# NORMALITY ANALYSIS OF DYNAMIC STRESSES IN BUSES DEPENDING ON STOP LENGTHS

By

P. MICHELBERGER, L. ILOSVAI, A. KERESZTES and T. PÉTER

Department of Transport Engineering Mechanics, Technical University Budapest

Received November 11, 1981.

Relying on measurements and statistical analyses, stochastic excitation by the road profile of vehicles traveling at constant speed can be considered as a normal process.

This is little surprising, since in practice, prevalence of the theory of central limit distribution involves frequent processes to be considered as normal at a fair approximation.

Normality is, however, of a high importance, since normal processes are easy to describe statistically. Normal is known to be called a process if it is of normal distribution for statistics of any order [1].

In case of statistics of order $n$, normal distribution of $n$ dimensions is unambiguously determined by the expected value vector and the correlation matrix.

Form the aspect of these analyses, it is advisable to point out some fundamental simplifying statements referring to normality [1], [2], [3], [4].

1. For a normal joint distribution of random variables $\xi_1, \xi_2, \xi_n$, picking out an arbitrary number $k$ of them $(1 \leq k \leq n)$, their entity forms a normal distribution of $k$ dimensions.

2. For a normal joint distribution, and pair-wise uncorrelation of radnom variables $\xi_1, \ldots, \xi_n$, these are independent of each other.

3. If input of a linear system is a normal process, then also its output is a normal process.

4. A normal process and its derivatives make up a normal process together.

5. If $\xi(t, \omega)$ is a continuously differentiable normal stationary process, and $M[\xi(t, \omega)] = 0$, then zero density $N_0$ of its concrete realization — in other words, the number of zero level intersections referred to unit time — can be determined ($t \in T$ being a set of parameters, and $\omega \in \Omega$ being set of elementary events).

Different terms of identical meaning are:

$$\bar{N}_0 = \frac{1}{\pi}\left[\frac{-R''_{\xi\xi}(0)}{R_{\xi\xi}(0)}\right]^{0,5} = \frac{1}{\pi}\left[\frac{\int_0^\infty f^2 S_{\xi\xi}(f)df}{\int_0^\infty S_{\xi\xi}(f)df}\right]^{0,5} = \frac{1}{\pi}\frac{D(\xi')}{D(\xi)}$$

(Rice's formula)

where $R_{\xi\xi}(\tau)$ — autocorrelation function of $\xi$

$R''_{\xi\xi}(\tau)$ — second derivative of $R_{\xi\xi}(\tau)$ with respect to $\tau$;

$S_{\xi\xi}(f)$ — spectral density function of $\xi$

$D(\xi)$ — standard deviation of $\xi$

$D'(\xi')$ — standard deviation of the first derivative of $\xi$.

In the following, the travel process of local transit buses between two stops, approximated by a so-called "trapezium" speed-time travel diagram has been examined, taking also nodding vibrations at start and slowing down into consideration [5].

Suspension characteristics of the tested bus were linearized [6]. Samplings by equidistant divisions along the road length of the excitation by the road profile of a specified spectral density generated by a computer showed a normal distribution. Investigations aimed at determining the stop lengths that under the described circumstances permit to consider input (exclusively the excitation by the road profile) and output of the vibrating system of a vehicle as an approximately normal processes.

1. Digital simulation analysis of vibrations

The model is seen in Fig. 1. The vibration process of local transit buses following from their special mode of operation has been examined earlier [5].

For model parameter values see [5]. The program written in ALGOL language for a digital computer then available has since much been developed.

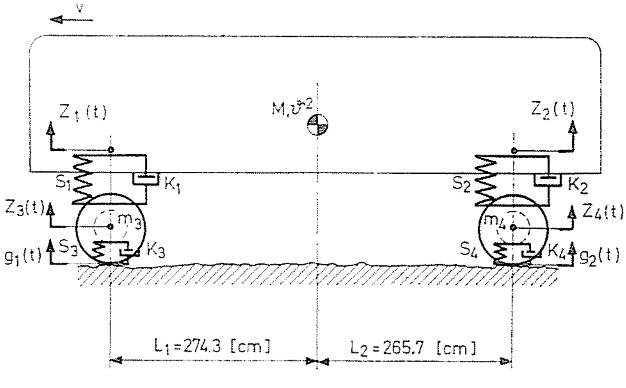



*Fig. 1*

In the actual study, for the normality analysis of output signals of the vibrating system, numerical determination of empirical distribution functions was indispensable.

Actually, computer outputs of the trapezium diagram of travel invariably indicated acceleration values in 1 $(m/s)^2$, and deceleration values in 2 $(m/s^2)$.

Stop distances varied from $L = 400$ (m) to 1600 (m), with $\Delta L = 200$ (m) increments.

In simulation runs, time interval for the numerical integration was chosen as $\Delta t = 0.02$ (s).

## 2. Analysis of the input normality

Road excitation was simulated in a digital computer as decribed in [5] and [6]. Spectral density function of excitation is identical with that of real, measured road profiles.

In our tests, the bus was driven on asphalt pavement.

One method of normality analysis is the graphic one, applied by us in preliminary examinations. Empirical distribution function or road excitation has been plotted on Gaussian paper (Fig. 2). $N/m, \sigma$ is known to be the distribution function of a normal distribution of expected value $m$ and standard deviation $\sigma$, for an arbitrary $m$ and $\sigma$, represented by a straight line on Gaussian paper. Distribution function of road excitation samples taken at intervals of 0.02 ($s$) for a constant travel speed $V = 50$ (km/h) — plotted in smooth line — is rather close to this straight line.

Distribution function plotted in dash line in Fig. 2 referring to road excitations sampled at intervals 0.02 (s) for the shortest theoretical trapeziodal travel diagram perspicuously deviates from the straight line.

In this case, with varying speed along the road length, the spacing of sampling spots varies. At the start of the vehicle, road excitations were densely sampled, followed by increasing sample spacings, finally, at constant speed, samplings became equidistant. Upon braking, sampling spots density towards the stop.

In the case of a fixed, single trapezoidal speed vs. time travel diagram, this phenomenon biasses the input statistics. Normality is offset by too close start and stop points. Estimated fitting test result of normality versus stop length $L$ are seen in Fig. 3. Now, only curve "$g$" referring to the input (road excitation) normality examination will be considered.

Normality has been checked by $\chi^2$ test. Accelerations $a_1$, decelerations $a_2$ and maximum speed parameters $V_{\max}$ of the trapezoidal travel diagram have throughout been recorded: $a_1 = 1$ [m/s$^2$], $a_2 = 2$[m/s$^2$], $V_{\max} = 50$ [km/h]. Thereafter road excitation samples belonging to trapezoidal travel diagrams for different stop lengths $L$ have been determined. Samples were applied to calculate $\chi^2$ values needed for normality examination.

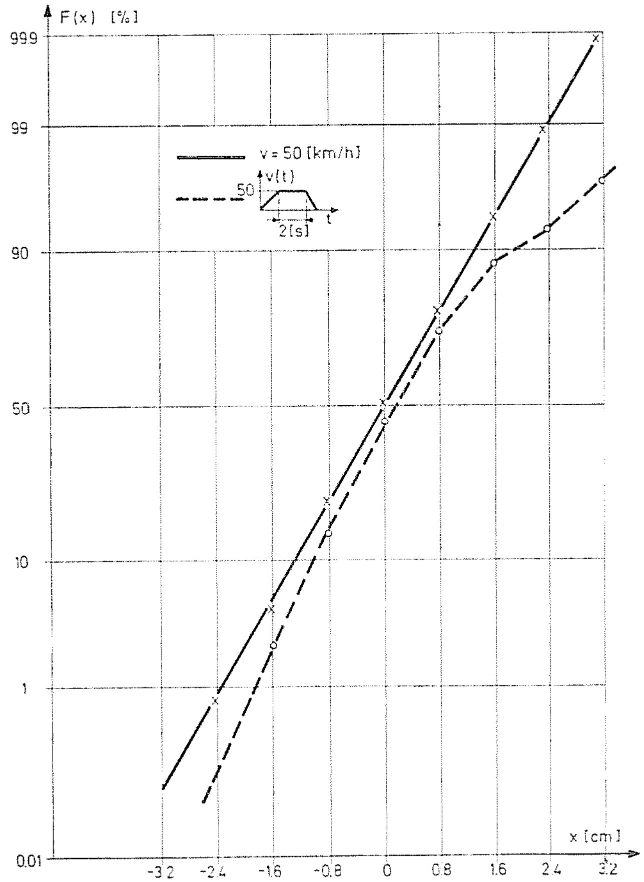
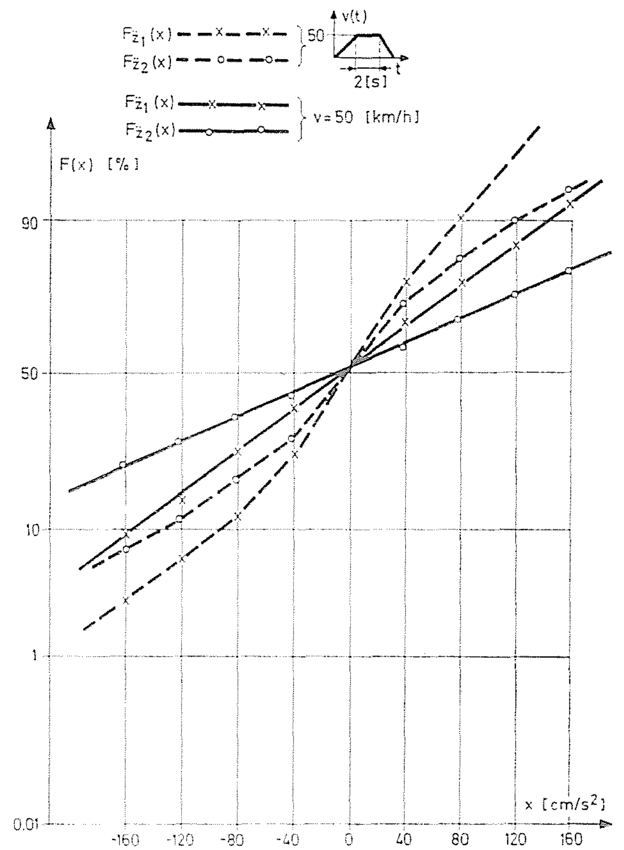



*Fig. 2*

Vertical axis of Fig. 4 shows probability percentages $p$ determined from the $\chi^2$ table belonging to our calculation result, permitting to draw conclusions on our hypothesis of normality. As expected, with increasing stop lengths, probability of the normality hypothesis to become true increases. Accepting the level $p = 70\%$, road excitation can be stated to be of normal distribution for a stop length of 800 m.

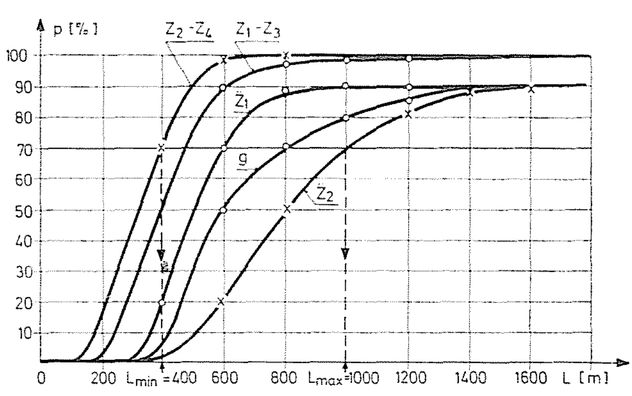
Throughout the examinations, stationarity and ergodicity are assumed, this is why in connection with the examination of a finite number of processes can be spoken of, that is, all other realizations can be concluded on [7].

The samples contained invariably more than 800 elements, at 30 degrees of freedom cn view of the high number of sample elements, the hypothesis was accepted over $p = 70(\%)$.



$F_{\ddot{z}_1}(x)$
$F_{\ddot{z}_2}(x)$
50
v(t)
t
2 [s]
$F_{\ddot{z}_1}(x)$
$F_{\ddot{z}_2}(x)$
v = 50 [km/h]
F(x) [%]
90
50
10
1
0.01
x [cm/s²]
-160 -120 -80 -40 0 40 80 120 160


*Fig. 3*



p [%]
$Z_2 - Z_4$
$Z_1 - Z_3$
$\ddot{Z}_1$
g
$\ddot{Z}_2$
100
90
80
70
60
50
40
30
20
10
0 200 $L_{min}$ = 400 600 800 $L_{max}$ = 1000 1200 1400 1600 L [m]


*Fig. 4*

### 3. Output normality examination

If the vibrating system really has only the road excitation as input, and the system itself is linear, then introductory statements 3 and 4 are valid. Though, concrete statistical tests have led to realizations of normalities different from that of the input as outputs. In the actual case, in addition to the road profile excitation, the vehicle is also excited by the speed variation along the travel direction. Hence, this phenomenon entrains the superposition of so-called nodding vibrations onto vibrations due to road excitation, another phenomenon justifying output normality examination.

The preliminary normality test for vibration acceleration of the car body is shown in diagram 3 plotted on Gaussian paper. $\ddot{Z}_1$ and $\ddot{Z}_2$ are vertical vibrational accelerations above the fore and aft axles, respectively. Smooth and dash lines in diagram 3 refer to vibration acceleration distribution functions for a constant speed $v = 50$ km/h and for the shortest theoretical trapezoidal travel diagram, respectively, plotted on Gaussian paper.

Preliminary examinations unambiguously demonstrated the output normality hypothesis to hold for even speeds, while it is to be discarded for short stop lengths of a few hundred m.

Stop length dependent normality examination results of some output displays checked by the $\chi^2$ test have been plotted in diagram 4.

Also in this case, the vehicle was driven according to the presented theoretical trapezoidal travel diagram.

$Z_1 - Z_3$, and $Z_2 - Z_4$ indicate $\chi^2$ test results for the relative displacements of the car body and the fore and aft axle, respectively. This test is of importance for the suspension spring stress analysis.

With increasing stop lengths, normality probabilities of output displays are seen to increase differently.

Again, it can be stated that normality can be spoken of even for idealized travel diagrams, for stop lengths of 400 to 1000 m.

## Conclusions

1. Statistic examinations of normality demonstrated in case of theoretical trapezoidal travel diagrams the input of vehicle vibrating systems (road excitation) not to be normal any more for $L < 800$ m, because of the uneven sampling.
2. Considering a single theoretical travel diagram, the output itself is other than normal for $L < 400$ m.
3. Diagram 4 informs on the access to normality, that can already be assumed for stop lengths $L = 400$ to $1000$ m.

4. The main goal of our investigations was to determine an upper bound of stop lengths in the matte of normality. In real travel diagrams also accelerations, decelerations and maximum speeds vary, hence they can be considered as random variables.

Thus, the central limit distribution may be assumed to prevail in e.g. a real travel diagram referring to several stop lengths of 400 m. Accordingly, a joint statistics involving several stop lengths of 400 m is likely to be of a stronger normality than the result of our investigation on a single, idealized travel diagram. Of course, exact confirmation of this assumption requires to perform further examinations.

## Summary

The Gaussian process character of stochastic input and output is a question of importance from several aspects of the dynamic design of vehicle structures. This hypothesis is true for the stochastic input (road excitation) of vehicles traveling at uniform speed. Vibration systems exhibiting a Gaussian input have also a Gaussian process at output.

Over road lengths between two stops in urban traffic, processes of acceleration at start and of deceleration before stopping bias the Gaussian process character of input and output. Tests made with increasing stop lengths aimed at finding the stop length where input and output of the bus vibrating system can again be considered as Gaussian processes.

## References

1. FREY, T.: Stochastic Processes, Tankönyvkiadó, Budapest (1970).
2. PRÉKOPA, A.: Theory of Probability, Műszaki Könyvkiadó, Budapest (1972).
3. BENDAT, J. S. Ph. D.: Principles and Applications of Random Noise Theory. John Wiley Sons, Inc. New York.
4. JÁNOSDEÁK, E.: Járművek, Mezőgazdasági gépek *24* (1977), 8.
5. ILOSVAI, L.—KERESZTES, A.—MICHELBERGER, P.—PÉTER, T.: Periodica Polytechnica, Vol. 7. No. 2. (1979).
6. PÉTER, T.: Doctor Techn. Thesis, Budapest, (1977).
7. FARKAS, M.—FRITZ, J.—MECHELBERGER, P.: Acta Technica Hung. 91 (1981) №. 3—4. (In press).

Prof. Dr. Pál MICHELBERGER
Prof. Dr. Lajos ILOSVAI
Dr. Albert KERESZTES
Dr. Tamás PÉTER

H-1521 Budapest