THEORETICAL STUDY OF DISTURBED BRAKING MANEUVERS*

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Received Apr. 9, 1981

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Introduction

During braking maneuvers the vehicle may lose its directional stability under different conditions, for instance if the slip ratio of rear wheels is greater than of the front ones, and its speed is greater than critical, this critical speed depends on the vehicle design parameters and the operating conditions [1].

In the present work the vehicle is treated as a one mass system with three degrees of freedom. The vehicle is forced to move in horizontal plane and is subjected to small external disturbances. The analysis procedure has been described in [1].

The study does not aim at finding the stability criteria, but at investigating vehicle behaviour, either stable or unstable, during disturbed braking maneuvers, where the vehicle characteristic equation is analysed similar to that of a "spring mass" system with linear damping.

Notations

m = vehicle mass, G = mg.
u = longitudinal speed
v = side-slip velocity
r = yawing rate
F_x = braking force
F_y = side forces
a = distance between the vehicle centre of gravity and the front wheel centre.
b = distance between the vehicle centre of gravity and the rear wheel centre.
l = wheel-base
μ = road coefficient of friction
S_F = front slip-ratio
S_R = rear slip-ratio
C = tyre lateral stiffness
C_s = tyre longitudinal stiffness
p = characteristic equation operator
t = time
φ = phase angle
$2n$ = damping coefficient
a_S^2 = spring constant
T = torque about the vertical axis path through the vehicle centre of gravity
I = inertia of the vehicle (centre of gravity)

* Research work made in the Department of Motor Vehicles, Technical University Budapest

Equations of Motion

The vehicle is simplified to have 3 degrees of freedom, the equations of motion are

$$m(\dot{u} - vr) = \Sigma F_x$$
$$m(\dot{v} + ur) = \Sigma F_y \tag{1}$$
$$I\dot{r} = \Sigma T.$$

Assuming controlled slip-ratios at the front and rear wheels, the longitudinal force F- is constant. According to the assumption of small disturbances in the side-slip velocity, for a yawing rate r the equations become [1];

$$m(\dot{v} + ur) = \Sigma F_y \tag{2}$$
$$I\dot{r} = \Sigma T$$

where

$$\Sigma F_y = F_{yf} + F_{yr}$$
$$\Sigma T = aF_{yf} - bF_{yr} \tag{3}$$

According to Dugoff's tyre model [2], the lateral forces at the front and rear tyres (F_{yf} and F_{yr} resp.) can be determined for $\lambda < 1$ (tyre slip) as follows [1];

$$F_{yf} = (C_\alpha/C_s)_f \cdot \psi_1 \cdot G \cdot \mu \cdot S_F^{-1} \cdot (v + ar)/u = K_F S_F^{-1} (v + ar)/u$$

and (4)

$$F_{yr} = (C_\alpha/C_s)_r \cdot \psi_2 \cdot G \cdot \mu \cdot S_R^{-1} (v - br)/u = K_R S_R^{-1} (v - br)/u$$

where

$$\psi_1 = b/l, \qquad \psi_2 = a/l \tag{5}$$

and

$$K_F = (C_\alpha/C_s)_f \cdot \psi_1 \cdot G \cdot \mu .$$
$$K_R = (C_\alpha/C_s)_r \psi_2 \cdot G \cdot \mu . \tag{6}$$

It is noted that the vertical forces at tyre contact patches are calculated according to the quasi-static assumption, or the ratio of the height of centre of gravity to the wheel-base is assumed to be small.

However, substituting Eqs (4) in Eqs (2), the following system of linear differential equations results.

$$\dot{v} + Av + Br = 0 \tag{7}$$

$$\dot{r} + Cr + Dv = 0 \tag{8}$$

where

$$A = (K_F S_F^{-1} + K_R S_R^{-1})/mu \tag{9}$$

$$B = u + (K_F a S_F^{-1} - K_R b S_R^{-1})/um \tag{10}$$

$$C = (a^2 K_F S_F^{-1} + b^2 K_R S_R^{-1})/Iu \tag{11}$$

$$D = (a K_F S_F^{-1} - b K_R S_R^{-1})/Iu \tag{12}$$

where

$$I = mK^2 \quad \text{and} \quad K^2 \approx ab.$$

Characteristic Equation of the System

To solve the system of linear differential equations (7) and (8) assume

$$v = e^{pt} \quad \text{and} \quad r = e^{pt}. \tag{14}$$

Substituting in Eqs (7), (8) leads to the characteristic equation

$$p^2 + 2np + a_s^2 = 0 \tag{15}$$

where

$$2n = A + C \tag{16}$$

$$a_s^2 = AC - BD. \tag{17}$$

It is interesting to note that Eq. (15) is similar to the characteristic equation of a "spring mass" with linear damping system, namely:

$$2n = \text{damping coefficient}$$

$$a_s^2 = \text{spring constant.}$$

Similar characteristic equation was derived by Ellis [3] for studying the natural frequency of steady-state vehicle motion. But in this equation the spring constant and the damping coefficient did not include the effects of the longitudinal stiffness, road condition or the slip-ratios of the tyres. These important parameters are found in the derived Eq. (15). The effect of these factors on the vehicle mode of motion has to be investigated.

However, the damping coefficient. Eq. (16), can be written as:

$$2n = \frac{\mu g}{u}\left[(C_\alpha/C_s)_f \psi_1 S_F^{-1}(1 + a/b) + (C_\alpha/C_s)_r \psi_2 S_R^{-1}(1 + b/a)\right] =$$

$$= \frac{\mu g}{u}\left[(C_\alpha/C_s)_f S_F^{-1} + (C_\alpha/C_s)_r S_R^{-1}\right] \tag{18}$$

where

$$\psi_1(1 + a/b) = \psi_1 + \psi_1 = 1$$
$$\psi_2(1 + b/a) = \psi_2 + \psi_1 = 1. \qquad (19)$$

Here it is recognized that the damping coefficient is always positive and much depends on the front and rear slip-ratios, lateral and longitudinal stiffness ratios of front and rear tyres, as well as on the speed and the road surface coefficient of adhesion.

The spring constant becomes

$$a_s^2 = \left(\frac{\mu g}{u}\right)^2 (C_\alpha/C_s)_f (C_\alpha/C_s)_r S_F^{-1} S_R^{-1} + \frac{\mu g}{l}\left[(C_\alpha/C_s)_r S_R^{-1} - (C_\alpha/C_s)_f S_F^{-1}\right] \qquad (20)$$

It is clear that the spring constant may be positive or negative. If $a^2 = -ev.$ value the motion will be unstable. On the other hand the motion will be always stable if condition

$$S_R^{-1}(C_\alpha/C_s)_r > (C_\alpha/C_s)_f S_F^{-1} \qquad (21)$$

is satisfied.

Even the stability condition complies with the behaviour of the response terms v and r, it mainly depends on the vehicle speed, tyre characteristics, road condition and slip-ratios. Here the vehicle behaviour can be examined as that of a vibrating system.

Discussion of the Characteristic Equation

The roots of characteristic equation (15) are written in the form

$$P_{1,2} = -n \pm \sqrt{1 - (a_s/n)^2}.$$

It is clear from Eq. 22 that the vehicle motion may be oscillatory or non-oscillatory, the mode of motion depends on the following conditions;

$$1 - (a_s/n)^2 < 1 \text{ non-oscillating}$$

In this case the side-slip velocity and the yaw rate take and exponentional form and the motion is stable if $(a_s/n)^2 > 0$. Here the motion and the time where the response terms vanish are highly dependent on the mentioned factors.

For example: if $S_R = 1$, $S_F = 0.1, = 1$, $u = 1o(m/s)$, $l = 4.5\,(m)$, $g = 9.81$ and $(C_\alpha/C_s)_f = (C_\alpha/C_s)_r = 1$.

Equations (18), (20) yield the values of the spring constant a_s^2, and of the damping coefficient, 2n:

$$a_s^2 = -10 \text{ (unstable motion)}$$

and $2n = 11$.

The motion is exponential, but diverging, where

$$(a_s/n)^2 = -0.33\,.$$

This means that the first root, $P_1 = P$ of Eq. (22) is positive and the disturbed response terms are increasing with time.

Under the same conditions and reducing the speed to 6 m/s, the motion tends to be stable, where

$$a_s^2 = +7.77 \text{ (stable motion)}$$

$$2n = 18.3.$$

It can be noted in the stable region that the time where the response time is nearly zero decreases as the speed decreases as a consequence of the increase in the damping coefficient $2n$ (see Fig. 1).

If the road coefficient of friction is reduced to 0.2, the speed where the motion will transform from stable to unstable will decrease, as indicated in Fig. 1.

(2) If $(a_s/n)^2 = 1$ (critical motion).

In this case the motion will change from oscillatory to exponential, and the damping coefficient is the critical damping. The motion is always stable as shown in Fig. 2. Here it is interesting to find the effect of vehicle speed on the slip-ratio, which is assumed to be equal between front and rear tyres.

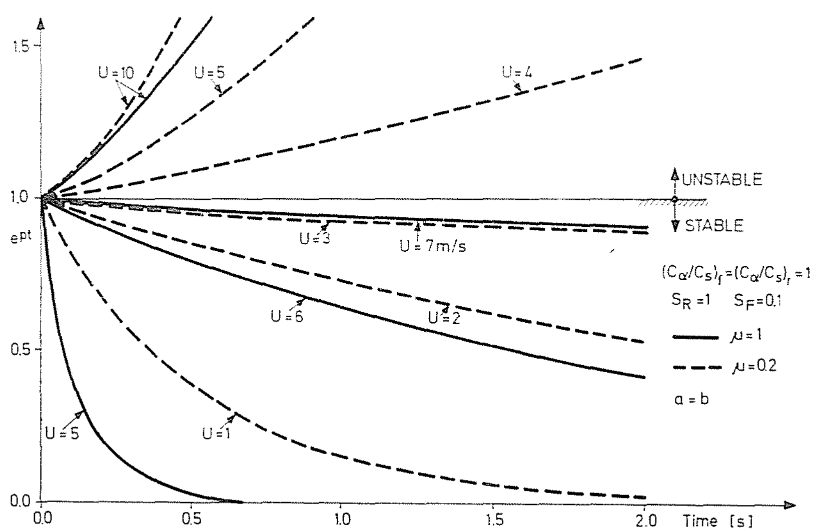

Fig. 1

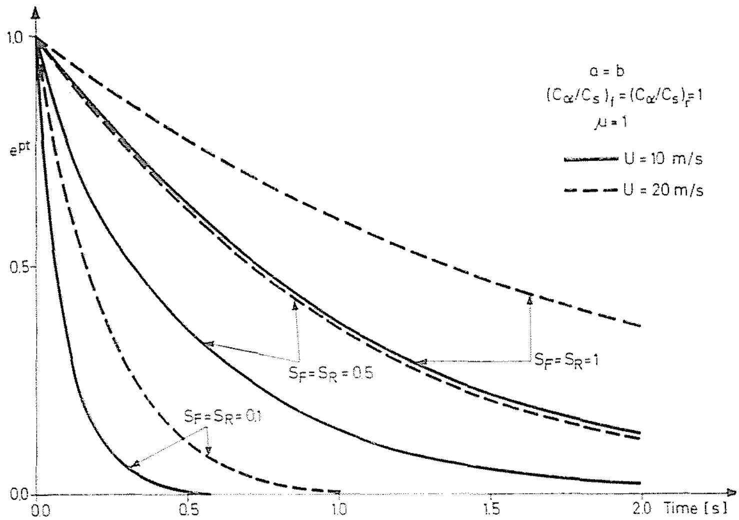

Fig. 2

For example, if

$$(C_\alpha/C_s)_f = (C_\alpha/C_s)_r = 1,\ u = 10\ (m/s),\ g = 9.81\ (m/s^2),\ l = 4.5\ (m)$$

$$a = b,\ m = 1000\ (\text{kg}),\ \text{and}\ S_F = S_R = 0.1$$

then

$$2n = 20$$

$$a_s^2 = 100\ \text{and}\ (a/n)^2 = 1.$$

If $S_F = S_R = 1$

then

$$2n = 2$$

$$a_s^2 = 1\ \text{and}\ (a/n)^2 = 1.$$

Here it is important to see, that the vehicle's damping coefficient and spring constant are highly reduced with increasing front and rear (equal) slip-ratios. This means that the disturbed lateral velocity and yawing rate vanish more quickly with increasing front and rear (equal) slip-ratios (see Fig. 2).

Fig. 2 shows that a result similar to increasing both front and rear slip-ratios is obtained by reducing the speed.

Fig. 3 shows the effect of tyre characteristics assuming constant speed and front and rear tyre slip-ratios. It can be noted that if $(C_\alpha/C_s)_f = 3$ and $(C_\alpha/C_s)_r = 0.3$ (ratios for actually available tyres [2]) the motion changes from stable to unstable at a certain critical speed; in this case, as seen in Fig. 3,

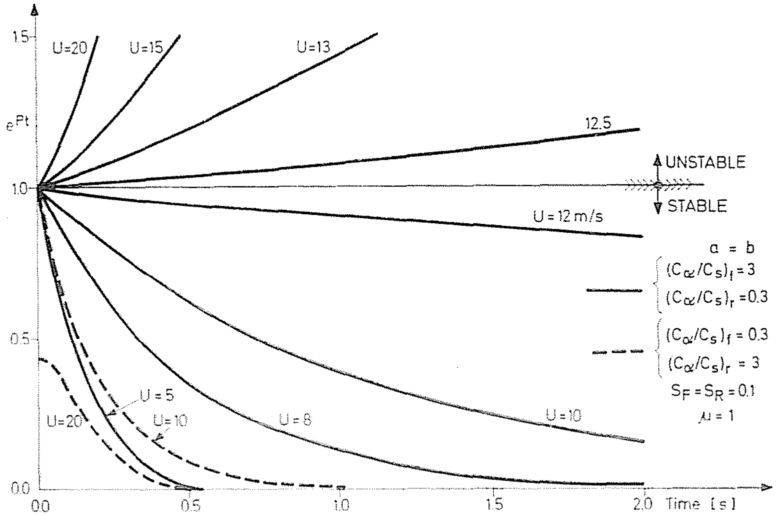

Fig. 3

this critical speed is in the range 12 to 12.5 m/s when $\mu = 1$ (dry asphalt surface). However, if the rear ratio of lateral to longitudinal stiffness is greater than the front one ($(C_\alpha/C_s)_f = 0.3$ and $(C_\alpha/C_s)_r = 3$), the motion is always stable. Here the speed decides if the motion is exponential or oscillatory, in other words, increasing the speed above a certain limit ($u > 15$ m/s) the motion will be oscillatory and below it the motion will be exponential.

The third case is where

$$3 - (a_s/n)^2 > 1 \text{ (Oscillatory motion).}$$

The motion will be oscillatory if the roots are complex, and stable if the real part is negative. In oscillatory motion the side-slip velocity and yawing rate can be represented by the assumed solution e^{pt} as follows:

$$e^{pt} = e^{-nt} \cdot \sin\left(\sqrt{a_s^2 - n^2}\, t + \varphi\right)$$

where

$$\varphi = \text{phase angle} = \tan^{-1}\sqrt{(a_s/n)^2 - 1}$$

and

$$P = -n + jn\sqrt{(a_s/n)^2 - 1}\,.$$

The motion is known to be stable if the slip ratio of the front wheels is greater than that of the rear ones ($S_F > S_R$) [1, 3]. Fig 4 illustrates this fact, where the vehicle behaviour (exponential or oscillatory) is highly affected by the road surface condition. For a high coefficient of friction ($\mu = 1$) the motion

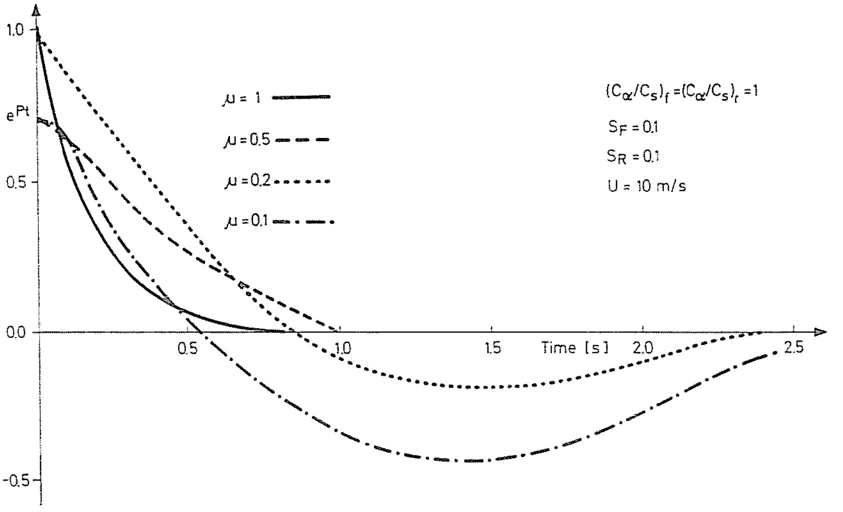

Fig. 4

was exponential and with a decreasing coefficient of friction the motion is oscillatory, with increased amplitudes. These results mean that, even if $S_F > S_R$ the vehicle will not move in straight line but deviate. The deviation highly depends on the road condition.

Summary

The behaviour of vehicle subjected to disturbed braking maneuvers is studied. The vehicle motion is analysed as a vibrating system with a spring constant and damping coefficient. The system is assumed of linear behaviour applying small disturbances in side-slip velocity and yawing rate.

The vehicle behaviour is investigated by plotting the assumed solution of the linear differential equation system.

The tyre characteristics, road surface, speed and slip-ratios have a great effect on changing the motion from exponential to oscillatory or from stable to unstable, enhancing the importance of the previously derived critical speed during braking.

References

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