TYRE RADIAL PROPERTIES*

T. A. NOSSEIR, M. EL-GINDY, and F. A. EL-SAIED

Ain Shams University, Faculty of Engineering, Energy and Automotive Dept., Cairo, Egypt.

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1. Introduction

In vehicle modelling the tyre is one of the most important elements which influences the accuracy of the calculated results. In case of vibration analysis of a vehicle manoeuvered on irregular road surface, it requires a more realistic e.g. viscoelastic tyre model. This model involves many variably parameters such as dynamic stiffness and damping coefficient which vary with the simulated frequency, vertical load. and inflation pressure of the tyre model. The viscoelastic model is a suitable one for the most important radial properties of a pneumatic type.

In the present work attempts are presented to simulate the tyre by a viscoelastic model and to investigate the radial properties under different operating conditions.

The dynamic stiffness and the damping coefficient for two types of tyres (cross-ply and radial-ply) have been calculated using mobility test data and also the transmissibility of both types have been investigated under different operating conditions.

2. Tyre model

The most widely used tyre model to obtain the response of the tyre consists of a mass and a linear elastic spring in parallel with. or without a viscous damping element [1—3]. A more realistic tyre model has been developed by Tielking [4]. He considered the tyre as a thin elastic cylindrical shell of finite width supported on radial springs. This model is not widely used in vehicle modelling.

Gehman [5] suggested a viscoelastic tyre model to simulate tyre performance. This model consists of static stiffness and frequency-dependent

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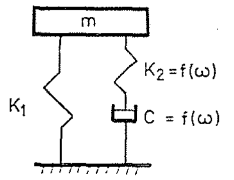


Fig. 1

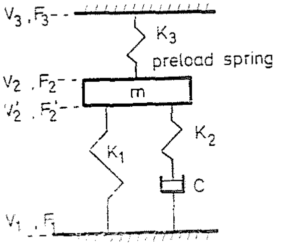


Fig. 2

spring stiffness and damping coefficient. In spite of that this model is more realistic than the others, it is not used in tyre modelling due to many difficulties associated with the determination of the dynamic behaviour of its elements.

Fig. 1 shows the suggested viscoelastic model. Fig. 2 shows the total model representing the tyre test arrangement, in which the tyre preload is simulated by a linear spring.

3. Theoretical analysis of the vertical response

The analysis is based on small displacement excitation applied to the tyre in order to neglect the non-linearity of the tyre characteristics [4]. In this case the tyre can be considered as a linear system and the mobility technique can be applied.

3.1 *Mobility technique*

The dynamic behaviour of idealized linear mechanical systems can be conveniently described in terms of mechanical impedance or mobility functions. The mobility M is defiend as [6]:

$$M = V/F \tag{1}$$

where

V — velocity

F — exciting force.

For a linear system, the input and output mobility functions can be defined in terms of four parameters such as:

The input force F_{in}

$$F_{in} = a_{11} F_{out} + a_{12} V_{out} . \tag{2}$$

The input velocity V_{in}

$$V_{in} = a_{21} F_{out} + a_{22} V_{out} \tag{3}$$

where

F_{out}, V_{out} — the output force and velocity, respectively;

$a_{11}, a_{12}, a_{21}, a_{11}$ — the four pole parameters.

The four pole paremeters for the basic elements (mass m; spring K; damper C) are

$$\text{For mass } m = \begin{bmatrix} 1 & J\omega m \\ 0 & 1 \end{bmatrix}$$

$$\text{For stiffness } K = \begin{bmatrix} 1 & 0 \\ J\omega/K & 1 \end{bmatrix} \tag{4}$$

$$\text{For viscous damper } C = \begin{bmatrix} 1 & 0 \\ 1/C & 1 \end{bmatrix}$$

where

ω — frequency.

The series elements, stiffness and damping are shown in the model (Fig. 2), their values depend on the exciting frequency. $K_2 = f(\omega)$ and $C = f(\omega)$. K_1 and K_3 are assumed to be constant.

Applying Eqs (2), (3) and (4) on the model:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{J\omega(J\omega C + K_2)}{K_1 K_2 + J\omega C(K_1 + K_2)} & 1 \end{bmatrix} \begin{bmatrix} F_2' \\ V_2' \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} F_2' \\ V_2' \end{bmatrix} = \begin{bmatrix} 1 & J\omega m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} F_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ J\omega/K_3 & 1 \end{bmatrix} \begin{bmatrix} F_3 \\ V_3 \end{bmatrix} \tag{7}$$

For the overall system, $V_3 = 0$ because there is no motion at the upper fundation. Solving Eqs (5), (6) and (7) yields the mobility $M = F_1/V_1$:

$$M = \frac{\omega^2 C K_2^2 + J\omega\,[K_1 K_2^2 + \omega^2 C^2 (K_1 + K_2)]}{K_1 K_2^2 + \omega^2 C^2 (K_1 + K_2)^2} + \frac{J\omega}{K_3 - \omega^2 m}. \tag{8}$$

Separating the real and imaginary parts of Eq. (8):

$$\text{Real part } R = \frac{\omega^2 C}{K_1^2 + \left(\frac{C\omega}{K_2}\right)^2 (K_1 + K_2)^2} \tag{9}$$

$$\text{Imaginary part } I = \omega \left[\frac{K_1 + \left(\frac{\omega C}{K_2}\right)^2 (K_2 + K_1)}{K_1^2 + \left(\frac{\omega C}{K_2}\right)^2 (K_2 + K_1)^2} + \frac{1}{K_3 - \omega^2 m} \right]. \tag{10}$$

In Eqs (9) and (10) there are two unknowns, K_2 and C. K_1, K_3 and m can be determined from a static test which will be described later.

Eqs (9) and (10) are non-linear simultaneous equations, the only physically valid solutions for the unknowns are

$$C = R\left[\left(\frac{K_1}{\omega}\right)^2 + \left(\frac{1-d}{R}\right)^2\right] \tag{11}$$

$$K_2 = K_1 \left[\frac{\left(\frac{K_1}{\omega}\right) + \left(\frac{1-d}{R}\right)^2}{\frac{d(1-d)}{R} - \left(\frac{K_1}{\omega}\right)^2} \right] \tag{12}$$

where

$$d = K_1 \left[\frac{I}{\omega} - \frac{1}{K_3 - m\,\omega^2} \right]. \tag{13}$$

3.2 *Transmissibility of a tyre (TR)*

The motion transmissibility across a tyre is a very important term in evaluating the vibrational behaviour of a tyre and its effect on the vehicle body vibrations. The transmissibility in the present model being defined as the modulus of V_2/V_1, from Eqs (5), (6) and (7) the transmisibility TR becomes

$$TR = |V_2/V_1| = \sqrt{\frac{K_1^2 + (\omega C/K_2)(K_1 + K_2)^2}{(K_1 + K_3 - m\,\omega^2)^2 + (\omega C/K_2)^2 (K_1 + K_2 + K_3 - m\omega^2)^2}}. \tag{14}$$

3.3 *Calculation of the tyre properties*

In order to predict the mobility M and the transmissibility TR, the values of C and K_2 must be determined vs. frequency by substituting experimental values of R, I and α into Eqs (11) and (12).

A FORTRAN computer program has been prepared to evaluate the values of R, I, C and K_2 under different operating conditions. The necessary input data to the computer program have been obtained from the experiments. The required data are the following measured parameters:

1. Static stiffness of the tyre (K_1)
2. Input velocity to the system (V_1)
3. Input force to the system (F_1).

4. Experimental work

4.1 *Static tests*

The static test has been performed to measure the static stiffness of the tyres used in the study. The test rig is shown in Fig. 3. In these tests the loading force and the tyre deflection have been measured for both cross-ply and radial-ply tyres at different inflation pressures.

The loading force has been measured by force transducer and the deflection by variable resistance. The force and deflection have been recorded simultaneously by a pen recorder.

4.2 *Dynamic tests*

Fig. 4 shows a general layout of the used test rig. It consists essentially of three parts:

1. Cam exciter. The cam is a disc with an eccentricity of 1 mm. The cam shaft rotates at different angular velocities to be controlled by means of a series of gear boxes.
2. Tyre clamping.
3. Preloading system, which consists of a single semi-elliptical leaf spring of 12.44 KN/m stiffness.

In the dynamic tests the following parameters have been measured and recorded simultaneously:

1. Input force and velocity F_1 and V_1, resp.
2. Output velocity V_2
3. Excitation frequency ω.

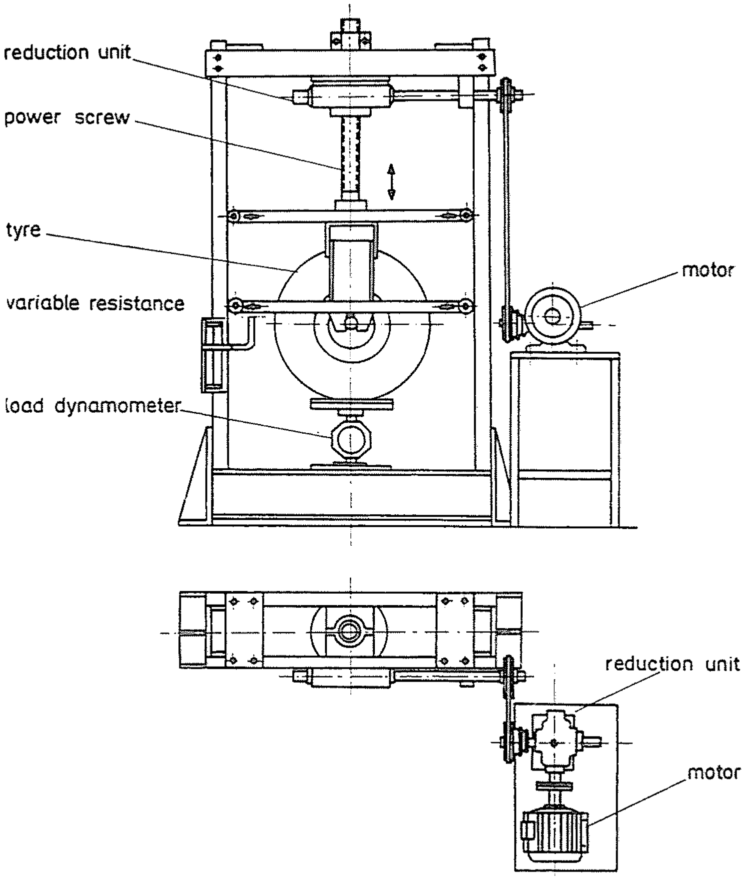


Fig. 3

5. Results and descussion

5.1 *Static stiffness K_1*

Fig. 5 shows a lattice plot of static load-deflection for the cross and radial tyres. This figure shows the effect of the radial load and inflation pressure on the tyre stiffness. It is evident that the static stifffness of the cross-ply is greater than that of the radial-ply, due to the greater shear stiffness of the former.

It is also evident that the cross-ply tyre exhibits marked non-linear characteristics whereas the radial-ply tyre is linear over most of its normal loading range.

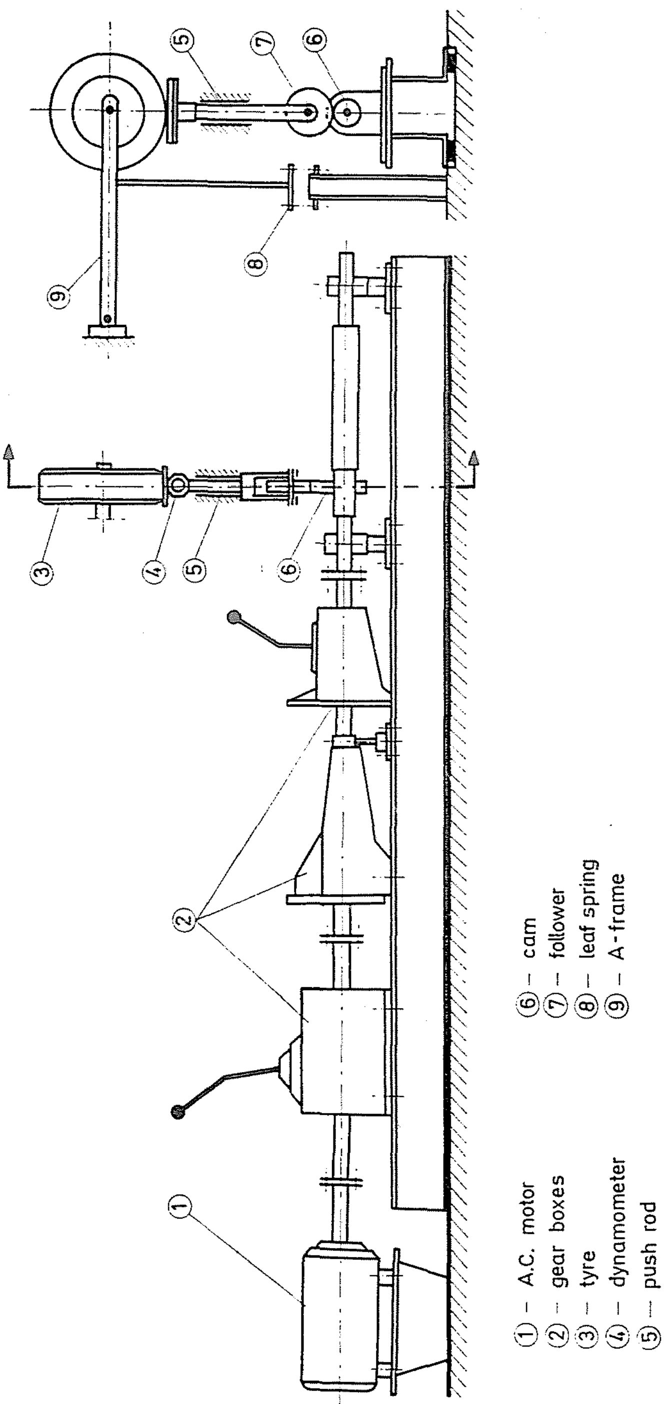
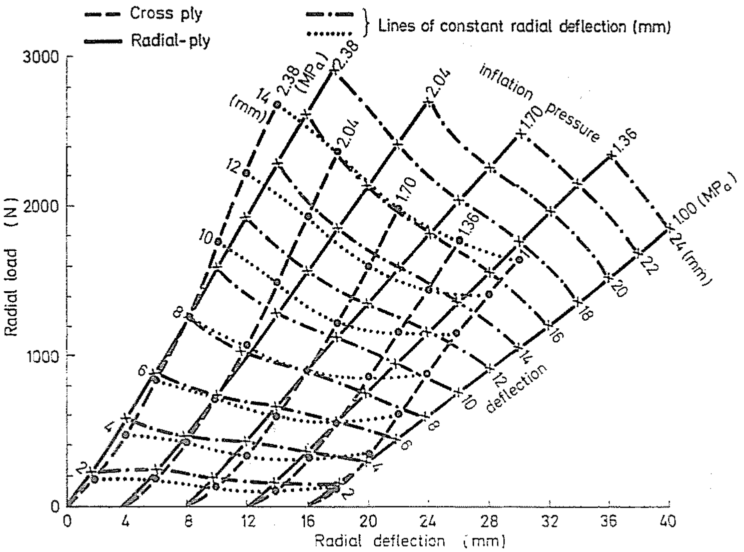
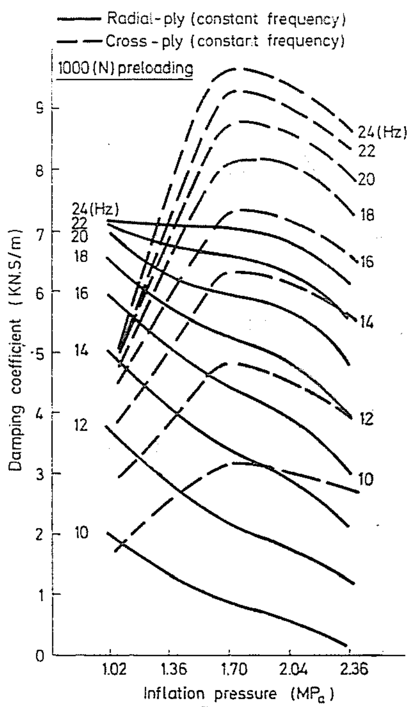

1 – A.C. motor
2 – gear boxes
3 – tyre
4 – dynamometer
5 – push rod
6 – cam
7 – follower
8 – leaf spring
9 – A-frame

Fig. 4



Cross ply
Radial-ply
Lines of constant radial deflection (mm)
Inflation pressure
2.38 (MPa)
2.04
1.70
1.36
1.00 (MPa)
14 (mm)
24 (mm)
deflection
Radial load (N)
Radial deflection (mm)

Fig. 5



Radial-ply (constant frequency)
Cross-ply (constant frequency)
1000 (N) preloading
24 (Hz)
Damping coefficient (KN.S/m)
Inflation pressure (MPa)

Fig. 6

5.2 *Damping Coefficient C*

Figs 6 and 7 show the variation of damping coefficient with the inflation pressure at constant frequencies varying from 10 to 24 Hz. and constant vertical loads (1000 and 2000 N). It can be noted that for cross-ply tyre the damping coefficient has a maximum value at a certain inflation pressure (1.7 MPa), above and below this pressure the damping coefficient decreases. For radial-ply the damping coefficient increases with the inflation pressure due to the high energy loss across the tyre at low inflation pressure (large hysteresis loops at low pressure).

Figs 7 and 8 point out that, radial-ply tyres under low preload at constant frequency have a higher damping coefficient than under high preloads at high inflation pressure. At low inflation pressure the difference between damping coefficients under low and high preloads becomes negligible. Cross-ply tyres have the same behaviour as radial-ply ones.

Figs 8 and 9 show the variation of damping coefficient with the exciting frequency at constant inflation pressure. For radial-ply tyres, the damping coefficient increases with the frequency, this can be explained referring to Pain [7]. He stated that "the energy lost per cycle in the system during forced oscillation at first increases as the frequency increases. Then it reaches a peak value at a certain frequency, beyond which the energy loss gradually diminishes with increasing frequency". This is so because at high frequency the share of

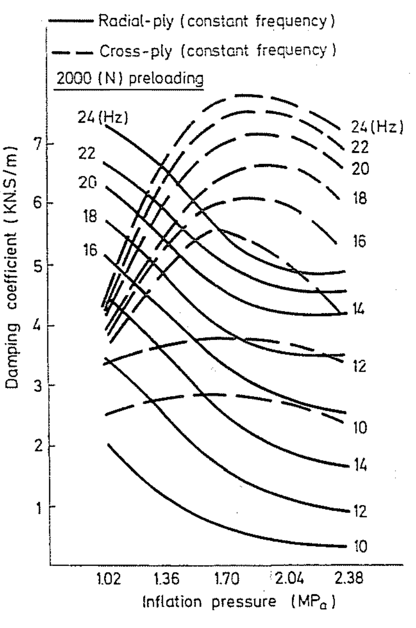


Fig. 7

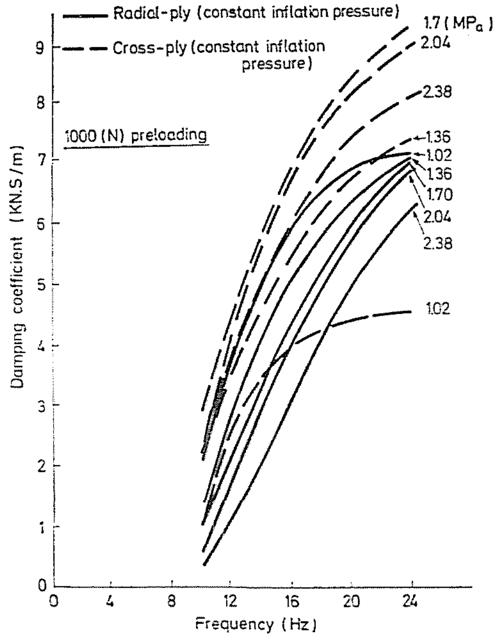

Radial-ply (constant inflation pressure)
Cross-ply (constant inflation pressure)
1000 (N) preloading
Damping coefficient (KN.S/m)
Frequency (Hz)
1.7 (MPa)
2.04
2.38
1.36
1.02
1.36
1.70
2.04
2.38
1.02

Fig. 8

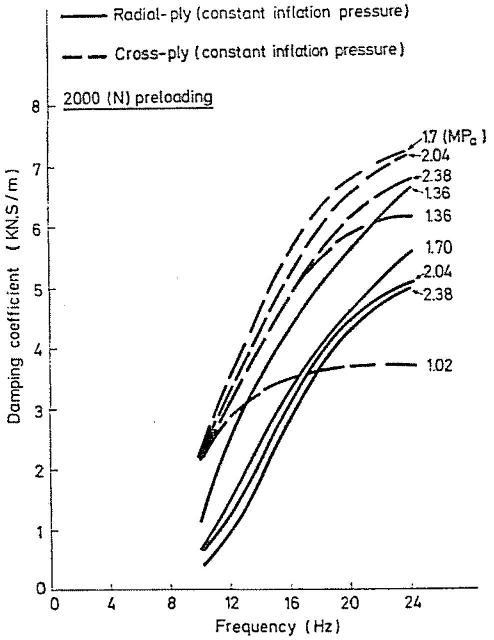

Radial-ply (constant inflation pressure)
Cross-ply (constant inflation pressure)
2000 (N) preloading
Damping coefficient (KN.S/m)
Frequency (Hz)
1.7 (MPa)
2.04
2.38
1.36
1.36
1.70
2.04
2.38
1.02

Fig. 9

displacement due to the viscous damper is small compared to the share of its companion spring.

Thus, it can be concluded that cross-ply tyres have a higher energy loss than have radial-ply ones. This is due to the higher damping coefficient of the cross-ply than of the radial-ply tyre.

5.3 *Dynamic stiffness*

Figs 10, 11 show the variation of dynamic stiffness with the inflation pressure at constant frequencies. For radial-ply the dynamic stiffness increases with the inflation pressure for frequencies over 19 Hz, but does not vary with the inflation pressure in the middle range of frequencies (between 16 and 19 Hz), and in the lower range (between 10 and 16 Hz) the dynamic stiffness increases as the inflation pressure decreases.

Figs 12, 13 show the variation of the dynamic stiffness with the exciting frequency at constant inflation pressure. For radial-ply, the dynamic stiffness increases as the frequency increases. This is due to the increase of the rubber elastic modulus from a minimum value at low frequency to a maximum at higher frequency.

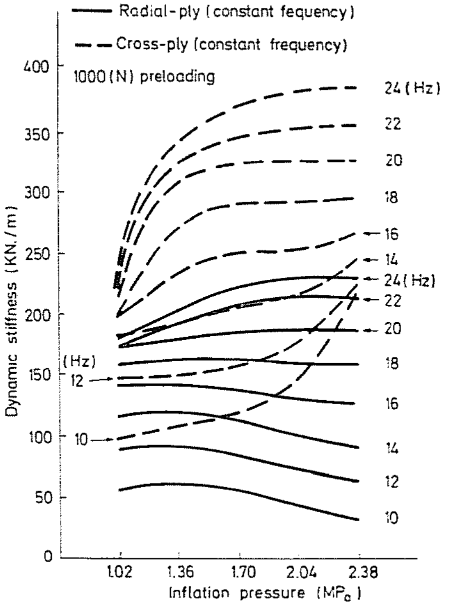


Fig. 10

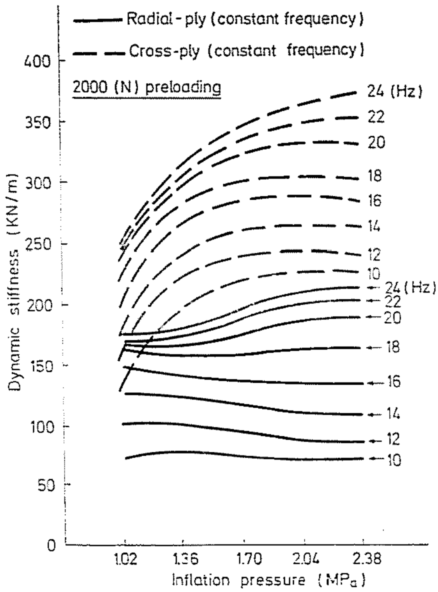


Fig. 11

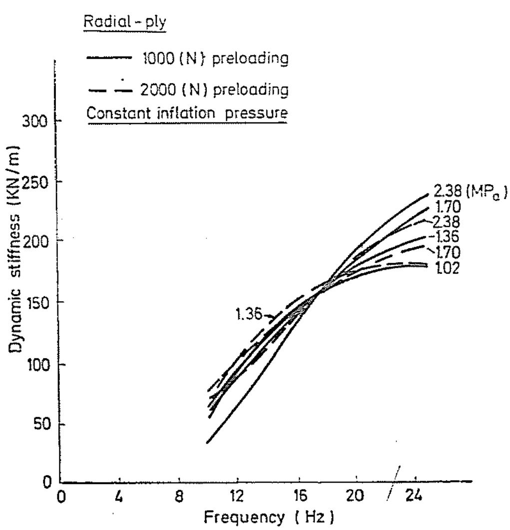


Fig. 12

5.4 *Transmissibility*

Fig 14 shows the transmissibility TR for both tyre types (cross-ply and radial-ply) at two different inflation pressures, 1.02 and 1.7 MPa. It can be noticed that in the range of low frequencies (below 18 Hz) the transmissibility

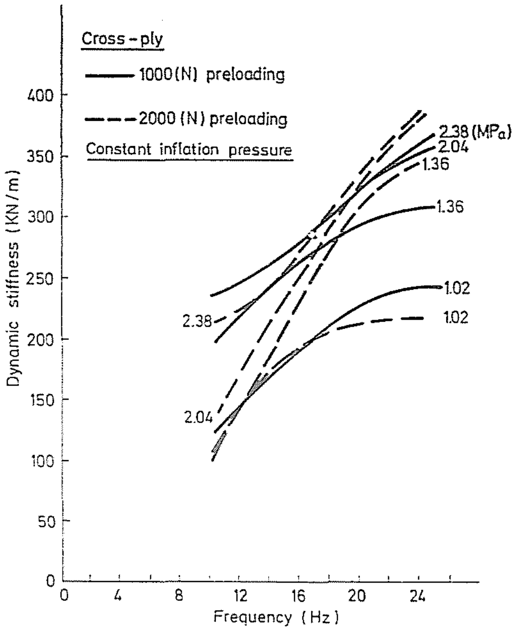


Fig. 13

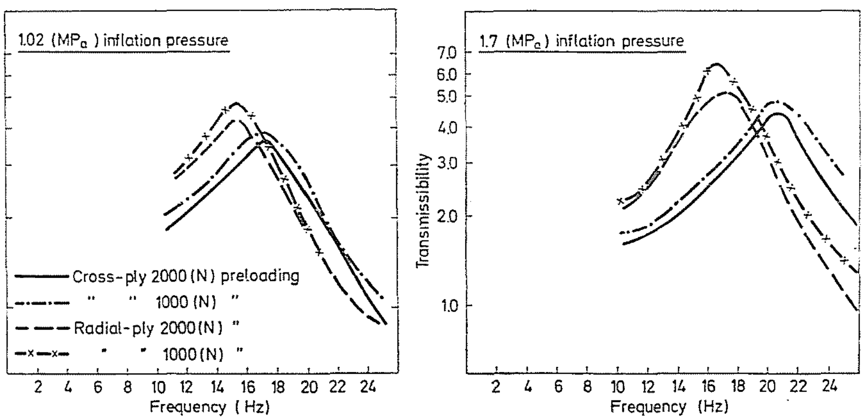


Fig. 14

level of the cross-ply is better (i.e. lower) than that of the radial-ply tyre. But in the range of high frequencies (over 18 Hz) the radial-ply tyre has a better transmissibility than the cross-ply one. This is due to the lower dynamic and static stiffness values of the radial-ply than the cross-ply tyre.

The figure also shows that the transmissibility of the radial-ply tyre has a peak value at resonance frequency lower than that of the cross-ply tyre.

The measured and calculated results show that

1. The cross-ply tyre exhibits marked non-linear characteristics whereas the radial-ply behaves linearly over most of its normal loading range.
2. Both the cross-ply and the radial-ply tyre exhibit the same behaviour of dynamic stiffness versus inflation pressure and frequency. But the cross-ply tyre has a higher dynamic stiffness level than has the radial-ply one.
3. Behaviour of the damping coefficients of cross-ply and radial-ply tyre versus inflation pressure and frequency are different. Here it can be concluded that cross-ply tyre show higher energy losses than radial-ply tyres do.
4. The transmissibility of the radial-ply tyre is better (i.e. lower) than that of the cross-ply one at frequencies higher than 18 Hz. And the resonance frequency of the radial-ply is lower than that of cross-ply one.

Summary

The radial properties of two types of tyres (cross-ply and radial-ply) have been investigated under variable operating conditions (inflation pressure, vertical load and frequency). The static and dynamic properties (dynamic stiffness and damping coefficient) have been calculated using a viscoelastic tyre model and data measured in mobility tests. Static and dynamic test rigs have been designed and built for the study purposes.

The investigations showed the viscoelastic tyre model to suit simulation of the dynamic behaviour of a pneumatic tyre.

Notations

TR — transmissibility of a tyre;
M — mobility (m/NS);
m — mass of tyre (kg);
K_1 — static stiffness of a tyre (N/m);
K_2 — dynamic stiffness of a tyre (N/m);
K_3 — preload spring stiffness (N/m);
C — damping coefficient of type (NS/m);
ω — ecxitation frequency (rad/s);
F — force (N);
V — velocity (m/s);
a — pole parameter;
J — $\sqrt{-1}$.

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Ass. Prof. Dr. T. A. Nosseir
Dr. M. El-Gindy
Eng. F. A. El-Saied

Ain Shams University, Faculty of Engineering, Energy and Auto. Dept., Abbassia, Cairo, EGYPT.