SOME ADDITIONAL REFLECTIONS ON THE VISCOUS FLOW IN A VORTEX CHAMBER

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1. As earlier pointed out [1] the flow within a vortex chamber is best characterized by the Navier—Stokes equations.

The vortex chamber actually constitutes a blockpulley (shell) where the primary fluid arrives tangentially and becomes mixed parallel to the shell axis with the secondary fluid sucked in by the arising vacuum, subsequently the mixture leaves at the flange of the circular opening in the centre of the blockpulley vortex chamber. In this paper the flow relations enabling the determination of the mixing ratio are investigated.

The Navier—Stokes equations may become basically simplified if certain rational approximations are used. In this paper the problems are discussed which arise if the variation of the viscosity according to position is neglected.

2. In a previous publication [1] a method has been developed to determine the peripheral velocity. According to this model the flow within the vortex has a cylindrical symmetry, consequently the equations are presented in a cylindrical co-ordinate system, whose position with respect to the chamber and the individual velocity components is depicted in Fig. 1.

According to the Newtonian axiom the conservation of the momentum of the fluid is expressed by the following equation:

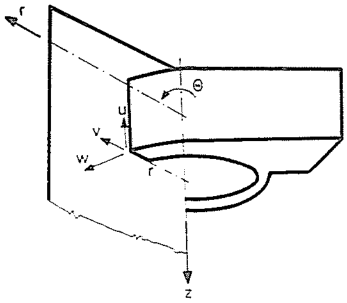
$$\frac{d\bar{c}}{dt} = \bar{g} + \frac{1}{\varrho} \operatorname{div} \bar{\bar{F}} . \tag{1}$$

As it is known [3] the left side term of the equation may be decomposed into two parts, the local and the convective acceleration

$$\frac{d\bar{c}}{dt} = \frac{\partial \bar{c}}{\partial t} + (\bar{c}\, \overline{\nabla})\, \bar{c} = \frac{\partial \bar{c}}{\partial t} + \bar{\bar{D}} \cdot \bar{c}. \tag{2}$$

The right side of the equation, on the other hand, may be reformulated

$$\bar{g} + \frac{1}{\varrho} \operatorname{div} \bar{\bar{F}} = \bar{g} + \bar{\bar{\Phi}} \cdot \overline{\nabla} \tag{3}$$



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where $\overline{\overline{\Phi}} = f(\overline{\overline{D}})$ represents a material equation. For a Newtonian flow one has [2]

$$\overline{\overline{\Phi}} = -p\overline{\overline{E}} + \mu(\overline{\overline{D}} + \overline{\overline{D}}^*). \tag{4}$$

Consequently by using (2) and (3) Eq. (1) can be rewritten to express a universal law of the conservation of momentum valid also for the generalized fluids:

$$\frac{\partial \bar{c}}{\partial t} + \overline{\overline{D}} \cdot \bar{c} = \bar{g} + \frac{1}{\varrho}\overline{\overline{\Phi}} \cdot \nabla. \tag{5}$$

In the case under discussion, however, the flow may be regarded as time constant, i.e.:

$$\frac{\partial \bar{c}}{\partial t} = 0.$$

Further on one knows from experiment and observation that the phenomenon has a cylindrical symmetry, in addition

$$\frac{\partial w}{\partial z} = 0$$

$$\frac{\partial v}{\partial z} = 0 \tag{7}$$

By neglecting the gravitational force the following component-equations are obtained from the Navier—Stokes equation (5):

$$\varrho\left(v\frac{\partial v}{\partial r} - \frac{w^2}{r}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(2\mu\frac{\partial v}{\partial r} - p\right) + \frac{1}{r}\left(p + 2\mu\frac{v}{r}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial r}\right) \tag{9}$$

$$\varrho\left(r\frac{\partial w}{\partial r} + \frac{v \cdot w}{r}\right) = \frac{\partial \mu}{\partial r}\left(\frac{\partial w}{\partial r} - \frac{w}{r}\right) + \mu\frac{\partial^2 w}{\partial r^2} - \mu\left(\frac{\partial w}{r\partial r} - \frac{w}{r^2}\right) + \frac{2}{r}\mu\left(\frac{\partial w}{\partial r} - \frac{w}{r}\right) \tag{10}$$

$$\varrho\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right) + \frac{\partial}{\partial z}\left(2\mu\frac{\partial u}{\partial z} - p\right). \tag{11}$$

Of course, in this equation system $\mu = \varrho\nu$ represents the dynamical and ν the kinetical viscosity. In our case the viscosity consists of two components, the Newtonian and the turbulent viscosity

$$\nu = \nu_0 + \nu_t \tag{12}$$

As a rule also in this case the first term is with several orders of magnitude smaller than the turbulent viscosity, for this reason one may write

$$\nu \cong \nu_t \,. \tag{13}$$

According to our investigations the shear force plays a considerably more important role in the peripheral than in the other direction, consequently the following assumption, i.e. approximation may be used

$$\frac{\partial v}{\partial r} = 0 \quad \text{and} \quad \frac{\partial v}{\partial z} = 0 \,. \tag{14}$$

As a consequence of Eq. (10) relations suitable for the quantitative analysis of the peripheral velocities are obtained. Of coures, also the continuity relation must be taken into account

$$\frac{\partial}{\partial z}(ru) + \frac{\partial}{\partial r}(rv) = 0 \,. \tag{15}$$

In Eq. (10) the term $\frac{\partial v}{\partial r}\left(\frac{\partial w}{\partial r} - \frac{w}{r}\right)$ refers to the fact that the viscosity is not a constant quantity. By neglecting this term, i.e. by assuming that the viscosity is in good approximation after all constant, one obtains the well-known equation describing the laminar flow [3].

Eq. (10) enables the evaluation of the turbulent viscosity from experimental data. If Eq. (10) is made dimensionless by relating the length to the radius of the chamber, and the velocity at the edge of the chamber to the peripheral velocity valid for the co-ordinates of the maximal radial direction one gets the equation

$$L'\left[W' - \frac{W}{R}\right] + L\left[W'' + \frac{W}{R} - \frac{W}{R^2}\right] - \left[VW' + \frac{VW}{R}\right] = 0 \tag{16}$$

which is for $L = \frac{v}{W_1 R_1}$ a first order, linear differential equation. This equation can be solved if the V and W functions are known from measurement.

The solution for an actual case is demonstrated as an example in Fig. 2 for the case $X = 1/3$ [1].

The figure contains also the approximation of the previously used broken-line viscosity distribution. Fig. 3 compares the mixing path-length values as calculated by Escuidier's formula with the experimental results.

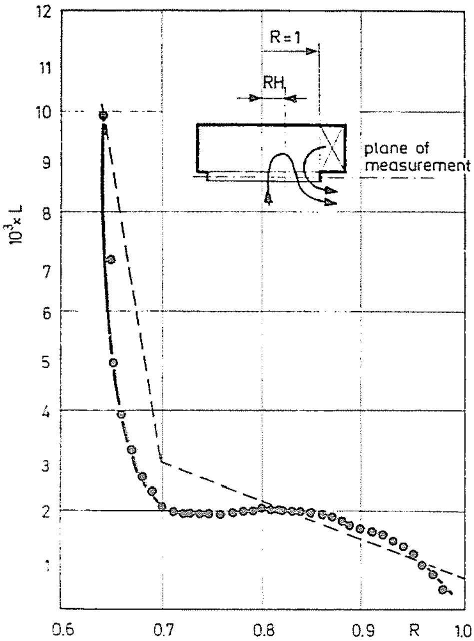

Fig. 2

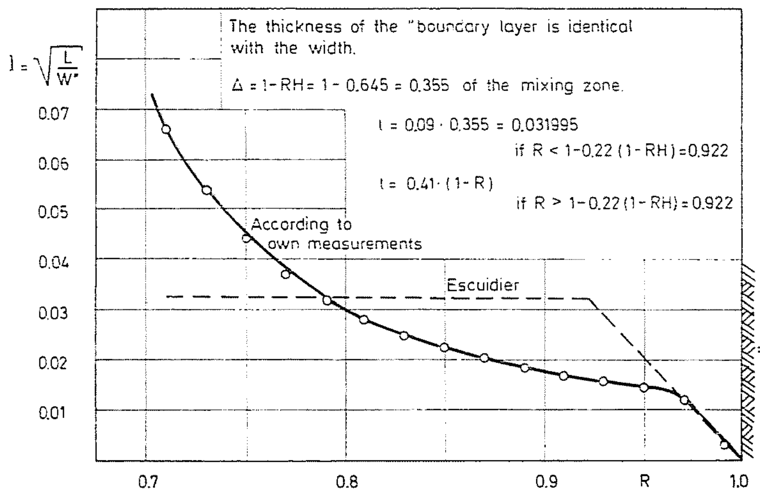

Fig. 3

The figure shows that Escuidier's formula yields acceptable approximations only in the range $0.7 < R < 1.0$. The rapid increase of the viscosity in the range $R < 0.7$ is not covered by Escuidier's treatment. Consequently the broken line approximation yields more exact results in calculating the peripheral velocity distribution.

The evaluation of the turbulent viscosity measurements were carried out by the following method. The figure shows that the viscosity distribution has a nearly constant range, which, however, if extended over the whole field, does not yield the total velocity distribution especially not at the peripheries.

If the viscosity is considered to be constant in a narrow, small section its values may be evaluated from Eq. (16) with the conditions $L =$ constant and $L' = 0$. One point from the viscosity distribution with $L =$ constant and $L' = 0$ evaluated this way and described in the previous paper [1] may be used as an initial value to solve Eq. (16).

Taking into consideration the viscosity values of the above method, satisfactory results are obtained in calculating the peripheral velocity not only in a narrow, but also for the whole range.

The actual curved distribution presented in Fig. 1 has been approximated be a broken line in the interest of an easier manageability. The approximation deviated only insignificantly in the calculation of the peripheral velocity.

Summary

The author describes equations of the mixing viscous component for the turbulent flow within a block pulley (shell) shaped vortex chamber. By rewriting the equations the paper presents the possibility to calculate the turbulent viscosity from the measurement of the velocity distribution. Evaluations of an actual example of the velocity distribution, compared with the results obtained by the distribution values of Escuidier's formula are presented, and a broken line method to approximate the turbulent viscosity is proposed.

Notations

w	peripheral velocity
u	axial velociy
v	radial velocity
r, Θ, z	cylindrical co-ordinates
W, U, V	dimensionless velocities
ν	turbulent viscosity
$\overline{\overline{D}}$	tensor derivation of the velocity field
$\bar{c}$	generalized velocity vector
$\bar{g}$	force field vector
$\overline{\overline{\Phi}}$	stress tensor as defined by equation (4)
$\overline{\nabla}$	Nabla operator vector
L	dimensionless viscosity
R	dimensionless co-ordinates
$\overline{\overline{F}}$	stress tensor

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