APPROXIMATIVE EVALUATION OF HEAT TRANSFER EXPERIMENTS IN THE TEST SECTION OF A DOUBLE-WALLED TUBE

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For designing heat exchange devices operating under normal conditions, the designer may choose between reliable engineering methods, offering, however, few data needed for calculating the temperature distribution in parts subjected to high mechanical and thermal stresses (as e.g. the rotor of electric revolving machines or blades of a gas turbine). Though, the aspects of utmost exploitation of insulating and structural materials and of safety can only be agreed in exact knowledge of place and extent of temperature peaks.

The primary condition for carrying out such calculations is information about the local values of the heat transfer coefficient.

The heat transfer process being in close interaction with the structure of the flow developing in the duct, the phenomenon is too complex to permit theoretical deduction of sufficiently exact information. All these point to the importance of model tests for determining the local heat transfer as a function of systematically varying hydrodynamic characteristics.

Some methological problems of the measurements will be analyzed, with special regard to conditions to be satisfied, permitting an important simplification of processing data, measured in a double-walled test section rather convenient in tests in revolving systems. Omitting consrtuction details, the test section is discussed only to an extent necessary for this purpose.

According to a definition befitting engineering calculations — either dimensioning or control — the local heat transfer coefficient $\alpha(\varphi; z)$ is the ratio of the heat flow density leaving the wall surface moistened by the flowing medium at a given point to the difference between the wall temperature at the same point and the characteristic medium temperature (see Eq. (6)).

The heated part — the test section — of the experimental channel serving for the measuring $\alpha(\varphi; z)$ is a double-walled tube (See Fig 1). The thin-walled lining — or insert tube — is made of stainless steel of a good thermal conductivity, and of a high strength, to act as load bearing structure of the test section. In a revolving system it can absorb also the load imposed

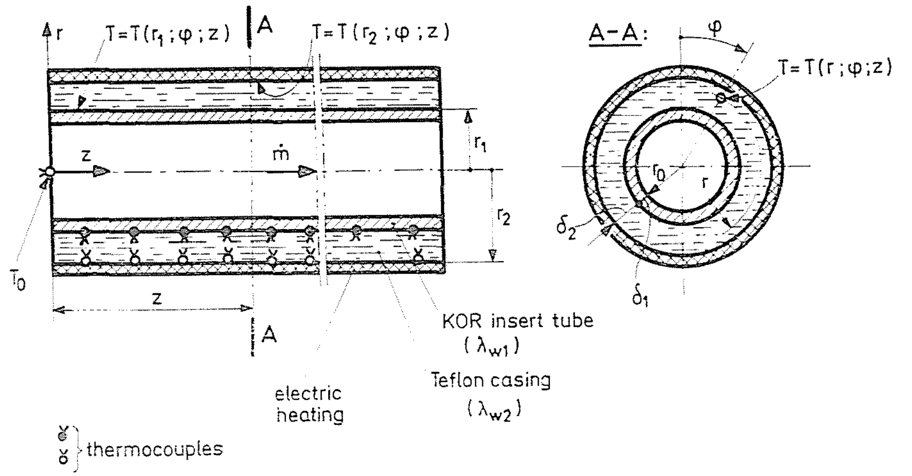

Fig. 1

by the centrifugal force. The heat flux is measured by the casing around the insert tube. To obtain an adequate accuracy, it is made from a synthetic material of poor thermal conductivity (e.g. Teflon) with a relatively large wall thickness. The thermocouples are placed on the internal and the external surfaces of the casing. To avoid technical difficulties, the internal thermocouples are in fact placed on the external surface of the insert tube containg the casing. Temperature data measured by them are used in forming both the heat flux and the temperature development in the channel wall. Thermal flux is generated by electric heating surrounding the casing. Temperature T_0 of the coolant entering the test section is measured by the thermocouple at the centerline channel. Measurement of the mass flow $\dot{m}$ outside the test section can be realized by any suitable method.

The local heat transfer coefficient as a test result becomes accessible only after having processed the primary measured data directly recorded by the measuring converters (metering orifice, thermometers etc.). Methods strictly pursuing the process are in general extremely labour-consuming. Namely the heat flux entering into the fluid has to be computed by forming the gradient of the tube wall temperature field on the surface bounding the flow, conditioned, in turn, by the previous solution of the differential equation describing the temperature field, taking the boundary conditions provided by the measurement data into consideration.

Making use of possibilities offered by the experimental device to get suitable simplifying assumptions, the work of data processing can be reasonably limited without impairing the expected exactness of the results.

In the actual case of chief problem is due to that in the test section the flow is not axisymmetrical resulting in a three-dimensional heat flow in the tube wall. Thus the temperature field of the wall in steady-state thermal condition is described in the cylindrical coordinate system $(r; \varphi; z)$ by the Laplacian differential equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{1}$$

to be solved under the full effect of the mentioned difficulties. The data processing would be much simplified by the approximation that the heat flux in the wall is one-dimensional with only radial components. This approximation can be accepted if the first term of the Laplacian equation much exceeds the second and third terms. Let us examine the conditions of the above.

As concerns the assessment of the orders of magnitude of the equation terms:

- $\mathcal{O}\{A\}$ is the symbol of the order of a quantity A.
- By order of the derivative of a variable the order of the change ratio is understood and its value — in lack of an analytical relationship — is approximated with the quotient of the supposed change by the range change.
- By order of product, the product of the order of the factors is understood.

For quite a rough approximation of the order of the ratio of the first to third terms of the Laplace equation, the radial temperature drop in the tube wall ΔT_r and the channel radius r_0 are chosen as units. Since the temperature drop ΔT_r occurs in the wall of thickness δ and the order of variable r is r_0, introducing the symbol $\Delta = \delta / r_0$, the order of the first term is estimated at:

$$\mathcal{O}\left\{\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right)\right\} = (1) \left(\frac{1}{\Delta}\right) (1) \left(\frac{1}{\Delta}\right) = \frac{1}{\Delta^2}.$$

(The orders of the coefficients are separately indicated in parentheses.)

The order of the third term is obtained considering that the change of the wall temperature along the channel length is considerable only in the thermal entrance length. Many researchers have found this length to be 10 to 15 times the tube diameter or more. For the estimation the most unfavourable instance is considered, where the axial temperature change ΔT_z reaches the order of the radial temperature drop halfway on the thermal entrance length rising section i.e. a length of $z = (10 \div 15)\ r_0$. Expressing variable z and change ΔT_z to scales of r_0 and ΔT_r, respectively, even in the most unfavourable instance the order of the third term of the Laplace equation is estimated at:

$$\mathcal{O}\left\{\frac{\partial^2 T}{\partial z^2}\right\} = \mathcal{O}\left\{\frac{\partial}{\partial z} \left(\frac{\partial T}{\partial z}\right)\right\} = \left(\frac{1}{10}\right) \left(\frac{1}{10}\right) = \frac{1}{100}.$$

Thus, rough calculations show for $r_0 = 5 \cdot 10^{-3}$ m the first term of the Laplace equation to be by four orders greater than the third one in an insert tube of wall thickness $\delta_1 = 0.25 \cdot 10^{-3}$ m ($\Delta = 1/20$) and about for hundred times that in the casing of wall thickness $\delta_2 = 2{,}5 \cdot 10^{-3}$ m.

To estimate the relation between orders of the first and second terms, a less formal consideration is applied than the previous one, reflecting better the physical principle.

Its result will be expressed in a form expliciting the condition to be satisfied by the measuring data to omit the effect of temperature change along the circumference in the Laplace equation.

The uneven temperature distribution with respect to the polar angle in any cross section is described by the difference ΔT_φ between the highest and least local value along the circumference with a radius r_1. If the flow structure in the tube cross section is symmetrical measured about any diameter, this difference is expected to develop on half the channel circumference (say in the angle range $0 \leq \varphi \leq \pi$) thus the second term in the Laplace equation is of the order:

$$\mathcal{O}\left\{\frac{1}{r^2}\,\frac{\partial^2 T}{\partial \varphi^2}\right\} = \frac{1}{r_1^2}\,\frac{\Delta T_\varphi}{\pi^2}\,.$$

For estimating the first term, the Fourier law can be used, expressing the relationship between the derivative $\partial T/\partial r$ and the radial heat flux. Obviously, the order of the radial heat flux equals the mean value of the heat flux density passing from the tube wall across the casing surface of radius r_0 to the flow. Its value is obtained from product $\alpha \cdot \Delta T_0$ where α is the mean heat transfer coefficient, ΔT_0 the difference between the average wall surface temperature and the characteristic temperature of the flow, the so-called temperature step. Thus, denoting the heat conductivity of the material of the insert tube by λ_{w1}, the order of the first term may be written as

$$\mathcal{O}\left\{\frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial T}{\partial r}\right)\right\} = \frac{1}{r_0}\,\frac{1}{\delta_1}\left(r_0\,\frac{\alpha\,\Delta T_0}{\lambda_{w1}}\right) = \frac{1}{\delta_1}\,\frac{\alpha\,\Delta T_0}{\lambda_{w1}}\,.$$

The estimation is valid also for the temperature field of the casing if δ_1 and λ_{w1} are replaced by the casing wall thickness δ_2 and its heat conductivity λ_{w2}, resp., as the radial heat flux is also of order $\alpha\Delta T_0$ in the casing wall. Obviously the importance of the second term of Laplace equation becomes insignificant compared to the first term if inequality

$$\frac{1}{r_1^2}\,\frac{\Delta T_\varphi}{\pi^2} \ll \frac{1}{\delta_i}\,\frac{\alpha\,\Delta T_0}{\lambda_{wi}}$$

exists. (For the casing $i = 2$, for the insert tube $i = 1$.) After some transformation, finally the condition

$$\frac{\Delta T_\varphi}{\Delta T_0} \ll A \left(\frac{r_1 \pi}{\delta_1}\right)^2 Bi \tag{2}$$

is obtained. Depending on whether the condition has to be referred to the temperature field of the insert tube or of the casing, coefficient A has to be taken as:

$$A = 1 \text{ (for the insert tube)},$$

$$A = \frac{\delta_1}{\delta_2} \cdot \frac{\lambda_{w1}}{\lambda_{w2}} \text{ (for the casing)}.$$

Coefficient Bi is the well-known non-dimensional characteristic of heat flow problems, the Biot number, defined as

$$Bi = \frac{\alpha \delta_1}{\lambda_{w1}} \tag{3}$$

(its physical meaning will be considered later).

Nonuniformity ΔT_φ has to satisfy condition (2) to neglect the wall temperature change along the circumference.

For an insert tube made of stainless steel ($\lambda_{w1} = 20$ W/mK) and casing of Teflon ($\lambda_{w2} = 0.23$ W/mK), in case of a mean value of $\alpha = 200$ W/m²K for the thermal conductivity ($Bi = 0.0025$), even a nonuniformity $\Delta T_\varphi/\Delta T_0 = 0.1$ along the circumference yields that the right-hand side of condition (2) is about 110 times the left-hand side for the insert tube and about 950 times for the casing. It is easy to understand that these numbers indicate at the same time the relative importance of the first term of the Laplace equation compared to the second term.

Remembering the ratio of estimated orders of the first to the third term, the order of magnitude analyses lead to the conclusion that

the heat flux in the test section wall is approximately radial and the temperature field is described by the ordinary differential equation

$$\frac{1}{r} \frac{d}{dr}\left(r \frac{dT}{dr}\right) = 0. \tag{4}$$

Let us consider now particulars of the data processing method. In the test section the thermocouples placed on the external surface (of radius r_2) and the internal surface (of radius r_1) of the casing are fitting tightly the insert tube measuring the temperature sets $T_2 = T(r_2; \varphi; z)$ and $T_1 = T(r_1; \varphi; z)$, respectively. These data cannot be directly used but for computing the heat flux density in the casing. Specifying the two sets — two numerical functions — as boundary conditions, the temperature field in the casing is obtained by solving the differential equation (4), from which the heat flux can be expressed in terms of Fourier's heat conductivity law. Omitting the details, in final account the value of the heat flux passing from the casing to the insert tube is:

$$\dot{q}(r_1; \varphi; z) = \lambda_{w2} \frac{T(r_2; \varphi; z) - T(r_1; \varphi; z)}{r_1 \ln \frac{r_2}{r_1}} . \tag{5}$$

The interpretation of the local heat transfer coefficient involves two characteristics based on temperature distribution $T(r_0; \varphi; z)$ at the flow-moistened surface of radius r_0. (But $T(r_0; \varphi; z)$ will not be measured, because of technical reasons.) Namely in the interpretation according to the generally applied definition

$$\alpha(\varphi; z) = \frac{\dot{q}(r_0; \varphi; z)}{T(r_0; \varphi; z) - T_f} \tag{6}$$

beside the reference temperature difference $\Delta T_0 = T(r_0; \varphi; z) - T_f$ also the heat flux

$$\dot{q}(r_0; \varphi; z) = \lambda_{w1} \frac{T(r_1; \varphi; z) - T(r_0; \varphi; z)}{r_0 \ln (r_1/r_0)} \tag{7}$$

passing to the flow across the moistened — reference — surface involves the surface temperature. However, this definition is to be retained, therefore, the measuring data processing formulae have to be developed in a way not to contain the temperature $T(r_0; \varphi; z)$.

Flux $\dot{q}(r_0; \varphi; z)$ is easy reduce to flux $\dot{q}(r_1; \varphi; z)$ containing only measuring data. Namely, since according to approximation (4) the local heat flux in the insert tube wall is inversely proportional to radius r and since in crossing the casing/insert tube boundary surface the radial component of the heat flux density vectors remains continuous, the surface heat flux becomes:

$$\dot{q}(r_0; \varphi; z) = \lambda_{w2} \frac{T(r_2; \varphi; z) - T(r_1; \varphi; z)}{r_0 \ln (r_2/r_1)} . \tag{8}$$

To eliminate the temperature step ΔT_0 a fictive heat transfer coefficient $\alpha_m(\varphi; z)$ is defined, the reference surface of which is the moistened surface in accordance with $\alpha(\varphi; z)$, but its reference temperature difference is the complet temperature step $\Delta T_t = T(r_1; \varphi; z) - T_f$.

Thus, be

$$\alpha_m(\varphi; z) \equiv \frac{\dot{q}(r_0; \varphi; z)}{T(r_1; \varphi; z) - T_f} . \tag{9}$$

Now it is shown that in data processing, $\alpha(\varphi; z)$ can be approximated by $\alpha_m(\varphi; z)$, indicating also the resultant systematic error.

Expressing from (9), (6) and (7) the complete temperature step ΔT_t, the temperature step ΔT_0, as well as the temperature drop $T(r_1, \varphi; z) - T(r_0; \varphi; z)$ across the insert tube wall, respectively, and taking into account that the complete temperature step is the sum of both latter, then obviously:

$$\frac{1}{\alpha_m} = \frac{1}{\alpha} + \frac{r_0 \ln (r_1/r_0)}{\lambda_{w1}} = \frac{1}{\alpha} + \frac{r_0 \ln \left(1 + \frac{\delta_1}{r_0}\right)}{\lambda_{w1}} .$$

Since with the insert tube dimensions $\delta_1/r_0 = 0.05 \ll 1$, the logarithmic function can be approximated with its first-order Taylor polynomial. With eqn. (3) thus becomes

$$\alpha = \alpha_m(1 + Bi). \tag{10}$$

The error from approximating the actual heat transfer coefficient by a fictive α_m value referred to the complete temperature step easy to measure, is seen to be equal to the Biot number, i.e. in average some permille as seen above. This low value of the Biot number indicates the very poor resistance of the insert tube wall to the heat flow passing from the casing to the flowing coolant, whereas the resistance of the convective heat transfer on the moistened surface is prevalent. This state of things further reduces the importance of the error committed by replacing Laplace equation (1) for the insert tube by Eq. (4) — as an exact computation would entrain but a slight change in the earlier estimated value.

After having substituted temperature $T(r_0; \varphi : z)$ of the moistened surface in every respect let us present the formula for computing the local heat transfer factor from measuring data:

$$\alpha(\varphi : z) = \frac{\lambda_{w2}}{r_0 \ln \frac{r_2}{r_1}} \frac{T(r_2; \varphi; z) - T(r_1; \varphi; z)}{T(r_1; \varphi : z) - T_f} . \tag{11}$$

Neglecting the Biot number compared to unity the formula was written by means of (8), (9) and (10).

Remind that the concept of the heat transfer coefficient calculated according to (11) means quotient of the heat flux passing to the coolant flow by the temperature T_0 of the flow entering the heated section or the mixed mean temperature $T_m(z)$. The first one is the actual temperature uniformly distributed in cross section $z = 0$, the second being the fictive temperature changing from cross section to cross section along the channel length, defined in terms of the enthalpy balance written for the part of the heated section up to the actual cross section.

$$\dot{m}c_p[T_m(z) - T_0] = r_0 \int_0^z \int_0^{2\pi} \dot{q}(r_0; \varphi; z)\, d\varphi\, dz.$$

($\dot{m}$ is the mass flow, and c_p its isobaric specific heat taken to be constant.) Substituting the integrand from (8), in data processing the mixed mean temperature of the fluid can be determined from:

$$T_m(z) = T_0 + \frac{\lambda_{w2}}{\dot{m}c_p \ln \frac{r_2}{r_1}} \int_0^z \int_0^{2\pi} [T(r_2; \varphi; z) - T(r_1; \varphi; z)] \, d\varphi \, dz. \tag{12}$$

In conclusion: for the experimental determination of local heat transfer coefficient $\alpha(\varphi; z)$ defined by Eq. (6) the following data have to be directly measured:

— temperature distribution $T(r_2; \varphi; z)$ on the external casing surface,

— temperature distribution $T(r_1; \varphi; z)$ on the internal casing surface fitting to the insert tube,

— temperature T_0 of the coolant before entering the heated test section,

— mass flow $\dot{m}$ of the coolant in the test section.

The value of $\alpha(\varphi; z)$ is computed from these data by means of Eq. (11). Characteristic temperature T_f is replaced either by T_0, or by $T_m(z)$ determined according to (12).

Summary

The laboriousness of the experimental determination of the local heat transfer coefficient is substantially reduced by simplifying the evaluation method. The steady-state temperature field of annular tube walls is obtained from the Laplace differential equation. For an other than constant convective heat transfer coefficient along the tube circumference, the temperature field is three-dimensional. By analyzing the order of magnitude of the terms in the Laplace equation referred to the cylindrical coordinate system $(r; \varphi; z)$ the conditions to be satisfied for an adequate approximation of the wall heat flux density using the one-dimensional Laplace equation have been established. A simple approximate method has been presented for processing temperature data registered in a double-walled test section, at a reasonable restriction of evaluation work.

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