ON THE DYNAMICS OF DRIVE-SYSTEM OF RAILWAY TRACTION VEHICLE UNDER STOCHASTIC TRACK EXCITATION

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Summary

Traction vehicles moving along railway tracks go through stochastically excited vertical vibrations owing to the vertical unevennesses always present in the track. These vibrations result in axle-load changes and so influencing the track direction froce transferred in the "wheel-rail" connection result dynamic loads revealed as stochastic process in each element of a drive system. The object of this study is the origination of the stochastic load characteristics of a drive system from the spectral density function of the track unevennesses.

Introduction

The vertical support and positive guidance of a pair of drivers of traction vehicles moving along tracks is realised on the spot of the wheel-rail connection. The tread motion in vertical plane of a pair of drivers is always determined by the vertical unevennesses always present in the track, which can be described by a stochastic process fluctuating around the theoretical tread-plane. So it can be said that the pair of drivers gets a vertical displacement excitation on the spot of the wheel-rail connection. The motion of the wheel-treads prescribed as a stochastic process is transferred into the vertical motion of the pair of drivers and into the vertical motion of the bogies and the vehicle superstructure through the suspension spring system. As a result of these vertical motions the vertical supporting forces of the wheel-tread also change stochastically. It is known [2], that the track-direction force (tractive effort) arising on the wheel-tread can be calculated as a product of the vertical wheel force by the connection coefficient.[1] As a consequence, stochastic fluctuations should be reckoned with concerning the tractive effort arising on the wheel-tread even in the case of a constant connection coefficient because of the fluctuation of the

[1] The concept of the connection coefficient is not identical to that of the friction coefficient. The former is also defined as a ratio of the circumferential force transferred to the track direction and the vertical wheel force in a formal way, but this ratio is formed from the actual momentan force values and doesn't characteristic to the frictional limit force. At a constant vehicle speed the connection coefficient is a function of the slipping speed realized between the wheel and the rail [1], [2].

vertical wheel force. But in the reality the connection coefficient can be considered as a steeply increasing linear function of the track-direction-creep at constant vehicle speed as long as macroscopic slip does not occur in the wheel-rail connection [1], [2], [3]. Thus the fluctuation of the circumferential tractive effort is increased also by the latter "gaining effect".

From the above said follows that the prime-mover side driving torque of the transmission chain is counteracted by the stochastically fluctuating torque of the tractive effort and so a non-zero angular acceleration in the transmission chain and consequently a dynamic excess load must be reckoned with even at a constant speed of the vehicle.

Since the fluctuations of the tractive effort transferred through the wheel-rail connection are basically due to vertical track unevennesses, it is directly evident that the improvement of the track quality is basically important as far as the increased reliability and service-life of the valuable tractive stock is concerned.

In this paper the spectral density function of the weakly stationary stochastic process of the vertical unevennesses in the track taken as known, a new procedure is introduced to give an approximative description of the dynamic loads of the tractive-unit-drive-system with the help of a small-scale planar linearized model. This procedure to be introduced here can be extended to the analysis of other models with greater degree of freedom, as well.

Planar dynamical model of a pair of drivers

The examination of the scope of problems outlined in the introduction is carried out on the basis of the dynamical model shown in Fig. 1. In our model the flat disc of mass m_k and moment of inertia Θ_k representing the pair of drivers of a vehicle is directly supported on the mass m_p representing the mass of the track moving together with the pair of drivers. The distance of the centre of masses (gravity points) m_k and m_p is determined by the stochastic process u_t of the vertical unevennesses in the track. The mass m modelling the vehicle's superstructure is connected to the centre of mass m_k from above through the parallel-connected linear spring of stiffness s_v and linear damper of damping coefficient k_v. The disc with moment of inertia Θ_k is connected to the disc with moment of inertia Θ through the torsion shaft of stiffness s_c and damping coefficient k_c representing the transmission chain. The driving torque M is directly applied upon the disc with moment of inertia Θ. On the spot of connection of masses m_k and m_p a vertical force of magnitude T arises. The circumferential tractive effort F_k arising in the wheel-rail connection can be written in the form of $F_k = T \cdot \mu$ as a product of the vertical force T by connection coefficient (adhesion coefficient) μ. The connection coefficient μ at a constant running

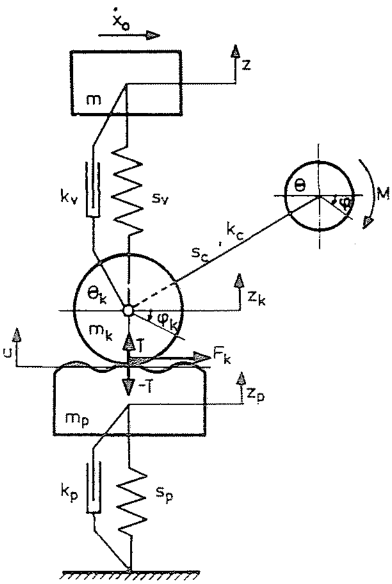

Fig. 1

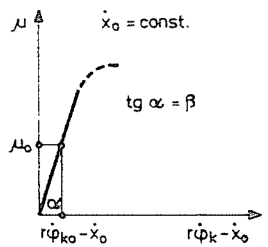

Fig. 2

speed $\dot{x}_0$ of the vehicle shown in Fig. 2 linearly depends on the elastic slip (creep) of track direction $r\dot{\varphi}_k - \dot{x}_0$ in the neighbourhood of the origin:

$$\mu = \beta(r\dot{\varphi}_k - \dot{x}_0)$$

where $\beta = \operatorname{tg}\alpha$ and r is the radius of the wheel and $\dot{\varphi}_k$ is the angular velocity of the wheel. In Fig. 2 the value of the connection coefficient μ_0 is shown as the characteristic of the undisturbed (stationary) state and realized in the case of elastic slip $r\dot{\varphi}_{k0} - \dot{x}_0$. The value of μ_0 can be easily determined with the knowledge of the train resistance as related to one of the pairs of drivers. Now let F_{k0} be the tractive effort required to overcome train resistance as related to one of the pairs of drivers. Then:

$$\mu_0 = \frac{F_{k0}}{T_0} \tag{1}$$

where T_0 designates the stationary vertical force arising in the wheel-rail connection. The elastic slip arising in an undisturbed state can also be determined:

$$r\dot{\varphi}_{k0} - \dot{x}_0 = \frac{\mu_0}{\beta} \tag{2}$$

hence the angular velocity in an undisturbed state is

$$\dot{\varphi}_{k0} = \frac{1}{r}\left(\frac{\mu_0}{\beta} + \dot{x}_0\right). \tag{3}$$

Torque M determined by the performance curve of the drive system is acting upon the disc with moment of inertia Θ modelling the driving side. In Fig. 3 the

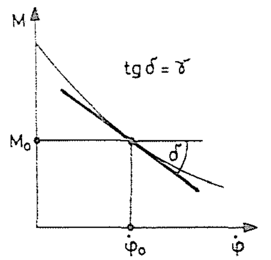

Fig. 3

diagram of torque $M(\dot{\varphi})$ is plotted transmitting drive to the pair of drivers where $\dot{\varphi}$ indicates the angular velocity of the driving side. In an undisturbed state the torque equilibrium: $M_0 = rF_{k0} = r\mu_0 T_0$ is in force while $\dot{\varphi}_0 = \dot{\varphi}_{k0}$. If the generally non-linear function $M(\dot{\varphi})$ is linearized in the neighbourhood of $\dot{\varphi}_0$, then for small disturbances around the stationary state

$$M = M_0 + \gamma(\dot{\varphi} - \dot{\varphi}_0) \tag{4}$$

will be yielded, where $\gamma = \left.\frac{dM}{d\dot{\varphi}}\right|_{\dot{\varphi}=\dot{\varphi}_0}$.

The track mass m_p found in the dynamical model is connected to the static ground plane through a system consisting of parallel connected spring of stiffness s_p and damper of damping coefficient k_p.

The stochastic process of the vertical wheel force

Concerning the drive-dynamics investigations, the description of the stochastic time function T_t of the vertical wheel forces developing in the dynamical system shown in Fig. 1 is basically important. In order to determine the characteristics of T_t, the equations describing the vertical motions of the system should be determined. For the sake of simplicity, the small displacements around the equilibrium state in the gravity field of the system should be examined. Consequently, the upward displacements z, z_k and z_p considered as positive ones, the stochastic time functions of z_t, z_{kt} and z_{pt} will be of zero expected value ($Ez_t = Ez_{kt} = Ez_{pt} = 0$). Furthermore, instead of examining the vertical wheel force process T_t, it is expedient to examine the process

$$\tilde{T}_t = T_t - T_0 \tag{5}$$

having zero expected value ($E\tilde{T}_t = 0$), since the expected value $ET_t = T_0$ has been subtracted from it.

The excitation is caused by the stochastic process u_t of the vertical unevennesses in the track. At a given running speed $\dot{x}_0$ the process u_t of zero expected value according to the definition is considered as weakly stationary

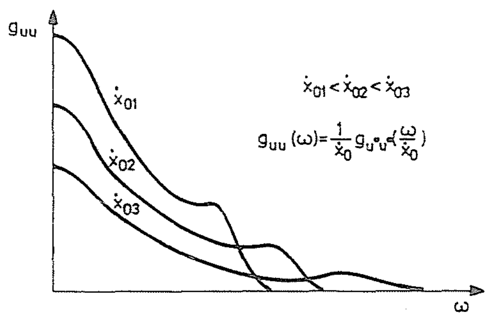

Fig. 4

whose spectral density function $g_{uu}(\omega)$ is available for us. Registrating the vertical unevennesses along the track, a realization of the stochastic process u_x^* can be obtained, hence the required excitation process will be obtained with the help of definition $u_t = u^*_{\dot{x}_0 t}$. Denoting the spectral density function of the process u_x^* by $g^{**}_{uu}(\omega)$, then:

$$g_{uu}(\omega) = \frac{1}{\dot{x}_0} g^{**}_{uu}\left(\frac{\omega}{\dot{x}_0}\right). \tag{6}$$

Under the above conditions the equations of motion to be written for the masses m, m_k and m_p will be as follows:

$$\begin{aligned} (z_{kt} - z_t)s_v + (\dot{z}_{kt} - \dot{z}_t)k_v &= m\ddot{z}_t \\ -(z_{kt} - z_t)s_v - (\dot{z}_{kt} - \dot{z}_t)k_v + \tilde{T}_t &= m_k\ddot{z}_{kt} \\ -\tilde{T}_t - z_{pt}s_p - \dot{z}_{pt}k_p &= m_p\ddot{z}_{pt} \end{aligned} \tag{7}$$

Expressing $\tilde{T}_t$ from the last equation of set (7) and considering the relationship of z_{pt} to the equality $z_{pt} = z_{kt} - u_t$, respectively, after some arrangement the following set of equations is yielded:

$$m\ddot{z}_t + k_v\dot{z}_t - k_v\dot{z}_{kt} + s_v z_t - s_v z_{kt} = 0$$

$$(m_p + m_k)\ddot{z}_{kt} - k_v\dot{z}_t + (k_v - k_p)\dot{z}_{kt} - s_v z_t + (s_v + s_p)z_{kt} = s_p u_t + k_p\dot{u}_t + m_p\ddot{u}_t \tag{8}$$

With the vector $Z_t = [z_t, z_{kt}]^*$ introduced and with the familiar matrix designations applied, equation (8) can be written in the following form:

$$\mathbf{M}\ddot{Z}_t + \mathbf{K}\dot{Z}_t + \mathbf{S}Z_t = F_t, \tag{9}$$

where: $F_t = [0, s_p u_t + k_p\dot{u}_t + m_p\ddot{u}_t]^*$.

The spectral density matrix $\mathbf{G}_F(\omega)$ of the excitation vector process F_t has the following form:

$$\mathbf{G}_F(\omega) = \begin{bmatrix} 0 & 0 \\ 0 & g(\omega) \end{bmatrix} \tag{10}$$

where $g(\omega)$ can be written with the knowledge of track-spectral density $g_{uu}(\omega)$

$$g(\omega) = (s_p^2 + \omega^2(k_p - 2s_p m_p) + \omega^4 m_p^2)g_{uu}(\omega) \tag{11}$$

With the knowledge of the spectral density matrix $\mathbf{G}_F(\omega)$, the spectral density matrix $\mathbf{G}_Z(\omega)$ of process Z_t can also be constructed using the fundamental theorem of statistical dynamics:

$$\mathbf{G}_Z(\omega) = \mathbf{H}(-i\omega)\mathbf{G}_F(\omega)\mathbf{H}^*(i\omega). \tag{12}$$

In (12) $\mathbf{H}(i\omega)$ is the frequency function matrix pertaining to equation (9):

$$\mathbf{H}(i\omega) = (\mathbf{M}(i\omega)^2 + \mathbf{K}(i\omega) + \mathbf{S})^{-1}. \tag{13}$$

With the knowledge of $\mathbf{G}_Z(\omega)$ the spectral density function of the process $\tilde{T}_t$ can also be determined. To this effect, it should be considered that on the basis of the second equation of set (7):

$$\tilde{T}_t = m_k\ddot{z}_{kt} + k_v\dot{z}_{kt} + s_v z_{kt} - s_v z_t - k_v\dot{z}_t. \tag{14}$$

It can be stated that owing to $Ez_{kt} = Ez_t = 0$, $E\tilde{T}_t = 0$ is also in force. Since stationarity would not be changed by linear operations, [5], [6], process $\tilde{T}_t$ contained in (14) is also weakly stationary. With the autocorrelation function $E\tilde{T}_t\tilde{T}_{t+\tau}$ written and the inverse Fourier-transformation carried out, after a lengthy deduction the following relationship is obtained:

$$g_{\tilde{T}\tilde{T}}(\omega) = A(\omega)g_{zz}(\omega) + B(\omega)g_{zz_k}(\omega) + C(\omega)g_{z_kz}(\omega) + D(\omega)g_{z_kz_k}(\omega) \tag{15}$$

i.e. $g_{\tilde{T}\tilde{T}}(\omega)$ will be reached as a function of the elements in spectral density matrix $\mathbf{G}_Z(\omega)$ where the coefficients are fixed polynoms of ω. Now, with the knowledge of the real-valued spectral density function $g_{\tilde{T}\tilde{T}}(\omega)$ determined in this way, the forced vibrations of the torsional drive-system can already be analysed.

The stochastic vibrational process of the transmission chain

In Fig. 5 the simple model of the torsional transmission chain is shown indicating the expressions of the torques applied on the discs as well. It is obvious that the stochastic excitation of the torsional system is originated from

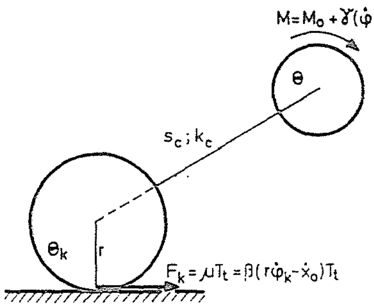

Fig. 5

the stochastic fluctuations-caused by T_t- of the circumferential force acting on the disc with moment of inertia Θ_k. For the motion relationships of the drive-system to be cleared up, the motion equation of the rotating masses with moment of inertia Θ_k and Θ, respectively, are written. Let φ_t denote the angular displacement of the drive side disc related to the static state, and let φ_{kt} denote the angular displacement of the disc modelling the pair of drivers. Now the equations of motion are[1]:

$$M_0 + \gamma(\dot{\varphi}_t - \dot{\varphi}_0) + s_c(\varphi_{kt} - \varphi_t) + k_c(\dot{\varphi}_{kt} - \dot{\varphi}_t) = \Theta\ddot{\varphi}_t, \tag{16}$$

$$-s_c(\varphi_{kt} - \varphi_t) - k_c(\dot{\varphi}_{kt} - \dot{\varphi}_t) - \mu T_t r = \Theta_k \ddot{\varphi}_{kt}.$$

For the sake of simpler treatment of the set of equations, on the one hand, the relative angular displacements Φ_t and Φ_{kt} around the position pertaining to the stationary torque M_0 will be introduced instead of the evolutive time

[1] Note that the more detailed examination of the influences acting upon the rotating masses (i.e. the influences of the bearing friction and the rolling resistance) can also be easily built into the set of equations (16).

functions φ_t and φ_{kt} and, on the other hand, the product μT_t will be linearized. The relative angular displacements — as it can easily be justified — are determined by the pair of relationships:

$$\Phi_{kt} = \varphi_{kt} - \dot{\varphi}_{k0} \cdot t, \quad \Phi_t = \varphi_t - \dot{\varphi}_0 \cdot t - \frac{M_0}{s_c} \tag{17}$$

while the linearization of the product μT_t will result in the expression

$$\mu T_t \approx \mu_0 T_0 + \mu_0(T_t - T_0) + T_0(\mu - \mu_0) \tag{18}$$

With the above said in section 2 in mind and the (17) taken into account, on the basis of relation:

$$\mu - \mu_0 = \beta(r\dot{\varphi}_{kt} - r\dot{\varphi}_{k0}) = \beta r \dot{\Phi}_{kt} \tag{19}$$

it can be written:

$$\mu T_t = \mu_0 T_0 + \mu_0 \tilde{T}_t + T_0 \beta r \dot{\Phi}_{kt}. \tag{20}$$

So the pair of equations (16) obtains the following simpler form:

$$\Theta \ddot{\Phi}_t + (k_c - \gamma)\dot{\Phi}_t - k_c \dot{\Phi}_{kt} + s_c \Phi_t - s_c \Phi_{kt} = 0$$

$$\Theta_k \ddot{\Phi}_{kt} - k_c \dot{\Phi}_t + (k_c + T_0 \beta r^2)\dot{\Phi}_{kt} - s_c \Phi_t + s_c \Phi_{kt} = -\mu_0 r \tilde{T}_t. \tag{21}$$

This in turn with the introduction of the vector $\psi t = [\Phi_t, \Phi_{kt}]^*$ and using the familiar matrix designations (21) can be written in the form:

$$\boldsymbol{\vartheta} \ddot{\psi}_t + \boldsymbol{\varkappa} \dot{\psi}_t + \boldsymbol{\sigma} \psi_t = R_t \tag{22}$$

where $R_t = [0, -\mu_0 r \tilde{T}_t]^*$. The spectral density function $\mathbf{G}_R(\omega)$ of the exciting stochastic vector process R_t is as follows:

$$\mathbf{G}_R(\omega) = \begin{bmatrix} 0 & 0 \\ 0 & \mu_0^2 r^2 g_{\tilde{T}\tilde{T}}(\omega) \end{bmatrix} \tag{23}$$

Let $\chi(i\omega)$ the frequency function of the system described by the differential equation system (22), i.e. the matrix function:

$$\chi(i\omega) = (\boldsymbol{\vartheta}(i\omega)^2 + \boldsymbol{\varkappa}(i\omega) + \boldsymbol{\sigma})^{-1} \tag{24}$$

then the spectral density matrix function of the process ψ_t similarly to (12) will be:

$$\mathbf{G}_\psi(\omega) = \chi(-i\omega)\mathbf{G}_R(\omega)\chi^*(i\omega). \tag{25}$$

With the knowledge of the spectral density matrix $\mathbf{G}_\psi(\omega)$ the characteristic quantities of the torque process loading the transmission chain can already be determined.

The dynamical torque load of the transmission system

The dynamical torque load acting on the elastic drive-shaft is examined starting from the relationship: $M_c = s_c(\varphi_k - \varphi)$ neglecting the torque demand of damping (the material damping factor k_c is very small). But on the basis of relationship (17), the expression:

$$M_c = M_0 + s_c(\Phi_k - \Phi) = M_0 + \tilde{M} \quad (26)$$

can be also considered. In (26) the dynamical torque load is due to the term $\tilde{M} = s_c(\Phi_k - \Phi)$ and so it is responsible for the stochastic fluctuation of the torque M_c around M_0. This fluctuation, in turn, can be characterised on the basis of the stochastic processes Φ_t and Φ_{kt} with spectral density matrix $G_\varphi(\omega)$ taken into consideration. The spectral density function of the zero-expectation torque process $\widetilde{M}_t = s_c(\Phi_{kt} - \Phi_t)$ can be determined from the auto-correlation function $E\widetilde{M}_t\widetilde{M}_{t+\tau}$ by means of inverse Fourier transformations. Since

$$E\tilde{M}_t\tilde{M}_{t+\tau} = s_c^2(E\Phi_{kt}\Phi_{k(t+\tau)} - E\Phi_{kt}\Phi_{t+\tau} - E\Phi_t\Phi_{k(t+\tau)} + E\Phi_t\Phi_{t+\tau}), \quad (27)$$

the spectral density $g_{\tilde{M}\tilde{M}}(\omega)$ is yielded from the elements of (25) in the following form:

$$g_{\tilde{M}\tilde{M}}(\omega) = s_c^2(g_{\Phi_k\Phi_k}(\omega) - g_{\Phi_k\Phi}(\omega) - g_{\Phi\Phi_k}(\omega) + g_{\Phi\Phi}(\omega)). \quad (28)$$

The variance of the torque load M_t will be determined by the area under the spectral density function [6], [7], so the following expression is obtained:

$$\sigma_{M_t}^2 = 2\int_0^\infty g_{\tilde{M}\tilde{M}}(\omega)d\omega. \quad (29)$$

The above said are demonstrated with the help of the system model plotted in Fig. 6 and in this way the fact is evidently explained that the spectral density (as well as the variance) of the dynamical torque loading of a vehicle transmission system can be originated ultimately from the spectral density function $g_{uu}(\omega)$ of the vertical unevennesses in the track with regard to the transfer properties of the track-vehicle dynamical system.

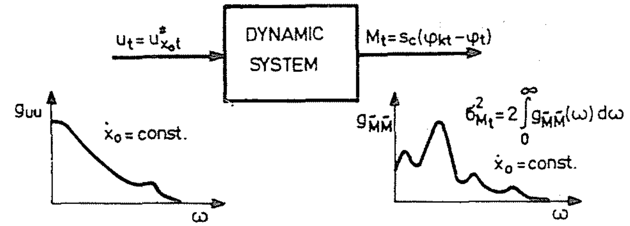

Fig. 6

The further characterisation of the dynamic-torque load can be given on the basis of Fig. 7, where the realization of the torque load and its density function approximated by Gaussian function are plotted. In this case, the maximum torque acting on the drive-shaft can be given with a probability of 0.9986 by means of the expression:

$$M_{t\max} = M_0 + 3\sigma_{M_t}.$$

So the dynamic factor related to the mean torque M_0 will be given in the form of

$$d = \frac{M_{t\max}}{M_0} = 1 + 3\frac{\sigma_{M_t}}{M_0}. \qquad (30)$$

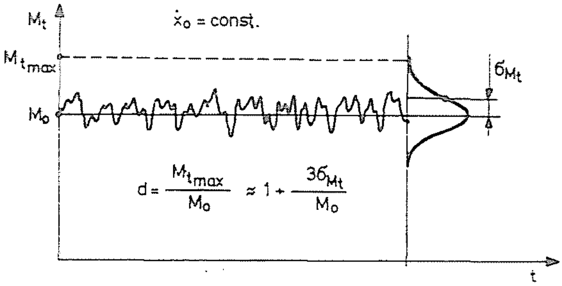

Fig. 7

In connection with relationship (30) it is emphasized again that — since σ_M is originated from the spectrum of the process u_t — consequently the dynamic factor d can be traced back to the unevennesses in the track on the basis of the transfer properties of the track-vehicle dynamical system.

Concluding remarks

The train of thoughts introduced in the foregoing can be extended naturally to the case of more complex track-vehicle models as well. The degrees of freedom can be increased both in the vertical dynamical system and in the transmission system but the relationships to be applied will be of the same structure like the ones dealt here within the frame of the linear system model but the size of the matrices and the filling in and location of its elements will be changed.

Within the scope of problems forming the object of this paper, there are some sub-problems which can be separated as self-important ones and the solution of them greatly contributes to the more exact process-analysis. Three of such sub-problems are mentioned below:

1. the identification of the elastic and dissipative characteristics of the railway track, as well as that of the effective masses moving together with the wheel-set,

2. the regard of the non-linearity of the track-vehicle system: the examination of the changing speed of the vehicle motion and that of the macroscopic slips in the wheel-rail connection, as well as their influence on the dynamic processes,

3. the examination of the inherent time-dependencies in the system. On the one hand, the examination of the inherent deterministic time dependencies in the coefficients of the descriptive system-equations (parametrical excitations e.g. in the gear-mesh of the transmission gears and the Hook-joint drive, resp.). On the other hand, the analysis of the inherent stochastic perturbations in the coefficients (e.g. stochastic variations of the elastic and dissipative track-characteristics along the track).

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