubber cord interaction in the complex body we can state that the mechanical behaviour of the elastic system is directed by the reinforcing material.

In literature, several publications deal with the test of orthotropic mechanical bodies reinforced with unstretchable cord ($\lambda^k = 1$).

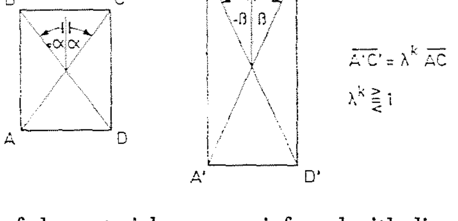

Fig. 4. Strain of the material system reinforced with diagonal cord

In R. S. Rivlin's tests the reinforcement is unstretchable cord in this model the task of the matrix is to connect the layers of the frame. Because of the geometrical condition ($\lambda^k = 1$) introduced arbitrarily the result of the test is suitable to describe partial strain [1].

A. E. Green and J. E. Adkins also tested the stress and strain of plates reinforced with unstretchable cord frame [4]. Their results are suitable only for the description of partial strain because of the positive condition $\lambda^k = 1$, the formulae are not suitable for the total energetic test of the complex body because of the elimination of the strain energy accumulated in the cord. Because of $\lambda^k = 1$, force does not develop in the cord, it is in contrast with the mechanical behaviour of real rubber-cord systems.

The mechanical behaviour of conveyor belts can be tested by continuum models of different structures and physical characteristics. Such an ideal body does not exist in nature so the continuum is only an abstraction but according to experience its production is useful because, for example with its help, the mechanical behaviour of the materials of the belts can be described in accordance with our aims.

The homogeneous orthotropic elastic continuum is the basis of tests in all cases when the orthotropic character of the belt is emphasized but the physical behaviour of the components, the effect of rubber and cord exerted on each other and the material structure of the system may be neglected.

The physical equation of the homogeneous orthotropic continuum is given in the following form

$$\sigma_r = \mathbf{M}_{rs}\varepsilon_s \tag{3}$$

$$(r, s = 1, 2, \ldots, 6)$$

where

σ_r is the column vector of stresses

ε_s is the column matrix of the characteristics of strain

$\mathbf{M}_{rs} = \mathbf{M}_{sr}$ is the symmetrical matrix that is the function of elastic material characteristics.

Matrix Eq. (3) describes the mechanical behaviour of the material of the ideally elastic body: it states mutually a univocal functionality between σ_r and ε_s. Thus the homogeneous, linear connection between the vectors of the stress and strain characteristics can be described in the knowledge of matrix $\mathbf{M}_{rs}$.

From Eq. (3) we obtain

$$\varepsilon_r = \mathbf{M}_{rs}^{-1}\,\sigma_s \tag{4}$$

Equation (4) reflects the reversible connection between ε_r and σ_s that is a consequence of the elastic behaviour of the ideal body.

The other type of belt model describes the mechanical behaviour of the complex body by the physical characteristics of the components, this model is the inhomogeneous orthotropic continuum. In case of a belt, the cord frame in the rubber and the mechanical effect of components exerted on each other can be tested with this kind of model.

The physical equation of inhomogeneous orthotropic continuum

$$\sigma_r = (\mathbf{M}^g_{rs} + \mathbf{M}^k_{rs})\ \varepsilon_s \tag{5}$$

where

$\mathbf{M}^g_{rs}$ is the rigidity matrix of the rubber and

$\mathbf{M}^k_{rs}$ is the rigidity matrix of the cord frame

σ_r and ε_s mean the same as in Eq. (3).

In Eq. (5) the elements of matrices $\mathbf{M}^g_{rs}$ and $\mathbf{M}^k_{rs}$ are not independent of each other but are in close connection with each other on the basis of geometrical equations describing the strain interworking of rubber and cord frame.

The mechanical behaviour of the material of real belts is, however, characterized neither by linearity nor by elastic characteristics by themselves. In connection with them the previous ideal material characteristics appear only partly and together with other characteristics. If a physical equation referring to linearly elastic solid continuum (3) (5) is used in the claculations, a laboratory test is necessary to state the deformation limit referring to the rubber-cord system until the basic Eq. (4), (5) is valid with the mistake stated in advance. This latter procedure, however, belongs to the sphere of material testing.

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