THE LOCATION OF THE CENTRE OF GRAVITY OF A TRAILER AND THE EFFECT OF THE OPTIMIZATION OF THE SUSPENSION PARAMETERS ON VEHICLE VIBRATIONS

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Summary

Our investigations show that in case of our vehicle combination, the optimal location of the centre of gravity of the trailer is the point above the axle of the trailer. Studying the complicated swinging couplings of various vehicle combinations requires determining the elements $[\mu_{ij}]$ of the reduced mass proportion matrices.

The optimization of the vehicle combination shows also that optimizing individually the passenger car and the trailer leads to a considerable improvement of the swinging characteristics of the optimized component. The couplings depend on the mass proportion factors μ_{ij} of the system and can be neglected if the μ_{ij} values are small.

Introduction

The effects of a trailer on the vibrations of the towing vehicle were studied in our paper [1]. In particular, we investigated the effects of the damping of the trailer from the point of view of both the towing vehicle and the trailer. Our analysis has led to the conclusion that even with a fully undamped trailer, driving the towing vehicle is more comfortable for the driver than driving the same vehicle without the trailer. On the other hand, the dynamical load on the undamped trailer is twice as high as with a suitable damping facility.

The lack of a suitable damping of a trailer results similar worsening effects on the dynamical load of the supporting springs of the trailer and on the running stability. Analysing mathematically the decomposition of a vehicle swinging system with six degrees of freedom into independent swinging systems we have shown that choosing appropriate coordinates the necessary and sufficient condition of such a decomposition can be fulfilled even for nonlinear systems. These conditions are independent of the nonlinear characteristics of the springs and shock absorbers and require only an appropriate choice of the geometric and mass parameters of the system.

The allocation of swinging systems into equivalence classes is described in [2]. This paper deals with the appropriate transformation of the system of differential equations of the nonlinear jointed model with six degrees of freedom

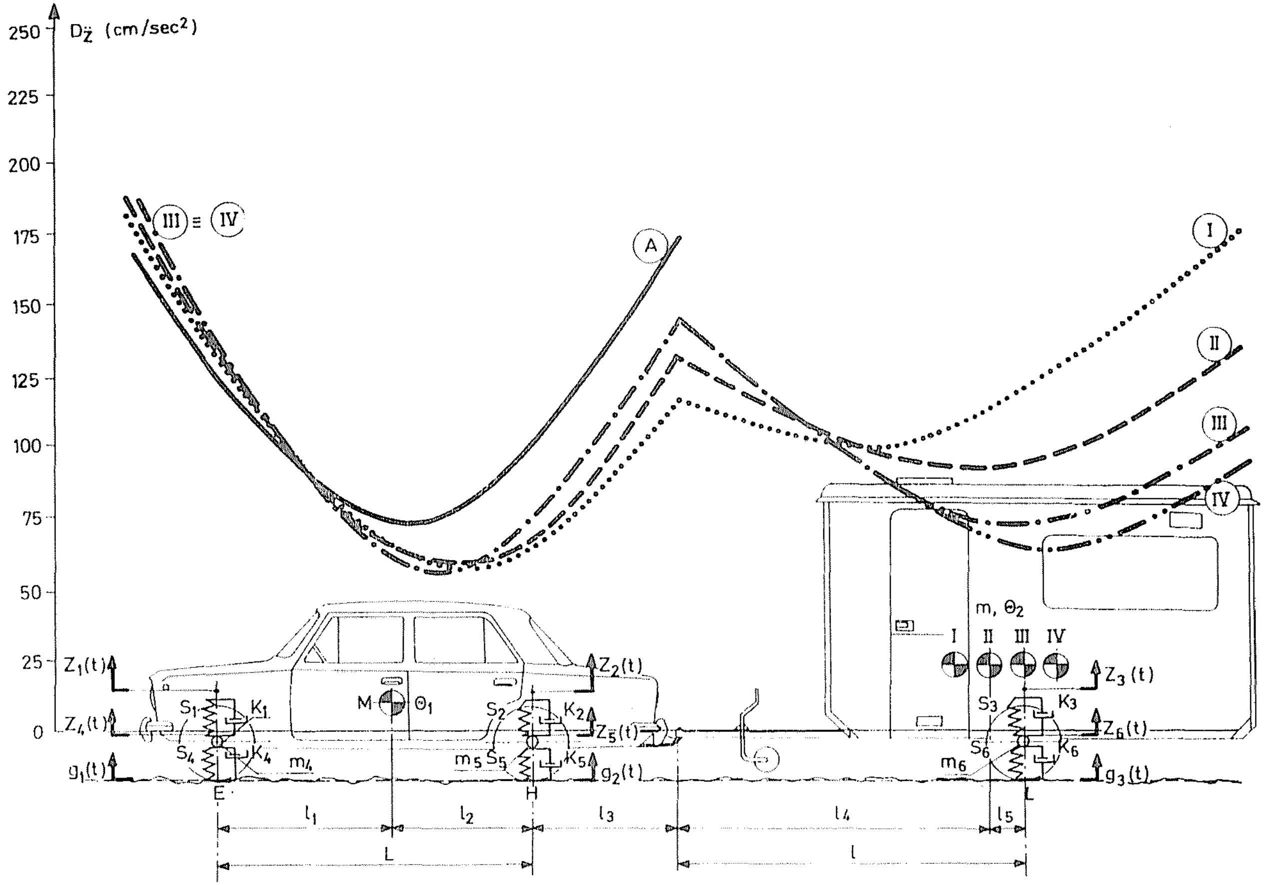
D$\ddot{z}$ (cm/sec^2)
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Fig. 1

by which the vibrational equivalence of two systems can be verified. The paper contains also an optimization method and applications for single vehicles.

Our present study is a continuation of the papers [1] and [2] and the mathematical models and parameters are taken from the just mentioned papers.

The location of the centre of gravity of the trailer

Figure 1 shows the investigated vehicle combination. The corresponding system of differential equations is as follows:

$$\begin{bmatrix} m_1 & m_{12} & m_{13} & 0 & 0 & 0 \\ m_{21} & m_2 & m_{23} & 0 & 0 & 0 \\ m_{31} & m_{32} & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix} \begin{bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \\ \ddot{Z}_3 \\ \ddot{Z}_4 \\ \ddot{Z}_5 \\ \ddot{Z}_6 \end{bmatrix} + \begin{bmatrix} K_1\{\dot{Z}_1 - \dot{Z}_4\} \\ K_2\{\dot{Z}_2 - \dot{Z}_5\} \\ K_3\{\dot{Z}_3 - \dot{Z}_6\} \\ K_4\{\dot{Z}_4 - \dot{g}_1\} - K_1\{\dot{Z}_1 - \dot{Z}_4\} \\ K_5\{\dot{Z}_5 - \dot{g}_2\} - K_2\{\dot{Z}_2 - \dot{Z}_5\} \\ K_6\{\dot{Z}_6 - \dot{g}_3\} - K_3\{\dot{Z}_3 - \dot{Z}_6\} \end{bmatrix} +$$

$$+ \begin{bmatrix} S_1\{Z_1 - Z_4\} \\ S_2\{Z_2 - Z_5\} \\ S_3\{Z_3 - Z_6\} \\ S_4\{Z_4 - g_1\} - S_1\{Z_1 - Z_4\} \\ S_5\{Z_5 - g_2\} - S_2\{Z_2 - Z_5\} \\ S_6\{Z_6 - g_3\} - S_3\{Z_3 - Z_6\} \end{bmatrix} = 0 \tag{1}$$

where:

$$a = \frac{l_3}{L}; \quad m_1 = \frac{1}{L^2}(Ml_2^2 + \Theta_1) + \frac{a^2}{l^2}(ml_5^2 + \Theta_2);$$

$$m_2 = \frac{1}{L^2}(Ml_1^2 + \Theta_1) + \frac{(1+a)^2}{l^2}(ml_5^2 + \Theta_2);$$

$$m_3 = \frac{1}{l^2}(ml_4^2 + \Theta_2); \quad m_{12} = m_{21} = \frac{1}{L^2}(Ml_1l_2 - \Theta_1) - \frac{a(1+a)}{l^2}(ml_5^2 + \Theta_2);$$

$$m_{13} = m_{31} = \frac{a}{l^2}(\Theta_2 - ml_4l_5); \quad m_{23} = m_{32} = \frac{1+a}{l^2}(ml_4l_5 - \Theta_2).$$

In our study only one parameter is changing, namely the distance of the centre of gravity of the trailer from the axle. In Fig. 1, this parameter is denoted by l_5. Table 1 contains the value of l_5 in four different relative positions of the towing vehicle and the trailer.

In the original vehicle combination we have $l_5 = 28$ [cm]. Curve "A" in Fig. 1 corresponds to the original single vehicle without trailer.

Table I

	I.	II.	III.	IV.
l_5 (cm)	56	28	0	−28

Our investigations have led to the conclusion that a translation of the centre of gravity forward improves slightly the swinging comfort of the towing vehicle but increases considerably the dynamical load on the trailer. Comparing situations III. and IV. we see that a translation of the centre of gravity backward does not influence the swinging comfort of the towing vehicle but decreases slightly the dynamic load on the trailer.

To characterise numerically the wheel-road connection of the vehicle, we introduce the following stability index:

$$ST = \frac{D(Z_t - g)}{Z_s} \cdot 100[\%] \tag{2}$$

where:

Z_t — is the displacement of the given axle
g — is the road excitation of the axle
Z_s — is the static depression of the tyre on the given axle.

Table II shows that a backward translation of the centre of gravity improves the running stability (ST_1) of the front wheels of the towing vehicle but decreases the stability (ST_2 and ST_3) of the middle and rear wheels.

Table II

Vibration parameters	I.	II.	III.	IV.
$D(\ddot{z}_1)$ [cm/s²]	135.75	139.10	140.84	140.73
$D(\ddot{z}_2)$ [cm/s²]	63.58	71.19	75.29	75.27
$D(\ddot{z}_3)$ [cm/s²]	121.37	95.85	74.48	65.45
$r(z_1, z_4)$	0,571	0.575	0.570	0.573
$r(z_2, z_5)$	0.494	0.478	0.475	0.473
$r(z_3, z_6)$	0.607	0.594	0.620	0.613
$D(\alpha)$ [rad]	0.01188	0.01290	0.01396	0.01331
$D(\beta)$ [rad]	0.00795	0.00887	0.01021	0.01104
ST_1 [%]	16.42	15.95	15.42	14.78
ST_2 [%]	12.60	14.29	16.30	18.60
ST_3 [%]	7.65	7.95	8.30	9.68
$r(z_4, g_1)$	0.967	0.966	0.966	0.966
$r(z_5, g_2)$	0.965	0.964	0.963	0.963
$r(z_6, g_3)$	0.987	0.988	0.990	0.989
S_{1eff} [N]	902.3	919.1	926.6	922.4
S_{2eff} [N]	576.9	620.6	642.6	640.1
S_{3eff} [N]	557.9	499.0	425.3	419.2
K_{1eff} [N]	279.4	281.5	283.1	283.2
K_{2eff} [N]	287.8	289.6	292.5	292.6
K_{3eff} [N]	253.5	242.2	231.9	228.7
P_1 [W]	62.53	63.30	64.13	64.05
P_2 [W]	62.23	63.54	64.90	64.91
P_3 [W]	49.22	44.62	40.65	39.33
P_Σ [W]	192.08	189.50	187.78	186.42

Moving the centre of gravity backward increased slightly the load on the springs and shock absorbers of the towing vehicle but decreased considerably the load on the springs and shock absorbers of the trailer (S_{ieff}, K_{ieff}, $i = 1, 2, 3$)

Translating the centre of gravity backward decreased slightly also the power loss ($P_{ö}$) of the swinging system.

Let us investigate now the effects of changing the position of the centre of gravity of the trailer by using the system of differential equations of the swinging system.

In the nonlinear system (1) of our model a translation of the centre of gravity modifies only the elements of the symmetric mass matrix M.

The structure of the mass matrix is the following:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{22} \end{bmatrix} \tag{3}$$

where:

$$\mathbf{M}_{11} = \begin{bmatrix} m_1 & m_{12} & m_{13} \\ m_{21} & m_2 & m_{23} \\ m_{31} & m_{32} & m_3 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{22} = \langle m_4, m_5, m_6 \rangle$$

In our investigations the elements of $\mathbf{M}_{22}$ were kept fixed at

$$m_4 = 60, \; m_5 = 80, \; m_6 = 50 \qquad [\text{kg}]$$

Thus in cases I—IV only the elements of the submatrix $\mathbf{M}_{11}$ were changing. (As our matrices are symmetric, in Table III. it suffices to give the elements of the lower left submatrices.)

We see that the masses m_1, m_2, m_3 in the main diagonal (the so called main masses) are playing a dominating role: numerically they are much larger than all other elements. Hence their increase and decrease influence directly the

Table III

Cases I—IV	$\mathbf{M}_{11}$: [kg]
I. $l_5 = 56$ [cm]	$\begin{bmatrix} 683.76 & & \\ -103.74 & 953.10 & \\ 9.29 & -29.72 & 507.50 \end{bmatrix}$
II. $l_5 = 28$ [cm]	$\begin{bmatrix} 679.83 & & \\ -91.16 & 912.86 & \\ 28.95 & -92.63 & 613.03 \end{bmatrix}$
III. $l_5 = 0$ [cm]	$\begin{bmatrix} 678.52 & & \\ -86.97 & 899.44 & \\ 54.37 & -173.99 & 731.24 \end{bmatrix}$
IV. $l_5 = -28$ [cm]	$\begin{bmatrix} 679.83 & & \\ -91.16 & 912.86 & \\ 95.56 & -273.79 & 862.12 \end{bmatrix}$

swinging comfort and the dynamic load on the frame. Moving the centre of gravity backwards causes a slight decrease of m_1 and a larger decrease of m_2 (in case IV this tendency is not present); m_3 increases substantially in all cases. We see that a decrease (resp. increase) of the main masses causes a proportional increase (resp. decrease) of the dynamic load, in accordance with the relative softening or hardening of the springs.

Of course the non-diagonal mass elements play also some role in the investigated phenomenon. Although this role is rather involved, it can be understood by using the method of [2].

To this end we transform our system (1) into the following form:

$$\begin{bmatrix} \mu_{11} & \mathbf{0} \\ \langle\mu\rangle\mu_{11} & \mathbf{E} \end{bmatrix} \begin{bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \\ \ddot{Z}_3 \\ \ddot{Z}_4 \\ \ddot{Z}_5 \\ \ddot{Z}_6 \end{bmatrix} + \begin{bmatrix} \psi_1(\dot{Z}_1 - \dot{Z}_4) \\ \psi_2(\dot{Z}_2 - \dot{Z}_5) \\ \psi_3(\dot{Z}_3 - \dot{Z}_6) \\ \psi_4(\dot{Z}_4 - \dot{g}_1) \\ \psi_5(\dot{Z}_5 - \dot{g}_2) \\ \psi_6(\dot{Z}_6 - \dot{g}_3) \end{bmatrix} + \begin{bmatrix} \varphi_1(Z_1 - Z_4) \\ \varphi_2(Z_2 - Z_5) \\ \varphi_3(Z_3 - Z_6) \\ \varphi_4(Z_4 - g_1) \\ \varphi_5(Z_5 - g_2) \\ \varphi_6(Z_6 - g_3) \end{bmatrix} = 0 \tag{4}$$

where:

$$\mu_{11} = \begin{bmatrix} 1 & \mu_{12} & \mu_{13} \\ \mu_{21} & 1 & \mu_{23} \\ \mu_{31} & \mu_{32} & 1 \end{bmatrix}, \quad \mu_{ij} = \frac{m_{ij}}{m_i}; \quad (i, j = 1, 2, 3)$$

$$\langle\mu\rangle = \langle\mu_{41}, \mu_{52}, \mu_{63}\rangle = \left\langle \frac{m_1}{m_4}, \frac{m_2}{m_5}, \frac{m_3}{m_6} \right\rangle;$$

$$\psi_i\{\} = \frac{K_i\{\}}{m_i}, \qquad \varphi_i\{\} = \frac{S_i\{\}}{m_i} \; (i = 1, 2, \ldots, 6)$$

Using the so obtained reduced mass proportion parameters, in Fig. 2 we sketched the action transmission of the linearized model of a jointed system with six degrees of freedom. We see the complicated role of the nondiagonal mass proportion elements μ_{ij} in the coupling and the conditions of the separation.

Optimization of the suspension parameters of the vehicle combination

In the optimization procedure our main problem is to determine which effect the optimization of the individual elements of the vehicle combination has on the swingings of the coupled articulated or trailer system of six degrees of freedom.

Curve "A" in Fig. 3 concerns the original single vehicle, curve "B" concerns the vehicle combination obtained by coupling the original vehicle with

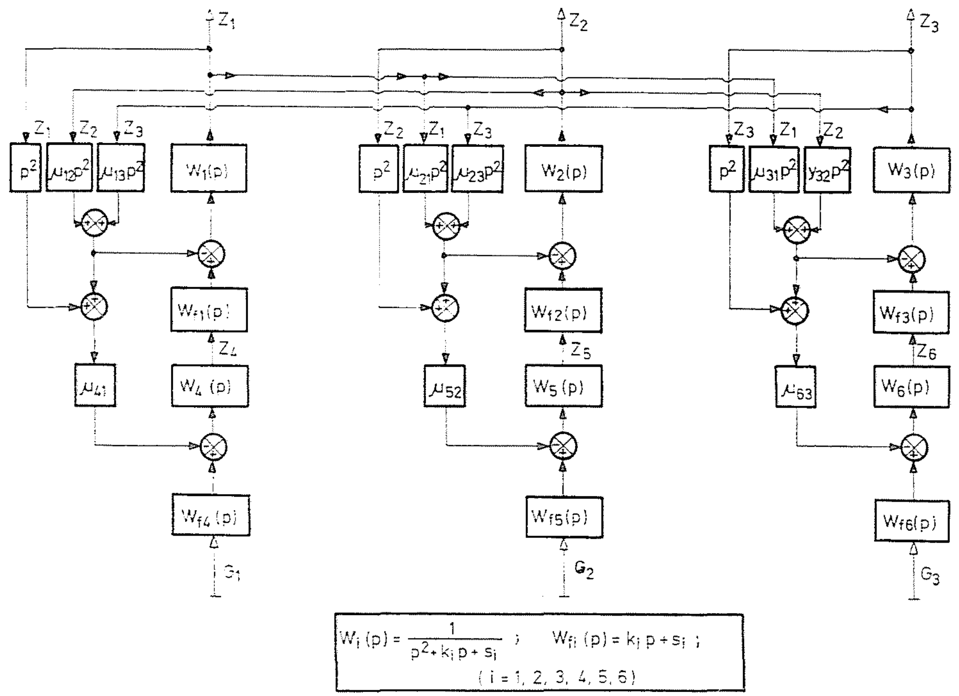


Fig. 2

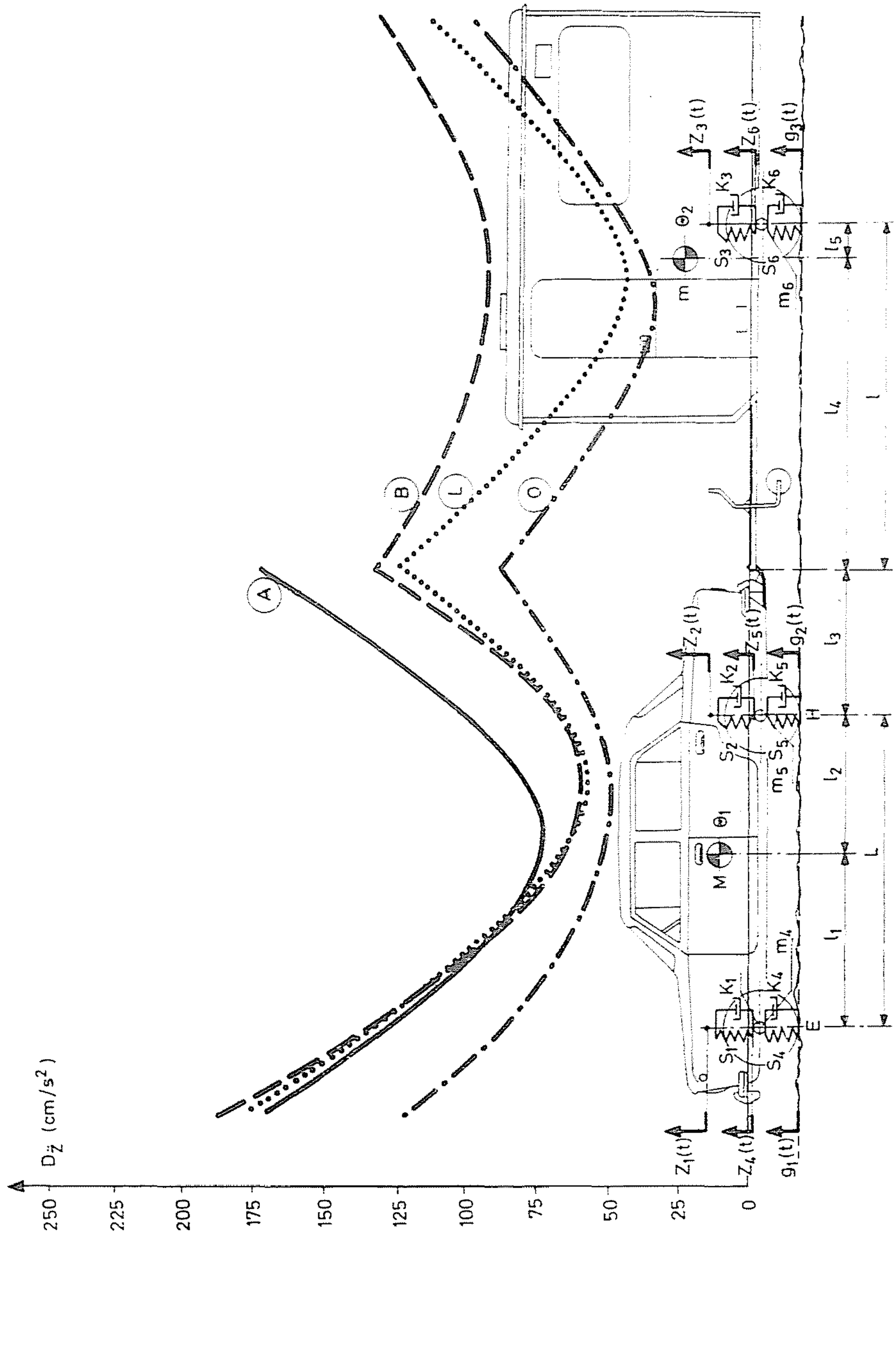
the original trailer. In case of curve "L" the original towing vehicle is coupled with a trailer having optimized suspension parameters and finally, curve "**0**" concerns the vehicle combination obtained by coupling the optimized towing vehicle with the optimized trailer.

Our method of optimization, the objective function and the spring and shock absorber characteristics of the optimized single vehicle were discussed in detail in [2], hence we repeat them here only briefly.

In course of the optimization we neglected the couplings between the swinging systems above the individual axles. The results show that we get considerably better swinging characteristics even if we replace the original system by the so obtained optimized characteristics. Hence the problem is reduced to optimizing a swinging system of two degrees of freedom. We proceed here as follows:

1. We choose a road section spectral density function fitting best to the expected stress.

2. As a first approximation we consider our model linear and determine the amplitude transfer characteristics $|W_z(i\omega)|$ playing an important role in our investigations.


D_z̈ (cm/s²)
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Z1(t)
Z4(t)
g1(t)
Z2(t)
Z5(t)
g2(t)
Z3(t)
Z6(t)
g3(t)
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Fig. 3

3. The optimization is carried out for different speed values v by minimizing objective functions equal to linear combinations of the variances of the output signals most important for the vibrations. Thus we get the optimal damping and spring rigidity factors belonging to different v's.
4. The optimal nonlinear suspension characteristics are determined by looking for those nonlinear characteristics which, statistically linearized at various speeds, best approximate the linear optimum parameters computed in step 3.

In the optimization we considered the bearing spring characteristics of the trailer to be linear around the working point.

For three different linear spring characteristics, namely for those yielding the self angular frequencies $\omega_f = 5, 10, 15$ [rad/s] on the trailer frame, we determined the optimal damping characteristics. These characteristics are symmetric and degressive, see Fig. 4. In our present investigations we used the value $\omega_f = 5$ [rad/s], optimal for the load of the superstructure.

Figure 3 shows that the optimization of the trailer alone does not improve the swinging comfort of the passenger car but decreases considerably the load on the trailer (Curve "L").

On the other hand, curve "O" shows that an optimization of the single passenger car decreases markedly the dynamic load on the trailer. (Of course, the swinging comfort of the single passenger car improves also considerably after the optimization.)

Table IV. shows that while the optimization of the trailer does not influence considerably the other swinging parameters of the towing vehicle, an optimization of the single passenger car improves the running stability of the

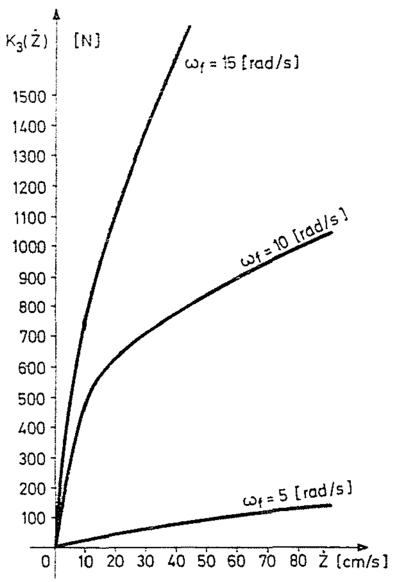

Fig. 4

Table IV

Vibration parameters	B.	L.	O
$D(\ddot{z}_1)$ [cm/s²]	139.10	135.59	86.07
$D(\ddot{z}_2)$ [cm/s²]	71.19	68.09	55.49
$D(\ddot{z}_3)$ [cm/s²]	95.85	51.03	44.24
$r(z_1, z_4)$	0.575	0.572	0.624
$r(z_2, z_5)$	0.478	0.496	0.615
$r(z_3, z_6)$	0.594	0.337	0.349
$D(\alpha)$ [rad]	0.01290	0.01215	0.007813
$D(\beta)$ [rad]	0.00887	0.01354	0.010621
ST_1 [%]	15.95	15.74	9.26
ST_2 [%]	14.29	14.19	8.09
ST_3 [%]	7.95	15.91	10.65
$r(z_4, g_1)$	0.996	0.967	0.976
$r(z_5, g_2)$	0.964	0.964	0.974
$r(z_6, g_3)$	0.988	0.956	0.957
S_{1eff} [N]	919.1	902.4	463.8
S_{2eff} [N]	620.6	599.8	391.7
S_{3eff} [N]	499.0	322.2	285.0
K_{1eff} [N]	281.5	279.3	404.6
K_{2eff} [N]	289.6	287.3	380.5
K_{3eff} [N]	242.2	56.2	55.6
P_1 [W]	63.30	62.46	65.49
P_2 [W]	63.54	62.59	57.96
P_3 [W]	44.62	16.16	15.79
$P_{\ddot{o}}$ [W]	189.50	164.63	158.56

trailer (ST_3) and decreases further the dynamic load on the bearing springs of the trailer (S_{3eff}).

The optimization has also a favourable effect on the total power loss ($P_{\ddot{o}}$) of the vehicle combination.

References

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Appendix

Symbol	Unit	Value	Definition
M	kg	1260	Single car body mass
m	kg	600	Van body mass
m_4	kg	60	Front axle mass
m_5	kg	80	Rear axle mass
m_6	kg	50	Van axle mass
Θ_1	kg · cm^2	1.8×10^7	Moment of inertia of the single motor car body about its centroid
Θ_2	kg · cm^2	8.8739×10^6	Moment of inertia of the van body about its centroid
ϑ_1^2	cm^2	15 000	$\vartheta_1^2 = \frac{\Theta_1}{M}$-square of the inertia radius
ϑ_2^2	cm^2	14 790	$\vartheta_2^2 = \frac{\Theta_2}{m}$-square of the inertia radius
l_1	cm	120.3	Distance of the passenger car body centroid from the front axle
l_2	cm	121.7	Distance of the passenger car body centroid from the rear axle
l_3	cm	110.0	Distance of the rear axle from the drawhead
l_4	cm	247.0	Distance of the drawhead from the van body centroid
l_5	cm	56, 28, 0, −28	Distance of the van body centroid from the van axle
L	cm	242.0	$L = l_1+l_2$
l	cm	275.0	$l = l_4+l_5$
$Z_1(t)$	cm	—	Displacement of the single motor car body above the front axle
$Z_2(t)$	cm	—	Displacement of the single motor car body above the rear axle
$Z_3(t)$	cm	—	Displacement of the van body above the axle
$Z_4(t)$	cm	—	Displacement of the front axle
$Z_5(t)$	cm	—	Displacement of the rear axle
$Z_6(t)$	cm	—	Displacement of the van's axle
$g_1(t)$	cm	—	Road excitation on the first wheel
$g_2(t)$	cm	—	Road excitation on the middle wheel
$g_3(t)$	cm	—	Road excitation on the van wheel
$\alpha(t)$	rad	—	Angular displacement of the passenger car body about its centroid
$\beta(t)$	rad	—	Angular displacement of the van body about its centroid
$\ddot{Z}_i(t)$	cm/s^2	—	Acceleration of the i^{th} displacement coordinate
$D(Z_i)$	cm	—	Standard deviation of the i^{th} displacement coordinate
$D\ddot{z}$	cm/s^2	—	Standard deviation of the vertical acceleration of body points
$r(z_i, z_j)$	—	—	Correlation coefficient of variables Z_i and Z_j
ST_i	%	—	($i = 1, 2, 3$) Stability of the first, second and third axles
S_{ieff}	N	—	Effective mean value of spring forces in the i^{th} suspension ($i = 1, 2, 3$)
K_{ieff}	N	—	Effective mean value of the damping forces in the first and the second suspension ($i = 1, 2$)
P_i	W	—	Effective power absorption in the i^{th} suspension damping ($i = 1, 2$)
P_δ	W	—	Effective power absorption of all the dampings